A Model of Aggregate Demand, Labor Utilization, and Unemployment

Pascal Michaillat and Emmanuel Saez

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Abstract

We present a model in which aggregate demand affects unemployment through labor utilization. Unemployment and underutilization of employed workers arise in equilibrium in the presence of matching frictions on the labor and product markets. Aggregate demand determines the probability that firms find customers and hence labor utilization, which in turn determines the profitability of employed workers and hence labor demand and unemployment. Unemployment is composed of three components: frictional unemployment, caused by high recruiting costs; Keynesian unemployment, caused by a low labor utilization; and classical unemployment, caused by a high real wage. We use the model and new empirical series for labor utilization and recruiting costs to re-examine the origin of employment fluctuations. We reach three conclusions: (1) price and real wage are not fully flexible because we observe large fluctuations in labor utilization and labor market tightness; (2) employment fluctuations are mostly driven by labor demand shocks and not labor supply shocks because we observe a positive correlation between employment and recruiting costs; and (3) labor demand shocks mostly reflect aggregate demand shocks and not technology shocks because we observe a positive correlation between output and labor utilization.

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1 Introduction

Unemployment remains a major issue in modern economies. The unemployment rate in the US remained above 7% for a five-year period between 2009 and 2013. To devise policies that attenuate unemployment fluctuations over the business cycle, it is necessary to understand the origins of these fluctuations.

While a vast literature in macroeconomics explores the origins of unemployment fluctuations, the Great Recession underlines a number of issues that remain to be clarified. Major sectoral shocks occurred (for instance to the construction and financial sectors), which revived the hypothesis that mismatch may be a major reason behind the increase in unemployment.\(^1\) The initial increase in unemployment triggered important extensions of unemployment insurance benefits, which led to concerns that unemployed workers may not search sufficiently, making it difficult for firms to hire workers and contributing to unemployment.\(^2\) Finally, many empirical macroeconomists noticed a sharp increase in TFP during the Great Recession, at the same time as unemployment kept on increasing.\(^3\) This observation poses a problem for the basic matching model in which fluctuations in technology lead to fluctuations in unemployment.

In this paper, we develop a new theoretical framework and provide a set of new empirical series that we use to continue exploring the sources of unemployment fluctuations. We start from a standard matching model of the labor market. This model, which is widely used by macroeconomists, has many desirable properties: it is simple, admits a tractable equilibrium representation, delivers useful comparative statics, and is amenable to policy analysis. It is true that the standard version of the model with Nash bargaining has unrealistically small unemployment fluctuations, but introducing a small amount of wage rigidity immediately generates realistic fluctuations in unemployment [Hall, 2005; Shimer, 2005]. Even though it is an equilibrium model, the model is also useful to think about rationing and about the nonlinearities created by slack and tight conditions [Michaillat, 2012, 2014]. Finally, the model accurately represents the mechanics of the labor market: the time required by workers to find jobs, the resources required by firms to fill vacancies and recruit

\(^1\)For a recent discussion of the role of mismatch shocks, see Andolfatto and Liborio [2012], Barlevy [2011], Diamond [2013], Lazear and Spletzer [2012], Ravn and Sterk [2013], or Sahin et al., [2011]. For an earlier discussion, see Lilien [1982], Abraham and Katz [1986], and Blanchard and Diamond [1989].

\(^2\)See for instance the discussions in Valletta and Kuang [2010], Farber and Valletta [2013], Elsby, Hobijn and Sahin [2010], or Rothstein [2011].

\(^3\)See for instance Fernald [2012].
workers, and the existence of long-term employment relationships.

However, to be able to study the origins of unemployment fluctuations, we need to improve the structure of the labor demand in the standard matching model. The labor demand does not account for the fact that it may be difficult for firms to sell their production. In the standard model, the product market is competitive so it is easy for firms to sell their production. The asymmetry between the product market, where it is easy for firms to sell their production, and the labor market, where it is difficult for workers to sell labor, seems artificial. The asymmetry also has important macroeconomic implications. In the standard model, the only factor that can shift the labor demand is a movement in the ratio between real wage and technology. When the real wage-technology ratio is high, unemployment should be high; when the real wage-technology ratio is low, unemployment should be low. This implication is counterfactual. We show that in the data, the opposite pattern emerges: when the wage is high, unemployment is low and when the wage is low unemployment is high. In addition, the presence of a competitive product market eliminates any role for aggregate demand as a source of unemployment fluctuations.

In this paper, we improve the labor demand structure by relaxing the assumption of perfect competition on the product market. Instead, we assume that there are matching frictions on the product market that make it difficult for firms to find customers. Our theory of labor demand establishes a link between aggregate demand and unemployment through a simple mechanism. With low aggregate demand, it is harder for firms to find customers so that labor utilization is lower. Hence labor profitability is lower. This in turns reduces labor demand and hence increases unemployment. To summarize, our model includes not only frictional unemployment due to labor market frictions and classical unemployment due to the level of real wages, but also Keynesian unemployment due to low labor utilization caused by low aggregate demand. So far as we know, no other model allows for such a decomposition of unemployment.

Of course, we could have assumed other product market structures to depart from perfect competition and allow aggregate demand to influence unemployment. We opted for the matching framework for three reasons. First, this assumption offers a certain symmetry to the model: product and labor markets are treated using the same market structure. This symmetry simplifies various aspects of the analysis. Second, this assumption allows us to write a model that retains many of the desirable properties of the standard matching model—simplicity, tractable comparative statics,
intuitive equilibrium representation, rationing on the product and labor markets, and sharp non-linearities in tight and slack economies. Third, as documented in Section 2, the model seems to capture a number of features of the product market: it is difficult for firms to sell all their production, as indicated by incomplete capacity utilization; it is difficult for firms to buy goods from other producers, as indicated by the number of employees devoted to procurement; and most firms and customers form long-term customer relationships, presumably to alleviate matching frictions.

We begin in Section 3 by presenting a basic model in which self-employed workers sell and purchase labor services on a market with matching frictions. Labor and produced good are a single good so that labor and product markets are a single market. We use this simple model to introduce our concepts of aggregate demand and labor utilization, and to describe the equilibrium concept. We also show that a matching equilibrium can be defined by analogy to the standard Walrasian equilibrium.

To introduce aggregate demand effects, we follow Hart [1982] and introduce a competitively traded, nonproduced good in addition to the produced good and labor, which are traded on markets with matching frictions. The nonproduced good is necessary to obtain an interesting concept of aggregate demand, because without it, consumers would mechanically spend all their income on the produced good. In our model consumers allocate their income between consumptions of produced and nonproduced good, and aggregate demand is the desired consumption of produced good. This nonproduced good could represent land, real wealth available for future consumption, or real money balances—as in Barro and Grossman [1971] or Blanchard and Kiyotaki [1987].

Because of the matching frictions, workers cannot sell all their services and are idle part of the time. The rate of idleness, that can be interpreted as one minus capacity utilization, is a negative function of market tightness. We obtain a number of results that foreshadow the results in the model with product and labor markets. First, under Nash bargaining or efficient pricing, aggregate demand shocks are fully absorbed by price adjustments. In contrast, supply shocks affect output but still have no effect on tightness. Second, under rigid prices, aggregate demand shocks matter. We show that tightness and output are positively correlated under aggregate demand shocks but negatively correlated under aggregate supply shocks.

In Section 4, we introduce firms that mediate between workers and consumers by hiring workers on a labor market with matching frictions, employing these workers to produce goods, and
selling the production on a product market with matching frictions. Following Michaillat [2012], we assume that firms are large, face a production function with diminishing marginal returns to labor, and maximize profits taking labor market tightness and real wage as given. In this richer model, slack is present at two levels: there are people who are not utilized on the job (less than full labor utilization), and there are people who do not have a job (positive unemployment). Unemployment can be decomposed into three components: frictional unemployment due to the presence of matching frictions, classical unemployment due to the level of real wages, and Keynesian unemployment due to the level of aggregate demand that affects labor utilization of employed workers and hence the marginal productivity of labor. We derive comparative statics under labor supply shocks (labor force participation or mismatch) and labor demand shocks (technology or aggregate demand). Section 5 then extends this model to a dynamic environment. The dynamic model provides a better mapping with the data and will be useful for the empirical analysis. The dynamic model also clarifies how long-term employment relationships and long-term customer relationships on the labor and product markets are formed and destroyed.

In Section 6, we use our model and new empirical series for labor utilization and recruiting costs to re-examine the origin of employment fluctuations. While labor market tightness series are well established, there are no corresponding measures of product market tightness or labor utilization. We construct a proxy for labor utilization, based on the series of capacity utilization from the Survey of Plant Capacity administered by the Census Bureau. All the effect of labor supply shocks on employment should appear through higher recruiting cost. To assess the importance of labor supply shocks, we therefore construct a proxy for recruiting cost, based on quarterly employment statistics for the recruiting industry from the Current Employment Statistics survey administered by the Bureau of Labor Statistics. We also use real wage data from the BLS and utilization-adjusted TFP data constructed by Fernald [2012] to have all the information required to assess the labor cost faced by firms. We reach three conclusions: (1) price and real wage are not fully flexible because we observe large fluctuations in labor utilization and labor market tightness; (2) employment fluctuations are mostly driven by labor demand shocks and not labor supply shocks because we observe a positive correlation between employment and recruiting costs; and (3) labor demand shocks mostly reflect aggregate demand shocks and not technology shocks because we observe a positive correlation between output and labor utilization.
2 Some Empirical Evidence On Matching Frictions

The matching framework incorporates the following three stylized facts: (i) selling is uncertain for sellers not engaged in a relationship; (ii) there are costs of buying that are sunk at the time of trading; and (iii) buyers and sellers engage in long-term relationships to alleviate frictions. Okun [1981] was convinced that matching frictions and bilateral trade were prevalent both on the product market and on the labor market. However, even if it is now accepted that the matching framework provides a realistic description of the labor market, most researchers are not used to thinking about the product market with this framework. In this section, we present evidence from a broad range of sources supporting the existence of matching frictions and bilateral trade on the product market. We contrast the evidence for the product market with similar evidence on the labor market and argue that the presence of matching frictions seem at least as pronounced on the product market as on the labor market. Part of the evidence is depicted in Figure 1. We mostly focus on the US.

Before we start reviewing the evidence on matching frictions, we display the share of employment in the production of goods and services in Figure 1(a). In 2014, 86% of current US output is in the service sector with only 14% for the manufacturing sector. These numbers imply that most production in the US is in labor services. The border between labor and production is blurry when production is labor services. Hence, we should not be surprised that the same market structure applies to both markets.

2.1 Difficulties to Sell

Workers obviously face difficulties selling their labor. Some unemployment prevails at all time. Figure 1(b) displays the unemployment rate in the US for the 1974:Q1–2013:Q2 period. The average unemployment rate is 6.5%.

Firms also face difficulties selling their labor. Some unused capacity prevails at all time. Figure 1(b) displays the rate of unused capacity (one minus the rate of capacity utilization) in the US.

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4 Of course, not all exchanges are conducted through bilateral trade, especially in the product market. As pointed out by Okun [1981], while most goods are exchanged through bilateral trade, some goods are indeed exchanged on auction markets: financial assets, agricultural commodities, mining products, or art work.

5 The unemployment rate is the seasonally-adjusted monthly series constructed by the Bureau of Labor Statistics (BLS) from the Current Population Survey (CPS).
Figure 1: Empirical evidence of matching frictions on labor and product markets

For the manufacturing sector, unused capacity is measured for the 1974:Q1–2013:Q2 period. The average rate of unused capacity in manufacturing is 26.4%. For the service sector, unused capacity is measured for the 1999:Q4–2013:Q2 period. The average rate of unused capacity in manufacturing is 14.8%. Since capacity utilization is computed by comparing the actual level of production to a level of production that would maximize profits, such a large share of unused capacity suggests that firms face difficulties in selling their production.

There is additional evidence that firms face difficulty in selling their production in the US. Foster, Haltiwanger and Syverson [2012] use microdata from the Census of Manufactures, a dataset constructed by the Census Bureau. The dataset contains information on manufacturing plants’ output quantities and prices. They analyze data for 1977–1997, a total of 17,000 plant-year observations for producers of one of ten products. Their main finding is the importance of a customer base for firm expansion. While young plants’ technical efficiency levels are similar to established plants’ levels, new establishments grow only slowly because they face difficulties more selling their production than existing firms with existing customer bases. Their conclusion is that despite similar or lower prices, new factories grow slowly because they need time to attract new customers.

### 2.2 Costs to Buy

Buying goods or labor is labor intensive; therefore, the main cost of buying goods and recruiting workers for firms is a labor cost. Using data from the Occupational Employment Survey (OES) database constructed by the BLS, we show on Figure 1(c) the number of workers devoted to recruiting workers and purchasing goods in US firms.

We measure the number of workers whose occupation is in human resources with a focus on recruiting, screening, and interviewing. On average 543,200 workers were employed in such occupation between 1997 and 2012. This number includes human resource managers, human resource specialists, and human resource assistants. These people work on employment, recruitment, and
placement, but do not work on compensation, benefits, job analysis, training or development. The classification of occupations evolves from year to year so it is impossible to be completely consistent. This explains why the number of workers in recruiting occupations fluctuates so much from year to year. The goal of this section is only to provide an illustrative lower bound on the amount of resources devoted to recruiting by firms.

Similarly, we measure the number of workers whose occupation is in buying, purchasing, and procurement. On average, 560,600 workers were employed in such occupation between 1997 and 2012. This number includes purchasing managers, buyers and purchasing agents, and procurement clerks. The classification of occupation evolves from year to year but is more stable than for the recruiting occupations. This explains why the number of workers in purchasing occupations is more stable than that in recruiting occupations.

2.3 Long-Term Relationships

Figure 1(d) displays the share of workers in long-term employment relationships in eleven countries: Austria (AT), Belgium (BE), Germany (DE), Spain (ES), France (FR), Italy (IT), Luxembourg (LU), Portugal (PT), Sweden (SE), the UK, and the US. This share are obtained from the OECD database for the same countries for 2005. Across the eleven countries, the average share of workers in long-term employment is 87.1%.

The fact that employment contracts are long-term contracts appears clearly in US data. For instance, the average monthly job-separation rate is around 3.5%. This separation rate would imply that the average employment contract last for $1/0.035 = 29$ months. But employment is much more stable than this duration implies because most jobs separation come form a small fraction of the workforce that stays only a short time on jobs. The study of Hall [1982] shows that US labor market provides stable, near-lifetime employment to an important fraction of the labor force across the time period, the occupation codes are 11-3120/11-3049/11-3040/13005 for managers, 13-1071/21508–11 for specialist, and 43-3060/43-3061/55326 for assistants. The occupation codes are 11-3060/11-3061/13008 for managers, 13-1020/13-1021–23/21302–08 for agents, and 43-4160/43-4161/55314 for clerks. The OECD reports the share of workers in permanent employment and the share of workers in temporary employment. We find the average separation rate using the seasonally-adjusted monthly series for total separations in all non-farm industries constructed by the Bureau of Labor Statistics (BLS) from the Job Openings and Labor Turnover Survey (JOLTS) for the December 2000–February 2014 period.
force. Hall finds that the typical worker today is holding a job which has lasted or will last about eight years. 60% of all workers hold jobs which will last 5 years or more, and over 25% are holding jobs which will last 20 years or more. Jobs seem to be even more stable nowadays due to a decline in job-destruction rates after the early 1980s [Davis et al., 2010].

We do not have as much detail on the nature of long-term relationships on the product market. But a broad range of evidence points to the importance of these long term relationships on the product market. Following Blinder et al. [1998], a number of researchers have surveyed firms about the reasons for price rigidity and the type of relationships they have with customers. The surveys were usually administered by mail or in person to firm managers in charge of sales. These surveys elicit the fractions of sales usually conducted with repeat customers. Figure 1(d) displays the fraction of sales to long-term customers. Across the eleven countries, the average share of sales long-term customers is 77.5%.

For the US, there is a lot of additional evidence that seller-buyer relationships last a long time. Okun [1981] provides abundant evidence. He argues that professional buyers prefer long-term, continuing, relationships with sellers, even when they procure physically homogenous products. He gives the examples of the US steel and copper markets: US firms and their customers are in long-term relationships, despite the existence of a spot market for imported metal.

Goldberg and Hellerstein [2011] present evidence on long-term relationships in intermediate goods. They find that one third of all transactions are conducted under contract across industries, for both goods and services. The evidence is based on BLS data on firms’ contractual arrangements. This measure of contractual agreement certainly understates the incidence of implicit contracts with customers and is a lower bound on the prevalence of recurrent interactions.

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14 The BLS defines a contract as an agreement with multiple deliveries over more than one month. The measure includes verbal agreements, though it more often references written ones.
2.4 A Historical Perspective

A historical perspective on the US labor market is useful, because it shows how the market structure evolved over time from what was essentially an auction market to a market best described by a random matching framework. The historical work of Jacoby [1984] documents how, in the 20th century, the labor market evolved from impermanent and market-oriented to bureaucratic, rule-bound, and secure internal labor markets. Or an overview of the structure of internal labor markets, see Doeringer and Piore [1971] and Osterman [1984]. For a detailed analysis of the internal labor market in a large US firm, see Baker, Gibbs and Holmstrom [1994].

The product market evolved similarly. In the marketing literature, Arndt [1979] notes a change in market structure and develops the concept of “domesticated markets”, which replace competitive markets. He explains that the “competitive market is eroding”, and that “transactions are occurring in internal markets within the framework of long-term relationships”. This study analyzes changes in market structure and their implications for marketing theory and practice. Arndt describes a process in which many earlier competitive markets are structured as a result of voluntary, long-term, binding commitments among the organizations involved. Examples of interorganizational systems include conglomerates, franchising, vertical and horizontal integration, joint ventures, joint product development, and joint physical distribution plans.

3 Basic Model, Equilibrium, and Comparative Statics

This section presents a basic model in which aggregate demand influences labor utilization. The model is static. All workers are self-employed and sell services on a market with matching frictions. Because of the matching frictions, workers may not be able to sell all their services. Thus, workers may be idle part of the time in equilibrium and labor utilization measures the part of the time when they are actively working.

3.1 Model

A measure of identical self-employed workers sell services on a market with matching frictions. The capacity of each worker is $k$; that is, a worker would like to sell $k$ units of services. Workers
are also consumers of services, but they cannot consume their own services. Each consumer visits \( v \) workers to purchase their services. The number of trades between consumers and workers is given by a matching function with constant elasticity of substitution:

\[
y = \left( k^{-\eta} + v^{-\eta} \right)^{-\frac{1}{\eta}},
\]

where \( \eta > 0 \) determines the elasticity of substitution between inputs in the matching process.\(^{15}\) With \( \eta > 0 \), the number of trades is less than aggregate capacity \( k \), and less than the total number of visits \( v \). In each trade, a consumer buys one unit of service from a worker at price \( p > 0 \).

We define market tightness as the ratio of visits to capacity: \( x \equiv \frac{v}{k} \). With constant returns to scale in matching, market tightness \( x \) determines the probabilities that one unit of service is sold and that one visit leads to a purchase. Workers sell each unit of service with probability

\[
f(x) = \frac{y}{k} = \left(1 + x^{-\eta} \right)^{-\frac{1}{\eta}},
\]

and each visit leads to a purchase with probability

\[
q(x) = \frac{y}{v} = \left(1 + x^\eta \right)^{-\frac{1}{\eta}}.
\]

The matching probabilities, \( f(x) \) and \( q(x) \), are always between 0 and 1.\(^{16}\) We abstract from randomness at worker and consumer levels: a worker sells \( f(x) \cdot k \) units of services for sure, and a consumer purchases \( q(x) \cdot v \) units of services for sure. The function \( f \) is strictly increasing in \( x \) and the function \( q \) is strictly decreasing in \( x \). In other words, when the market is slacker, a larger fraction of workers’ capacity remains unsold and idleness rises while a larger fraction of consumers’ visits results in a purchase.

We model matching costs as follows: Each visit requires \( \rho \in (0,1) \) units of services that are dissipated and do not contribute to actual consumption \( c \). (For instance, one needs to purchase taxi services to get to the hair salon and purchase hairdressing services.) The \( \rho \cdot v \) units of services for

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\(^{15}\)This functional form is borrowed from den Haan, Ramey and Watson [2000].

\(^{16}\)With a standard Cobb-Douglas matching function, \( y = k^\eta \cdot v^{1-\eta} \), the trading probabilities \( f(x) \) and \( q(x) \) may be greater than 1, and the matching function needs to be truncated to ensure probabilities between 0 and 1. This truncation would complicate the analysis, which is why we use the matching function of den Haan, Ramey and Watson [2000].
Matching costs are purchased like the \( c \) units of services for consumption. Hence, the number of visits is related to consumption and market tightness by \( q(x) \cdot v = c + \rho \cdot v \). Therefore, the desired level of consumption determines the number of visits: \( v = c / (q(x) - \rho) \). Because of matching costs, consuming one unit of services requires buying \( 1 + (\rho \cdot v/c) = 1 + \tau(x) \) units of services, where

\[
\tau(x) \equiv \frac{\rho}{q(x) - \rho}.
\]

The function \( \tau \) is positive and strictly increasing as long as \( q(x) > \rho \), which holds in equilibrium.

In what follows, we focus on consumption decisions and relegate the matching process to the background. Consuming \( c \) requires purchasing \( (1 + \tau(x)) \cdot c \) in the course of \( (1 + \tau(x)) \cdot c/q(x) \) visits, which costs a total of \( \rho \cdot (1 + \tau(x)) \cdot c \). In sum, the matching cost, \( \rho \), imposes a wedge \( \tau(x) \) on the price of services. At the limit where \( \rho \) is zero, the wedge disappears.

Output is \( y = f(x) \cdot k \), and hence consumption is \( c = f(x) \cdot k / [1 + \tau(x)] = [f(x) - \rho \cdot x] \cdot k \). Effectively, \( \rho \cdot x \cdot k = \rho \cdot v \) services are dissipated in matching, creating a wedge between output \( y \) and consumption \( c \). Figure 2 depicts the supply side of our economy as a function of tightness \( x \): capacity \( k \), output \( y = f(x) \cdot k \), and consumption \( c = [f(x) - \rho \cdot x] \cdot k \). Output is a concave and increasing function of tightness. Consumption first increases and then decreases with tightness. The gap between consumption and output represents the cost of matching. The gap between output and capacity represents idleness.
We define the aggregate supply as the amount of consumption traded for a given tightness:

**DEFINITION 1.** The aggregate supply is a function of market tightness defined by

\[ c^s(x) = (f(x) - \rho \cdot x) \cdot k \]  

for all \( x \in [0,x^m] \), where \( x^m > 0 \) satisfies \( \rho = q(x^m) \). The function \( c^s \) is strictly increasing on \([0,x^*]\), strictly decreasing on \([x^*,x^m]\), \( c^s(0) = 0 \), and \( c^s(x^m) = 0 \). The constant \( x^* \) maximizes \((f(x) - \rho \cdot x) \cdot k\) so \( f'(x^*) = \rho \). The constant \( x^* \) depends only on \( \rho \) and \( \eta \).

When \( x \) is low, the matching process is congested by the amount of services for sale. Consequently, the additional visits to sellers resulting from an increase in \( x \) lead to a large increase in output and thus a large increase in \( f(x) \). Conversely when \( x \) is high, the matching process is congested by visits, and the additional visits resulting from the increase in \( x \) only lead to small increases in output and \( f(x) \). The value \( x^* \) maximizes \( f(x) - \rho \cdot x \).

Besides the market for services described above, consumers also trade a nonproduced good on a perfectly competitive market. Each consumer has an exogenous endowment \( \mu > 0 \) of nonproduced good. We take the nonproduced good as numéraire. We introduce a nonproduced good in the model to break Say’s law (the result that the supply of services automatically generates its own demand) and hence introduce an aggregate demand in this static framework. Concretely, this nonproduced good could represent land, gold, or money.\(^\dagger\)

Consumers have a constant-elasticity-of-substitution (CES) utility function given by

\[
\left[ \frac{\chi}{1 + \chi} \cdot c^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{1 + \chi} \cdot m^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}},
\]

where \( c \) is consumption of services, \( m \) is consumption of nonproduced good, \( \chi \in (0, +\infty) \) is a parameter measuring the taste for services relative to nonproduced good, and \( \varepsilon \) is a parameter measuring the elasticity of substitution between services and nonproduced good. To guarantee unicity of the equilibrium, we impose \( \varepsilon > 1 \).

\(^\dagger\)This modeling technique was common in the Keynesian literature of the 1970s and 1980s, when the nonproduced good was often money. See for instance Barro and Grossman [1971], Hart [1982], or Blanchard and Kiyotaki [1987]. Michaillat and Saez [2014] extend the current model to a dynamic macroeconomic model with money and savings in which the nonproduced good is replaced by real money or real wealth.
The consumer’s income comes from the sales of $\mu$ units of nonproduced good at price $1$ and $f(x) \cdot k$ units of services at price $p$. The consumer uses the income to purchase $m$ units of nonproduced good at price $1$ and $c$ units of services at price $(1 + \tau(x)) \cdot p$. Hence, the consumer’s budget constraint is

$$m + (1 + \tau(x)) \cdot p \cdot c = \mu + p \cdot f(x) \cdot k. \quad (5)$$

Given $x$ and $p$, the consumer chooses $m$ and $c$ to maximize (4) subject to (5). The optimal choice of consumption satisfies

$$\frac{1}{1+\chi} \cdot (1 + \tau(x)) \cdot p \cdot m^{-\frac{1}{p}} = \frac{\chi}{1+\chi} \cdot c^{-\frac{1}{p}}. \quad (6)$$

Equation (6) says that at the optimum, a consumer is indifferent between spending income on the nonproduced good or on the service good. We define the aggregate demand as the optimal choice of consumption for a given tightness and price, taking into account the market-clearing condition on the nonproduced good market ($m = \mu$):

**DEFINITION 2.** The aggregate demand is a function of market tightness and price defined by

$$c^d(x, p) = \frac{\chi^e \cdot \mu}{(1 + \tau(x))^e \cdot p^e} \quad (7)$$

for all $(x, p) \in [0, x^m] \times (0, +\infty)$, where $x^m > 0$ satisfies $\rho = q(x^m)$. The function $c^d$ is strictly decreasing in $x$ and $p$. We have $c^d(x = 0, p) = \chi^e \cdot [(1 - \rho)/p]^e \cdot \mu$, and $c^d(x^m, p) = 0$.

Aggregate demand $c^d$ is strictly decreasing in $x$ and $p$ because the effective price of services is $(1 + \tau(x)) \cdot p$ and an increase in effective price reduces the consumption of services relative to that of nonproduced good, fixed to $\mu$.

### 3.2 Equilibrium Representation

Before presenting a formal equilibrium concept, we introduce the equilibrium representation that we use in the paper. This equilibrium representation derives naturally from the formal equilibrium concept, and it is sufficient to obtain all the results in the paper.
An equilibrium consists of a triplet \((c,x,p)\) such that aggregate supply is equal to aggregate demand:

\[
\begin{align*}
    c^s(x) &= c^d(x,p) \\
    c &= c^s(x)
\end{align*}
\]

Figure 3(a) represents aggregate demand, aggregate supply, and the equilibrium in a \((c,x)\) plane. The aggregate demand curve slopes downward. The aggregate supply curve slopes upward for \(x \leq x^*\) and downward for \(x \geq x^*\). The equilibrium corresponds to the intersection of the two curves with positive consumption.\(^{18}\) The figure also shows capacity \(k\) and output \(y = f(x) \cdot k\). A fraction \(c/y = 1/(1 + \tau(x))\) of output is consumed, and a fraction \(1 - (c/y) = \tau(x)/(1 + \tau(x))\) of output is allocated to matching. The fraction \(u = 1 - (y/k) = 1 - f(x)\) of services that are not sold is idleness.

Figure 3(b) represents aggregate demand and aggregate supply in a \((c,p)\) plane. Aggregate supply does not depend on the price so the aggregate supply curve is vertical. The aggregate demand curve is downward sloping. The equilibrium corresponds to the intersection of the aggregate supply and aggregate demand curves in this plane as well.

Even though the matching cost, \(\rho\), plays an important role in the model, underutilization of labor would not necessarily disappear if the matching cost were arbitrarily small. What happens when the price is rigid and the matching cost becomes arbitrarily small can be illustrated on Figure 3(a). The aggregate supply curve takes the shape of the output curve and the aggregate demand curve becomes vertical with \(c^d = \chi \cdot \mu \cdot p^{-\epsilon}\). Hence, not all labor is utilized in equilibrium if the price is high enough: \(p > \chi \cdot \mu /k)^{1/\epsilon}\).\(^{19}\)

Since the equilibrium has three variables and two conditions, there is one more variable than equilibrium conditions. This property implies that infinitely many combinations of price and tightness are consistent with our equilibrium definition.\(^{20}\) To select an equilibrium, we will therefore

---

\(^{18}\)There is another equilibrium at the other intersection of the curves, but it has zero consumption and is unstable. Appendix A characterizes all the possible equilibria, with zero or positive consumption.

\(^{19}\)The labor market model of Michaillat [2012] exhibits the same property. In that model, when the wage is high enough, some unemployment remains even when the recruiting cost is arbitrarily small.

\(^{20}\)The property that there is one more variable than equation in matching models is well known. Edgeworth [1881] noted that the solution to the bilateral monopoly problem is indeterminate. More recently, Howitt and McAfee [1987], Hall [2005], Farmer [2008] argue that the price is indeterminate in matching models because each seller-buyer pair must share the positive surplus created by their pairing, thus deciding the price in a situation of bilateral monopoly.
consider several price-setting mechanisms. Before we do so, it is useful to discuss in more detail the equilibrium concept we use.

3.3 Formal Equilibrium Concept

Our equilibrium where both price and tightness equilibrate the market can be seen as a generalization of the Walrasian equilibrium. The Walrasian equilibrium is the special case of our equilibrium in which sellers can sell all their services with certainty. To our knowledge, defining the matching equilibrium by analogy to the Walrasian equilibrium is novel. This definition is useful to explain why the matching equilibrium is indeterminate, and why economic forces do not put pressure on prices.

Following Walrasian theory, we make the institutional assumption that a price, \( p \), and a market tightness, \( x \), are posted on the market for services, and we make the behavioral assumption that buyers and sellers of services take price and tightness as given. This assumption is justified as follows.

Market tightness is the ratio of aggregate buying effort (visits) to aggregate capacity. Thus, buyers and sellers take it as given as they are small relative to the size of the market. The issue is more complicated for the price since buyer and seller could bargain over the price once they are matched, implying that they have some control over the price. However, the actual transaction
price has no influence on search decisions once a match is realized; what matters is the price that buyers and sellers expect to trade at ex-ante. Since the transaction price depends on the other party and possibly other factors (for instance, custom, social norms, other buyers, and other sellers), we assume that each party takes the expected transaction price as given.

To clarify the definition, we do not use a representative buyer but index all buyers by \( i \in [0, 1] \). Sellers are passive in the sense that their individual capacity is uniformly fixed at \( k \).\(^{21}\) We come back to representative agents once we have presented the equilibrium concept.

**DEFINITION 3.** An equilibrium is a price \( p \), market tightness \( x \), aggregate consumption \( c \), aggregate output \( y \), a collection of visits \( \{v(i), i \in [0, 1]\} \) such that

1. Taking \( x \) and \( p \) as given, buyer \( i \in [0, 1] \) chooses the number of visits \( v(i) \) to maximize her utility subject to her budget constraint and to the constraint imposed by matching frictions:
   \[
   c(i) = v(i) \cdot q(x) / (1 + \tau(x)),
   \]
   where \( c(i) \) is consumption of buyer \( i \).

2. Actual labor market tightness is \( x = \int_0^1 v(i)di/k \)

3. The price \( p \) is pairwise Pareto efficient in all buyer-seller matches.

4. Aggregate consumption satisfies \( c = \int_0^1 c(i)di \) and aggregate output satisfies \( y = k \cdot f(x) \).

Let us discuss condition by condition how our equilibrium concept is a direct extension of the Walrasian equilibrium to a market with matching frictions.

As in a Walrasian equilibrium, Conditions (1) imposes that buyers and sellers behave optimally given the quoted price and tightness. A key difference between the two equilibrium concepts is that in a Walrasian equilibrium, buyers and sellers decide the quantity that they desire to buy or sell whereas in our equilibrium, buyers and sellers decide the buying effort that they desire to exert. These efforts lead to a trade with a probability determined by the market tightness. Consumers decide how many sellers of services to visit, knowing that each visit leads to a purchase with probability \( q(x) \).

In a Walrasian equilibrium, the market clears: at the quoted price, the quantity that buyers desire to buy equals the quantity that sellers desire to sell. This condition can be reformulated as

\(^{21}\)This correspond to an endowment economy. It is possible to endogenize the supply by allowing each seller to choose its capacity. Our concept of equilibrium easily carries over to that case.
a consistency requirement. Sellers and buyers make their decisions with the expectation that they will be able buy or sell any quantity at the equilibrium price. In other words, they expect that the probability to buy or sell an item is one. For the equilibrium to be consistent with the expectations of sellers and buyers, the quantity that buyers desire to buy must be equal to the quantity that sellers desire to sell such that anybody desiring to trade at the quoted price is able to trade in equilibrium. This condition can only be fulfilled if the market clears.

Condition (2) is the equivalent to this consistency requirement in presence of matching frictions. Once buyers have chosen \( \{v(i), i \in [0,1]\} \), the number of trades is given by

\[
\left[ \left( \int v(i)di \right)^{-\eta} + k^{-\eta} \right]^{-\frac{1}{\eta}} = k \cdot f \left( \frac{\int v(i)di}{k} \right) = \left( \int v(i)di \right) \cdot q \left( \frac{\int v(i)di}{k} \right).
\]

These equalities imply that the selling probability faced by sellers is \( f \left( \frac{\int v(i)di}{k} \right) \) and the buying probability faced by buyers is \( q \left( \frac{\int v(i)di}{k} \right) \). In equilibrium, we impose the consistency requirement that expected probability and actual probability are the same: \( f(x) = f \left( \frac{\int v(i)di}{k} \right) \) and \( q(x) = q \left( \frac{\int v(i)di}{k} \right) \). Equivalently, we impose, that the posted tightness equals the actual tightness: \( x = \int v(i)di/k \).

Last, Walrasian theory imposes that no mutually advantageous trades between two agents are available, which in turn imposes that buyers and sellers expect to trade with probability one. This is because without a matching function, buyers who do not trade with anybody but would like to trade at the current price can come together on the market place. If excess supply or demand existed at the market price, buyers or sellers could initiate new trades at a different price until all opportunities for pairwise improvement are exhausted. For example, if there is excess demand for a good, a buyer who is not receiving as much of the good as she desires could offer a slightly higher price and get sellers to sell the good to her first, making both buyer and seller better off. Condition (3) is the equivalent of this condition in our theory. This condition imposes that buyer and seller both receive a non-negative share of the surplus arising from the seller-buyer match. This surplus arises because workers remain idle and do not sell one unit of service if the match is broken, and also because buyer’s matching costs are sunk at the time of matching. However, in presence of matching frictions, the condition that no mutually advantageous trades are available only applies ex-post to agents who are matched. Importantly, even though the equilibrium is pairwise Pareto
efficiency, the equilibrium might not be Pareto efficient overall in the sense that changing the price or wage could improve everybody’s welfare as we shall see. Finally, note that a Walrasian equilibrium is composed of as many variables as equations because Condition (4) imposes that $x$ is such that the trading probabilities are one, thus adding one equation to the equilibrium system.

Under these equilibrium conditions, the budget constraints of all consumers are satisfied, and that sales equal purchases through the matching process. Thus, following the traditional Walras law, the market for nonproduced good also necessarily clears: $\int_0^1 m(i)di = \mu$, where $m(i)$ is consumption of nonproduced good by buyer $i$.

### 3.4 Equilibrium and Comparative Statics under Various Price Mechanisms

In this section, we describe the equilibrium and the comparative statics under three alternative pricing mechanisms that have been most widely used in the search literature: efficient pricing, Nash bargaining, and rigid pricing. In the comparative statics, we consider aggregate demand shocks and aggregate supply shocks. We parameterize an aggregate demand shock by a change in taste for services, $\chi$, or in endowment, $\mu$. We parameterize an aggregate supply shock by a change in capacity, $k$. The comparative statics are summarized in Table 1.

#### 3.4.1 Efficient Price

We first define and describe the efficient allocation and the efficient price:

**DEFINITION 4.** An efficient allocation is a pair $(x, c)$ of market tightness and consumption that maximizes welfare subject to the matching frictions, $c \leq (f(x) - \rho \cdot x) \cdot k$. The efficient price is the price that implements the efficient allocation.

Welfare is given by

$$\begin{align*}
\left\{ \frac{\chi}{1 + \chi} \cdot c^{(\varepsilon-1)/\varepsilon} + \frac{1}{1 + \chi} \cdot \mu^{(\varepsilon-1)/\varepsilon} \right\} ^{\varepsilon/(\varepsilon-1)}.
\end{align*}$$

Hence, maximizing welfare is equivalent to maximizing consumption:
**PROPOSITION 1.** The efficient allocation is \((x^*, c^*)\), where \(x^*\) and \(c^*\) satisfy \(f'(x^*) = \rho\) and \(c^* = [f(x^*) - \rho \cdot x^*] \cdot k\). The efficient price is

\[
p^* = \frac{1}{1 + \tau(x^*)} \cdot \chi \cdot \left( \frac{\mu}{c^*} \right)^{1/\epsilon}.
\]

The efficient allocation is the point that is furthest to the right on the aggregate supply curve. At this point, the aggregate supply function is maximized. The price \(p^*\) is such that the aggregate demand curve intersects the aggregate supply curve at the efficient allocation. This price necessarily exists because by increasing the price from 0 to \(+\infty\), the aggregate demand curve rotates around the point \((0, x^m)\) from a horizontal position to a vertical position. The efficient price ensures that the aggregate demand curve is always in the position depicted in Figure 4. The price \(p^*\) in Proposition 1 corresponds to the price characterized by Hosios [1990] in his seminal analysis of the efficiency properties of matching models.

Efficient pricing is a generalization of the Walrasian equilibrium to a situation with matching costs. Suppose the matching cost \(\rho\) goes to zero. In that case, the aggregate supply curve \(c^x\) is the same as the output curve \(y = f(x) \cdot k\). \(\tau(x) \equiv 0\) and hence the demand curve \(c^d\) is vertical. In that case, the efficient tightness is \(x^* = \infty\) so that \(f(x^*) = 1\) and \(f'(x^*) = 0\). Hence, the price is efficient.
When sellers can sell their services with probability one which is the standard Walrasian case.\footnote{Efficient pricing can be seen as a special case of Nash bargained pricing under the condition of Hosios [1990], or it can be obtained through the competitive search equilibrium of Moen [1997]. In directed search, an auctioneer posts various \((p,x)\) possibilities and buyers and sellers decide ex-ante which market to enter. In equilibrium, supply and demand are equalized across all markets. Sellers and buyers chose the market giving the highest utility. Hence, the only market that agents enter in equilibrium is the efficient market \((p^*,x^*)\).}

If the price is not efficient, the economy can be either too tight or too slack.

**DEFINITION 5.** The economy is slack if \(x < x^*\), tight if \(x > x^*\), and efficient if \(x = x^*\).

Figure 5 illustrates the regimes. In the slack regime, the price is above its efficient level so aggregate demand is too low and tightness is below its efficient level. Consumption and output are below their efficient level. In the tight regime, the price is below its efficient level so aggregate demand is too high and tightness is above its efficient level. Consumption is again below its efficient level but output is above their efficient level. In our model, higher consumption always implies higher welfare, which is not the case of higher output.

With an efficient price, equilibrium product market tightness is set at \(x = x^*\), where \(x^*\) satisfies \(f'(x^*) = \rho\). Since the level of product market tightness is solely determined by the matching function and the matching cost, product market tightness responds neither to aggregate demand nor aggregate supply shocks. A direct consequence is that labor utilization, \(f(x)\), does not respond either to aggregate demand or aggregate supply shocks. Output and consumption are given by \(y = f(x) \cdot k\) and \(c = [f(x) - \rho \cdot x] \cdot k\); thus, output and consumption do not respond to aggregate...
demand shocks (both $x$ and $k$ remain constant) but they do respond to aggregate supply shocks ($x$ remains constant but $k$ fluctuates). These results echo a number of studies that have used efficient pricing and typically found that they generate quantitatively very small fluctuations in tightness for realistic shocks [Blanchard and Galí, 2010; Shimer, 2005].

### 3.4.2 Nash Bargained Price

We assume that buyers and sellers use Nash bargaining to determine the price. This is the traditional way to set prices and select an equilibrium in the matching literature [Diamond, 1982; Mortensen, 1982; Pissarides, 1985].

Following previous work on Nash bargaining, we assume that consumers have a linear utility function $(\chi e + m)/(1 + \chi)$, which is the special case of the CES utility function when $\varepsilon \to +\infty$. The optimal consumption choice of consumers, given by (6), yields

$$
(1 + \tau(x)) \cdot p = \chi.
$$

Equation (9) shows that product market tightness is pinned down independently of quantity, so that aggregate demand is perfectly elastic with respect to $x$. Figure 6 represent the product market in a $(c,x)$ plane. The aggregate demand is represented by an horizontal curve.

With Nash bargaining, the price is given by

$$
p = \beta \cdot \chi,
$$

where $\beta \in (0,1)$ is the bargaining power of sellers. The price is the generalized Nash solution to the bargaining problem between a consumer and a firm with bargaining power $\beta$. The surplus to the consumer of buying one unit of produced good is $\mathcal{C}(p) = [\chi/(1 + \chi)] - [p/(1 + \chi)]$. The surplus to the worker of selling one unit of produced good is $\mathcal{F}(p) = p/(1 + \chi)$. The Nash solution maximizes $\mathcal{C}(p)^{1-\beta} \cdot \mathcal{F}(p)^{\beta}$, so $\mathcal{F}(p) = \beta \cdot [\mathcal{F}(p) + \mathcal{C}(p)] = \beta \cdot \chi$ and $p$ satisfies (10). Combining (9) and (10), we determine equilibrium tightness:

$$
\beta \cdot (1 + \tau(x)) = 1.
$$
Product market tightness, $x$, is determined independently of quantities. Consumption is given by the aggregate supply: $c = c^s(x)$.

With a bargained price, equilibrium product market tightness is determined by (11). The level of product market tightness is solely determined by the matching function, the matching cost, and the bargaining power. Hence, product market tightness responds neither to aggregate demand nor aggregate supply shocks. Following the same logic as with the efficient price, we can show that labor utilization responds neither to aggregate demand nor aggregate supply shocks, and that output and consumption respond only to aggregate supply shocks. Hence, Nash bargained prices can be seen as fully flexible prices that absorb entirely aggregate demand shocks and keep tightness constant although the price is not necessarily efficient.

Our comparative static results again are consistent a number of studies that have used Nash bargaining pricing models and typically found that they generate quantitatively very small fluctuations in tightness for realistic shocks, or no fluctuations at all [Blanchard and Gali, 2010; Michaillat, 2012; Millard, Scott and Sensier, 1997; Shimer, 2005, 2010].
3.4.3 Rigid Price

Let us assume that the price is a parameter of the model and that only market tightness equilibrates the market. In the definition of the equilibrium, the price becomes an exogenous variable; only tightness and quantity are endogenous variables such that the equilibrium is composed of two variables that satisfy two conditions. Furthermore, the price remains fixed in response to shocks in the comparative-statics analysis. We formally define and characterize the equilibrium with rigid price:

**DEFINITION 6.** Given a price \( p \), an *rigid-price equilibrium* consists of a pair \((x, c)\) of market tightness and consumption such that aggregate supply equals aggregate demand and consumption is given by the aggregate supply:

\[
\begin{align*}
    c^s(x) &= c^d(x, p) \\
    c &= c^d(x, p)
\end{align*}
\]

**PROPOSITION 2.** For any price \( p > 0 \), there exists a unique rigid-price equilibrium with positive consumption. Equilibrium tightness, \( x \), is the unique solution to

\[
(1 + \tau(x))^{x-1} \cdot f(x) \cdot k = \frac{\chi^e \cdot \mu}{p^e}.
\]

Equation (12) is obtained by manipulating the equilibrium condition \( c^s(x) = c^d(x, p) \).

Figure 7(a) illustrates a positive aggregate demand shock, corresponding to an increase in \( \chi \) or \( \mu \). The shock leads the aggregate demand curve to rotate outward and therefore increase market tightness and output. Since tightness increases, idleness falls. The impact on consumption depends on the regime: in the slack regime, consumption increases; in the efficient regime, consumption does not change; and in the tight regime, consumption falls. In the tight regime, a higher tightness reduces the output devoted to consumption even though it increases total output because it increases sharply the share of output dissipated in matching frictions.

Figure 7(b) illustrates a positive aggregate supply shock, corresponding to an increase in \( k \). Several researchers assume that the wage is a parameter or a function of the parameters. See for instance Hall [2005], Blanchard and Galí [2010], and Michaillat [2012].
The shock leads the aggregate supply curve to expand, raising consumption but reducing market tightness. Since tightness decreases, idleness increases. Since $k$ increases but $x$ falls, the impact on output $y = f(x) \cdot k$ is not obvious. Equation (12) implies, however, that $y = f(x) \cdot k = \left(\chi^e \cdot \mu / p^e\right) / [1 + \tau(x)]^{e-1}$. As $x$ falls, $(1 + \tau(x))^{e-1}$ falls since $e > 1$, and hence output $y$ falls.

Interestingly, when the economy is in the efficient regime, shifts in aggregate supply do influence consumption whereas shifts in aggregate demand have no first-order effects on consumption.

Aggregate supply and aggregate demand shocks generate different correlations between variables. Market tightness and output are positively correlated under aggregate demand shocks but negatively correlated under aggregate supply shocks. An implication is that idleness decreases after a positive aggregate demand shock but increases after a positive aggregate supply shock. The intuition is simple. After a positive aggregate demand shock consumers want to consume more services so workers sell a larger fraction of a fixed amount of services available. Hence, output and market tightness are higher. On the other hand, after a positive aggregate supply shock workers offer more services for sale but consumers do not desire to consume more at a given price, so workers sell a smaller fraction of a larger amount of services available. Hence, market tightness is lower. Since tightness is lower in equilibrium, the effective price faced by consumers, $(1 + \tau(x)) \cdot p$, is lower, stimulating consumers to purchase more services and increasing output.
Table 1: Comparative statics in the basic model (Section 3)

<table>
<thead>
<tr>
<th>Increase in:</th>
<th>Effect on:</th>
<th>Output</th>
<th>Tightness</th>
<th>Labor utilization</th>
<th>Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$y$</td>
<td>$x$</td>
<td>$f(x)$</td>
<td>$c$</td>
</tr>
<tr>
<td><strong>A. Efficient pricing</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aggregate demand</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Aggregate supply</td>
<td></td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td><strong>B. Nash bargaining</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aggregate demand</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Aggregate supply</td>
<td></td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td><strong>C. Rigid pricing</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aggregate demand</td>
<td></td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>(slack)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0 (efficient)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>− (tight)</td>
</tr>
<tr>
<td>Aggregate supply</td>
<td></td>
<td>+</td>
<td>−</td>
<td>−</td>
<td>+</td>
</tr>
</tbody>
</table>

Notes: The comparative statics are derived in Section 3.4. An increase in aggregate demand results from an increase in endowment, $\mu$, or an increase in taste for services, $\chi$. An increase in aggregate supply results from an increase in capacity, $k$.

4 Labor Utilization and Unemployment

This section builds a model in which firms hire workers on a labor market with matching frictions, employ these workers to produce goods, and sell the production on a product market with matching frictions. The model allows us to study how aggregate demand shocks propagate from the product market to the labor market and how they affect unemployment. It also allows us to describe the effects of a variety of supply-side shocks: technology shocks, labor force participation shocks, and labor market mismatch shocks.

The product market has the same structure as in Section 3. The only difference is that the capacity of firms, $k$, is not exogenous but determined endogenously from the production decision of firms. The labor market has a similar structure with matching frictions modeled as in the product market. Following Michaillat [2012], we assume that firms are large, face a production function with diminishing marginal returns to labor, and maximize profits taking labor market tightness and real wage as given. This section omits the description of the product market (as it is identical to
that of Section 3) and focuses on the labor market.

### 4.1 Labor Market and Firms

The economy has a measure $l$ of identical firms and a measure $h$ of identical households. Households own the firms and receive their profits. Household members pool their income before jointly deciding consumption. An exogenous number $h \in (0, 1)$ of household members is in the labor force, and a number $1 - h$ is out of the labor force. There are matching frictions on the labor market. All labor force participants are initially unemployed and search for a job. Each firm posts $v$ vacancies to hire workers. The number $l$ of workers who are hired is given by a matching function taking as argument initial unemployment and vacancies:

$$l = (h - \hat{\eta} + \hat{\nu} - \hat{\eta}) - \hat{\eta}.$$ 

The parameter $\hat{\eta} > 0$ influences the curvature of the matching function. Labor market tightness is defined as the ratio of vacancy to initial unemployment:

$$\theta \equiv \frac{v}{h},$$

Labor market tightness determines the probabilities that a jobseeker finds a job and a vacancy is filled.

Jobseekers find a job with probability $\hat{f}(\theta) = l/h = (1 + \theta - \hat{\eta})^{-\frac{1}{\hat{\eta}}}$, and a vacancy is filled with probability $\hat{q}(\theta) = l/\hat{\nu} = (1 + \theta \hat{\eta})^{-\frac{1}{\hat{\eta}}}$. We assume away randomness at firm and household levels: each firm hires $\hat{v} \cdot \hat{q}(\theta)$ workers for sure, and $\hat{f}(\theta) \cdot h$ household members find a job for sure. The function $\hat{f}$ is increasing in $\theta$ and the function $\hat{q}$ is decreasing in $\theta$. That is, when the labor market is slacker, the probability to find a job is lower but the probability to fill a vacancy is higher.

The representative firm hires $l$ workers. Some of the workers are engaged in production while others are engaged in recruiting. More precisely, $n < l$ workers are able to produce a quantity $k$ of output according to the production function $k = a \cdot n^\alpha$. The parameter $a > 0$ measures the technology of the firm and the parameter $\alpha \in (0, 1)$ captures decreasing marginal returns to labor. Because of the product market frictions, the firm only sells a fraction $f(x)$ of its capacity $k$.

Posting a vacancy requires a fraction $\hat{\rho} > 0$ of a worker’s time. The idea is that it takes human resources to hire workers. Thus, the firm devotes $l - n = \hat{\rho} \cdot \hat{v} = \hat{\rho} \cdot l/\hat{q}(\theta)$ workers to recruiting a total of $l$ workers. The number $n$ of producers is therefore related to the number $l$ of workers by $l = (1 + \hat{\tau}(\theta)) \cdot n$, where $\hat{\tau}(\theta) \equiv \hat{\rho}/(\hat{q}(\theta) - \hat{\rho})$ measures the number of recruiters for each producer. The function $\hat{\tau}$ is positive and strictly increasing as long as $\hat{q}(\theta) > \hat{\rho}$. The firm pays its $l$ workers a real wage $w$ (the nominal wage is $p \cdot w$), and the wage bill of the firm is $w \cdot l = (1 + \hat{\tau}(\theta)) \cdot w \cdot n.$
From this perspective, matching frictions in the labor market impose a wedge \( \hat{\tau}(\theta) \) on the wage of producers.

Given \( \theta, x, p, \) and \( w, \) the firm chooses \( n \) to maximize profits

\[
\Pi = p \cdot f(x) \cdot a \cdot n^\alpha - (1 + \hat{\tau}(\theta)) \cdot p \cdot w \cdot n.
\]

Hence, the optimal number of producers \( n \) satisfies:

\[
f(x) \cdot a \cdot n^{\alpha - 1} = (1 + \hat{\tau}(\theta)) \cdot w.
\] (13)

At the optimum, the real marginal revenue of one producer equals the real marginal cost of one producer. The real marginal revenue is the marginal product of labor, \( a \cdot \alpha \cdot n^{\alpha - 1} \), times the selling probability, \( f(x) \). The real marginal cost is the real wage, \( w \), plus the marginal recruiting cost, \( \hat{\tau}(\theta) \cdot w \).

### 4.2 Equilibrium Representation

The equilibrium concept is the same as in Section 3. To obtain a convenient representation of the equilibrium, we define aggregate demand, aggregate supply, labor demand, and labor supply functions. The aggregate demand is given by (7). The aggregate supply is given by \( c^s(x, n) = (f(x) - \rho \cdot x) \cdot a \cdot n^\alpha \). By symmetry, we define labor supply and demand based on productive workers (instead of all workers producers plus recruiters). Labor supply and labor demand are defined as follows:

**Definition 7.** The labor demand is a function of labor market tightness, product market tightness, and real wage defined by

\[
n^d(\theta, x, w) = \frac{f(x) \cdot a \cdot \alpha}{(1 + \hat{\tau}(\theta)) \cdot w}^{\frac{1}{1-\alpha}}
\]

for all \((\theta, x, w) \in [0, \theta^m] \times (0, +\infty) \times (0, +\infty), \) where \( \theta^m > 0 \) satisfies \( \hat{\rho} = \hat{q}(\theta^m) \). The labor supply
is a function of labor market tightness defined for all for all $\theta \in [0,x^m]$ by

$$n^s(\theta) = (\hat{f}(\theta) - \hat{\rho} \cdot \theta) \cdot h.$$  

Labor demand and labor supply curves are depicted on Figure 8 in $(n,\theta)$ and $(n,w)$ planes. Labor demand gives the number of producers that satisfies the firm’s optimal employment choice, given by (13). Labor supply gives the number of producers employed after the matching process when a number $h$ of household members are in the labor force. Lemma 1 establishes a few properties of labor demand and labor supply:

**Lemma 1.** The function $n^d$ is strictly decreasing in $\theta$, strictly increasing in $x$, strictly decreasing in $w$, $n^d(\theta = 0, x, w) = [f(x) \cdot a \cdot \alpha \cdot (1 - \hat{\rho})/w]^{1/\alpha}$, and $n^d(\theta^m, x, w) = 0$. The function $n^s$ is strictly increasing on $[0, \theta^*]$, strictly decreasing on $[\theta^*, \theta^m]$, $n^s(\theta = 0) = 0$, and $n^s(\theta^m) = 0$. The constant $\theta^*$ maximizes $(\hat{f}(\theta) - \hat{\rho} \cdot \theta) \cdot h$ so $\hat{f}'(\theta^*) = \hat{\rho}$. The constant $\theta^*$ depends only on $\hat{\rho}$ and $\hat{\eta}$.

The behavior of the labor supply is the same as that of the aggregate supply because the matching process is similar on labor and product markets. The labor demand decreases with $w$ and $\theta$ because when either of them increases, the effective wage of a producer, $(1 + \hat{\tau}(\theta)) \cdot w$, increases and firms reduce hiring of producers. The labor demand increases with $x$ because when $x$ increases, the probability $f(x)$ to sell output increases and firms increase hiring of producers. Figure 8(a) represents labor demand and labor supply in a $(n,\theta)$ plane. The labor demand curve slopes downward. The labor supply curve slopes upward for $\theta \leq \theta^*$ and downward for $\theta \geq \theta^*$. The figure also shows the labor force, $h$, employment, $l = \hat{f}(\theta) \cdot h$, unemployment $u = h - l = (1 - \hat{f}(\theta)) \cdot h$, the number of producers, $n = l/(1 + \hat{\tau}(\theta))$, and the number of recruiters, $l - n = [\hat{\tau}(\theta)/(1 + \hat{\tau}(\theta))] \cdot l$. Figure 8(b) represents labor demand and labor supply in a $(n,w)$ plane.

Following the equilibrium representation of Section 3, we can show that an equilibrium consists
of a 6-tuple \((p, w, c, \theta, n)\) such that

\[
\begin{align*}
    c^s(x, n) &= c^d(x, p) \\
    c &= c^s(x) \\
    n^i(\theta) &= n^d(\theta, x, w) \\
    n &= n^d(\theta, x, w)
\end{align*}
\]

Since the equilibrium is composed of six variables that satisfy four conditions, there are two more variables than equilibrium conditions. To select an equilibrium, we consider several price- and wage-setting mechanisms.

### 4.3 Equilibrium under Various Pricing Mechanisms

#### 4.3.1 Equilibrium with Efficient Prices

We define and describe efficient allocation and efficient prices:

**DEFINITION 8.** The efficient allocation is the quadruplet \((x, \theta, c, n)\) that maximizes welfare, given by (8), subject to the matching frictions on the product market, \(c \leq (f(x) - \rho \cdot x) \cdot a \cdot n^\alpha\), and to the matching frictions on the labor market, \(n \leq (\hat{f}(\theta) - \hat{\rho} \cdot \theta) \cdot h\). The efficient price and
efficient real wage are the price and real wage that implement the efficient allocation.

**PROPOSITION 3.** The efficient allocation is \( (x^*, \theta^*, c^*, n^*) \), where \( x^*, \theta^*, c^* \) and \( n^* \) satisfy \( f'(x^*) = \rho, c^* = [f(x^*) - \rho \cdot x^*] \cdot k^* \) with \( k^* \equiv a \cdot (n^*)^\alpha \), \( f'(\theta^*) = \hat{\rho} \), and \( n^* = [f(\theta^*) - \hat{\rho} \cdot \theta^*] \cdot h \). The efficient real wage and efficient price are given by

\[
\begin{align*}
    w^* &= \frac{1}{1 + \hat{\tau}(\theta^*)} \cdot f(x^*) + a \cdot \alpha \cdot (n^*)^{\alpha - 1} \\
    p^* &= \frac{1}{1 + \tau(x^*)} \cdot \chi \cdot \left( \frac{\mu}{c^*} \right)^\frac{1}{\alpha}.
\end{align*}
\]

(14) (15)

The economy can be in five different regimes:

**DEFINITION 9.** The economy is efficient if \( \theta = \theta^* \) and \( x = x^* \), labor-slack and product-slack if \( \theta < \theta^* \) and \( x < x^* \), labor-slack and product-tight if \( \theta < \theta^* \) and \( x > x^* \), labor-tight and product-slack if \( \theta > \theta^* \) and \( x < x^* \), labor-tight and product-tight if \( \theta > \theta^* \) and \( x > x^* \).

The economy behaves differently in the five regimes because the aggregate supply and labor supply functions have different properties across regimes.\(^{24}\) Proposition 4 establishes the boundaries of the regimes in a \((w, p)\) plane:

**PROPOSITION 4.** There exist a function \( w \mapsto p^t(w) \) such that for any \( w > 0 \), the product market is slack \((x < x^*)\) if and only if \( p > p^t(w) \). There exist a function \( w \mapsto p^\theta(w) \) such that for any \( w > 0 \), the labor market is slack \((\theta < \theta^*)\) if and only if \( p > p^\theta(w) \). The function \( p^t \) is strictly decreasing for \( w \in (0, w^s) \) and strictly increasing for \( w \in (w^s, +\infty) \). The function \( p^\theta(w) \) is strictly decreasing for \( w \in (0, w^L) \) and such that \( p^\theta(w) = 0 \) for all \( w > w^L \), where \( w^L > w^s \) is a constant of the parameters. Furthermore, \( p^t(w^s) = p^\theta(w^s) = p^* \).

Figure 9 displays the five regimes in a \((w, p)\) plane. The labor market is slack above the curve \( p = p^\theta(w) \) and tight below. The product market is slack above the curve \( p = p^t(w) \) and tight below. Moreover, \( \theta = \theta^* \) on the curve \( p = p^\theta(w) \) and \( x = x^* \) on the curve \( p = p^t(w) \). As the price and wage implementing \((x^*, \theta^*)\) are unique, the curves \( p = p^\theta(w) \) and \( p = p^t(w) \) cross only once, at \((w^s, p^*)\).

\(^{24}\)The four non-efficient regimes we obtain in this model echo the four regimes from the traditional fix-price fix-wage model of Barro and Grossman [1971]. See Bénessy [1993] for a comprehensive review.
4.3.2 Equilibrium with Bargained Prices

We assume that consumers have a linear utility function \((\chi \cdot c + m)/(1 + \chi)\) and that firms have a linear production function \(a \cdot n\), which is the special case of our production function when \(\alpha = 1\). The optimal consumption choice of consumers yields (9) and the partial equilibrium on the product market is depicted in Figure 6. The optimal employment choice of firms, given by (13), yields

\[
(1 + \hat{\tau}(\theta)) \cdot w = a \cdot f(x).
\]

Equations (16) show that the labor demand is perfectly elastic with respect to \(\theta\). Figure 10 represents the labor market in a \((n, \theta)\) plane. Labor demand is represented by a horizontal curve.

With Nash bargaining, the price is by (10) and the wage is given by

\[
w = \hat{\beta} \cdot a \cdot f(x),
\]

where \(\hat{\beta} \in (0, 1)\) is the bargaining power of workers. The real wage is the generalized Nash solution of the bargaining problem between a worker and a firm. The surplus to the firm of hiring one worker is \(\mathcal{F}(w) = a \cdot f(x) - w\). The surplus to the worker of being hired is \(\mathcal{W}(w) = w\). The Nash solution maximizes \(\mathcal{F}(w)^{1-\hat{\beta}} \cdot \mathcal{W}(w)^{\hat{\beta}}\), so \(\mathcal{W}(w) = \hat{\beta} \cdot [\mathcal{W}(w) + \mathcal{F}(w)] = \hat{\beta} \cdot a \cdot f(x)\) and \(w\) satisfies (17).
Combining (9), (16), (17) and (10), we determine the equilibrium tightnesses:

\[
\beta \cdot (1 + \tau(x)) = 1
\]
\[
\hat{\beta} \cdot (1 + \hat{\tau}(\theta)) = 1.
\]

Employment and consumption are given by the supply equations \( n = n^s(\theta) \) and \( c = c^s(x, n) \).

### 4.3.3 Equilibrium with Rigid Prices

We assume that price and real wage are parameters of the model, and that only product and labor market tightnesses equilibrate the markets. We define and characterize the equilibrium with rigid prices:

**DEFINITION 10.** Given real wage \( w > 0 \) and product market tightness \( x > 0 \), a rigid-price labor market equilibrium consists of a pair \( (\theta, n) \) of labor market tightness and employment such that labor supply equals labor demand and employment is given by the labor demand:

\[
\begin{cases}
  n^s(\theta) = n^d(\theta, x, w) \\
  n = n^d(\theta, x, w)
\end{cases}
\]

Figure 10: Labor market with linear production function
Given price \( p > 0 \) and production employment \( n > 0 \), a rigid-price product market equilibrium consists of a pair \((x, c)\) of product market tightness and consumption such that aggregate supply equals aggregate demand and consumption is given by the aggregate demand:

\[
\begin{align*}
  c^s(x, n) &= c^d(x, p) \\
  c &= c^d(x, p)
\end{align*}
\]

Given prices \((p, w)\), a rigid-price general equilibrium consists of a quadruplet \((x, \theta, c, n)\) of tightnesses and quantities such that \((\theta, n)\) is a rigid-wage labor market equilibrium given \((x, w)\) and \((x, c)\) is a rigid-price product market equilibrium given \((n, p)\).

The rigid-price labor market and product market equilibria are partial equilibria because they take as given the tightness and quantity in the other market. The product market equilibrium can be represented as in Figure 3 with \( k = a \cdot n^\alpha \). Similarly, the labor market equilibrium is represented in Figure 8. In general equilibrium, the two partial-equilibrium systems hold simultaneously. The following proposition characterizes the general equilibrium:

**PROPOSITION 5.** For any \( p > 0 \) and \( w > 0 \), there exists a unique rigid-price general equilibrium with positive consumption. The equilibrium tightnesses, \((x, \theta)\), are the unique solution to the system

\[
\begin{align*}
  h^{1-\alpha} \cdot \hat{f}(\theta)^{1-\alpha} \cdot (1 + \hat{\tau}(\theta))^{\alpha} &= \frac{a \cdot \alpha}{w} \cdot f(x) & (20) \\
  h \cdot \hat{f}(\theta) \cdot (1 + \tau(x))^{\epsilon-1} &= \frac{\alpha}{w} \cdot \chi^x \cdot \frac{\mu}{p^\epsilon} & (21)
\end{align*}
\]

Equation (20) implicitly defines \( \theta \) as a strictly increasing function of \( x \) while equation (21) implicitly defines \( \theta \) as a strictly decreasing function of \( x \). These two functions intersect exactly once.

Equation (20) arises from the partial-equilibrium condition on the labor market combining (13) with \( n'(\theta) = (\hat{f}(\theta) - \hat{\rho} \cdot \theta) \cdot h = \hat{f}(\theta) \cdot h/[1 + \hat{\tau}(\theta)] \). Equation (21) arises from a combination of the partial-equilibrium conditions on the labor and product markets\(^{25}\) combining (12) with \( k = \]

\(^{25}\)In addition to the equilibrium with positive consumption, there exist two other equilibria with zero consumption. Appendix A extends Proposition 5 to characterize all the possible equilibria and to describe the domain and codomain of the functions implicitly defined by (20) and (21).
Product market tightness
Labor market tightness

Equation (13)

\[ m \leq 0 \]

Equation (14)

General equilibrium

Figure 11: Rigid-price equilibrium in the model of labor utilization and unemployment (Section 4)

\[ f(x) \cdot a \cdot n^\alpha = \hat{f}(\theta) \cdot h \cdot w / \alpha \] obtained from (13). Figure 11 represents the general equilibrium as the intersection of an upward-sloping and a downward-sloping curve in a \((x, \theta)\) plane. The upward-sloping curve is the locus of points \((x, \theta)\) that solve (20), and the downward-sloping curve is the locus of points \((x, \theta)\) that solve (21).

4.4 Comparative Statics

We use comparative statics to describe the response of the equilibrium to four types of shocks: aggregate demand shock, technology shock, labor market mismatch shock, and labor force participation shock. We contrast comparative statics under rigid, efficient, and bargained prices. The results are summarized in Table 2. We parameterize a positive aggregate demand shock by an increase in taste for services, \(\chi\), or in endowment, \(\mu\) as in Section 3. A technology shock is a shock in \(a\). A labor force participation shock is a shock in \(h\). We parameterize a mismatch shock by a change in matching efficacy on the labor market, along with a corresponding change in recruiting costs. An increase in mismatch leads to a fall in matching efficacy and a fall in matching cost: \(\hat{f}(\theta)\) and \(\hat{q}(\theta)\) become \(\lambda \cdot \hat{f}(\theta)\) and \(\lambda \cdot \hat{q}(\theta)\) with \(\lambda < 1\), and \(\hat{\rho}\) becomes \(\lambda \cdot \hat{\rho}\). Intuitively, with

---

26On Figure 11, the domain of the function that solves equation (21) is \([0, x^\text{m}]\). If the price, \(p\), is below some threshold, the function is only defined for \(x\) above some threshold (this threshold is necessarily below \(x^\text{m}\)). At the threshold, the function asymptotes to \(+\infty\). See Appendix A for more details.

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mismatch, a fraction of potential workers are not attractive to employers and these non-attractive workers can be spotted at no cost. Hence, only a fraction of job applicants are considered.

**Comparative Statics with Efficient Prices.** With efficient prices, product market tightness is \( x = x^* \) and labor market tightness is \( \theta = \theta^* \), where \( x^* \) satisfies \( f'(x^*) = \rho \) and \( \theta^* \) satisfies \( \hat{f}'(\theta^*) = \hat{\rho} \). Accordingly, the tightnesses do not respond to any of the shocks that we consider, not even the labor market mismatch shock (which affects \( \hat{f}(.) \) and \( \hat{\rho} \) by the same multiplicative factor \( \lambda \) and hence leaves \( \theta^* \) unchanged). A direct consequence is that labor utilization, \( f(x) \), does not respond to these shocks either. Total employment and employment of producers increase after an increase in participation but fall after an increase in mismatch, since \( l = \hat{f}(\theta) \cdot h \) and \( n = [\hat{f}(\theta) - \hat{\rho} \cdot \theta] \cdot h \). The level of unemployment rises after an increase in participation and an increase in mismatch, since \( u = [1 - \hat{f}(\theta)] \cdot h \) (the unemployment rate remains unchanged). Output and consumption rise after an increase in participation but fall after an increase in mismatch, since \( y = f(x) \cdot a \cdot n^\alpha \) and \( c = [f(x) - \rho \cdot x] \cdot a \cdot n^\alpha \). Total employment, employment of producers, and unemployment remain the same after an increase in technology. Output and consumption rise after an increase in technology.

**Comparative Statics with Bargained Prices.** With bargained prices, product market tightness and labor market tightness are determined by \( \hat{x} \) and \( \hat{\theta} \). The tightnesses solely depend on the matching costs and the matching functions, because they are determined by the functions \( \tau \) and \( \hat{\tau} \). Accordingly, the comparative statics with bargained prices are therefore the same as with efficient prices, even though the tightnesses may not be efficient with Nash bargaining. In particular, aggregate demand shocks have no effect at all under Nash bargaining.

**Comparative Statics with Rigid Prices.** First, we consider an aggregate demand shock. A positive aggregate demand shock leads to an upward shift of the downward-sloping curve in Figure 11. After the shock, labor market tightness and product market tightness increase. Unemployment decreases because \( u = h \cdot (1 - \hat{f}(\theta)) \). Output increases because equation (20) implies that \( y = f(x) \cdot a \cdot n^\alpha = (w/\alpha) \cdot h \cdot \hat{f}(\theta) \). The response of consumption and employment of producers depends on the regime. In the efficient regime, neither consumption nor employment respond to a marginal change in tightness. Employment of producers decreases in a labor-tight regime.

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but increases in a labor-slack regime. Consumption decreases in a product-tight and labor-tight regime but increases in a product-slack and labor-slack regime. In the two other regimes, $n^\alpha$ and $f(x) - \rho \cdot x$ move in opposite direction so it is not possible to determine the change in consumption. In the partial-equilibrium diagram of Figure 3, a positive aggregate demand shock leads to an upward rotation of the aggregate demand curve. This rotation raises product market tightness. In the partial-equilibrium diagram of Figure 8, the increase in product market tightness leads to an outward shift of labor demand because the probability to sell is higher. Labor market tightness increases as a result. Since the number of producers changes, aggregate supply adjusts in Figure 3. Product market tightness adjusts again, which feedbacks on the labor demand. Feedbacks between labor demand and aggregate supply continue until convergence to the new general equilibrium with higher labor market and product market tightnesses.

Second, we consider an increase in technology, $a$. A positive technology shock leads to an upward shift of the upward-sloping curve in Figure 11. After the shock, labor market tightness increases but product market tightness decreases. As with a positive aggregate demand shock, unemployment decreases, output increases, and the response of consumption and employment of producers depends on the regime. In the partial-equilibrium diagram of Figure 3, an increase in technology leads to an expansion of the aggregate supply curve. In the partial-equilibrium diagram of Figure 8, an increase in technology leads to an outward shift of the labor demand curve. The resulting changes in product market tightness and labor market tightness influence the probability to sell and employment of producers, which in turn feedback to the aggregate supply and labor demand curves. The new general equilibrium has higher labor market tightness and lower product market tightness.

Third, we consider an increase in labor force participation, $h$. This increase leads to downward shifts of the two curves in Figure 11. After the shock, labor market tightness decreases. Unemployment increases because the unemployment rate, $1 - \hat{f}(\theta)$, increases and the number of workers in the labor force, $h$, increases. The response of product market tightness, $x$, is ambiguous on the general-equilibrium diagram of Figure 11, but we can recombine (20) and (21) to determine the response of $x$. Exponentiating (21) to the power of $1 - \alpha$ and dividing (20) by the resulting
equation yields

\[(1 + \hat{\tau}(\theta))^\alpha = \left[ \frac{a \cdot \alpha}{w} \right] \cdot \left[ \frac{\alpha \cdot \chi^{\epsilon} \cdot \mu}{p^{\epsilon}} \right]^\alpha \cdot f(x) \cdot (1 + \tau(x))^{(1-\alpha)(\epsilon-1)}. \tag{22}\]

This equation implies that \( x \) decreases when \( \theta \) decreases after the shock to labor force participation. As a consequence, consumption, given by \( c = c^d(x, p) \), and output, given by \( y = (1 + \tau(x)) \cdot c^d(x, p) \), increase. Since \( y = f(x) \cdot a \cdot n^\alpha \), \( y \) increases, and \( f(x) \) decreases, it must be that employment of producers, \( n \), increases. Since \( x \) falls and \( l = h \cdot \hat{f}(\theta) \), equation (21) implies that total employment increases.

Finally, we consider a labor market mismatch shock. The function \( \hat{\tau} \) remains the same so the labor demand curve does not change. The efficient labor market tightness, \( \theta^* \), also remains the same. However, an increase in mismatch shifts inward the labor supply and employment curves. The increase in mismatch leads to an upward shift of the two curves in Figure 11. After the shock, labor market tightness increases. The response of product market tightness, \( x \), is ambiguous on the general-equilibrium diagram of Figure 11. However, (22) implies that \( x \) increases when \( \theta \) increases. Since \( x \) increases and \( l = h \cdot \hat{f}(\theta) \), equation (21) implies that total employment decreases. Unemployment therefore increases. Since \( l \) decreases and \( 1 + \hat{\tau}(\theta) \) increases, the number of producers, \( n = l / [1 + \hat{\tau}(\theta)] \), decreases. Since \( x \) increases after the mismatch shock, consumption, given by \( c = c^d(x, p) \), and output, given by \( y = (1 + \tau(x)) \cdot c^d(x, p) \), decrease.

### 4.5 Keynesian, Classical, and Frictional Unemployment

Our model is the first that allows for the existence of the three canonical types of unemployment: frictional unemployment, classical unemployment, and Keynesian unemployment. Equilibrium employment satisfies

\[ l = \left[ \frac{f(x) \cdot a \cdot \alpha}{w} \right]^\frac{\alpha}{\alpha - 1} \cdot \left[ \frac{1}{1 + \hat{\tau}(\theta)} \right]^\frac{\alpha}{\alpha - 1}, \]

where \( x, w, \) and \( \theta \) are equilibrium product market tightness, real wage, and labor market tightness. The model captures frictional unemployment through \( \hat{\tau}(\theta) > 0 \). Employment is lowered because firms incur a cost to fill vacancies. The model captures Keynesian unemployment through \( f(x) < 1 \).
Table 2: Comparative statics in the model of labor utilization and unemployment (Section 4)

<table>
<thead>
<tr>
<th>Increase in:</th>
<th>Effect on:</th>
<th>Output</th>
<th>Labor utilization</th>
<th>Labor market tightness</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$y$</td>
<td>$f(x)$</td>
<td>$\theta$</td>
<td>$u$</td>
</tr>
<tr>
<td>A. Efficient pricing</td>
<td>Aggregate demand</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Technology</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Labor force</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>Mismatch</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>B. Nash bargaining</td>
<td>Aggregate demand</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Technology</td>
<td>+</td>
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<td></td>
<td>Labor force</td>
<td>+</td>
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<tr>
<td></td>
<td>Mismatch</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>C. Rigid pricing</td>
<td>Aggregate demand</td>
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<td>+</td>
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<td></td>
<td>Technology</td>
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<tr>
<td></td>
<td>Mismatch</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Notes: The comparative statics are derived in Section 4.4. An increase in aggregate demand results from an increase in endowment, $\mu$, or an increase in taste for services, $\chi$.

Employment is lowered because firms may not be able to sell all their production, or equivalently, to utilize their labor fully. The model also captures classical unemployment through $w > a \cdot \alpha \cdot f^\alpha$. Employment is lowered because firms may pay a real wage that is above the marginal product of labor of the last worker in the labor force.

In fact, we can decompose equilibrium unemployment in three categories. This decomposition is useful because it allows us to distinguish between various models of unemployment in the literature. It also allows us to assess the potential effectiveness of various macroeconomic policies.

Equilibrium employment is

$$ l^A = \left( \frac{f(x) \cdot a \cdot \alpha}{w} \right)^{\frac{1}{1-a}} \left[ \frac{1}{1 + \bar{\tau}(\theta)} \right]^{\frac{\alpha}{1-a}}. $$
Employment without recruiting cost is

$$ l^B = \min \left\{ 1, \left[ \frac{f(x) \cdot a \cdot \alpha}{w} \right]^{\frac{1}{\alpha}} \right\} $$

Employment without recruiting cost and with a competitive product market is

$$ l^C = \min \left\{ 1, \left[ \frac{a \cdot \alpha}{w} \right]^{\frac{1}{\alpha}} \right\} $$

Hence, we can define frictional unemployment as $u^F = l^B - l^A > 0$, Keynesian unemployment as $u^K = l^C - l^B \geq 0$, and classical unemployment as $u^C = 1 - l^C \geq 0$. In standard matching model of the labor market, as explained in Michaillat [2012], $u^F > 0$ but $u^K = u^C = 0$. In the model of Michaillat [2012], jobs are rationed because wages are too high, so $u^F > 0$ and $u^C > 0$ but $u^K = 0$. In this model, we add Keynesian unemployment: $u^F > 0$ and $u^C > 0$ and $u^K > 0$. These components do not have direct welfare implications but they are useful to understand the mechanics of the model.\(^{27}\)

### 5 Dynamic Model

In this section we embed the static model into a dynamic environment. This extension allows to improve the mapping between theoretical and empirical quantities. At the limit without discounting, the comparative steady states in the dynamic model are exactly the same as the comparative statics in the static model.

For tractability, we work in continuous time. The main difference between static and dynamic models is that in the dynamic model, buyers engage in long-term customer relationships with sellers and firms engage in long-term employment relationships with workers.\(^{28}\)

\(^{27}\)Earlier Keynesian models such as Barro and Grossman [1971] had Keynesian unemployment and classical unemployment depending on the regime but no frictional unemployment.

**Matching.** On the labor market at time $t$, there are $h$ workers in the labor force, $l(t)$ employed workers, and $u(t) = h - l(t)$ unemployed workers. Firms post $\hat{v}(t)$ vacancies and labor market tightness is $\theta(t) = \hat{v}(t)/u(t)$. Employed workers become unemployed at rate $\hat{s} > 0$. Unemployed workers find a job at rate $\hat{f}(\theta(t))$ and a vacancy is filled at rate $\hat{q}(\theta(t))$. The functions $\hat{f}$ and $\hat{q}$ have the same expression as in the static model. The number of new employment relationship at time $t$ is given by

$$\dot{l}(t) + \hat{s} \cdot l(t) = \hat{f}(\theta(t)) \cdot (h - l(t)) = \hat{q}(\theta(t)) \cdot \hat{v}(t).$$

On the product market at time $t$, firms have a capacity $k(t) = a \cdot n(t)^{\alpha}$ and output is $y(t) < k(t)$. Each firm and consumer who transact are engaged in a long-term customer relationship. The long-term relationship is supported by the positive surplus that arises because customers do not need to incur shopping cost and firms do not face uncertainty when selling to customers. The customer relationships are destroyed at rate $s$.

Consumers who are not in a customer relationship engage in $v(t)$ visits to find a new seller. The product market tightness is $x(t) = v(t)/(k(t) - y(t))$. The $k(t) - y(t)$ units of services available at time $t$ are sold at rate $f(x(t))$ and the $v(t)$ purchasing visits are successful at rate $q(x(t))$. The functions $f$ and $q$ have the same expression as in the static model. The number of new customer relationship at time $t$ is given by

$$\dot{y}(t) + s \cdot y(t) = f(x(t)) \cdot (k(t) - y(t)) = q(x(t)) \cdot v(t).$$

**Consumers.** The utility function of the representative consumer is given by

$$\int_{t=0}^{+\infty} e^{-\sigma t} \cdot \left[ \frac{X}{1+\chi} \cdot c(t)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{1+\chi} \cdot m(t)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{1}{\varepsilon-1}} dt, \quad (23)$$

where $\sigma > 0$ is the time discount factor, $c(t)$ is consumption at time $t$, and $m(t)$ is holding of nonproduced good at time $t$. Each visit costs $\rho$ units of good so purchases and consumption are
related by
\[ y(t) = c(t) + \frac{\rho}{q(x(t))} \cdot [y(t) + s \cdot y(t)]. \tag{24} \]

The nonproduced good does not depreciate and plays the role of an asset. The law of motion of this asset is given by
\[ m(t) = p(t) \cdot w(t) \cdot l(t) - p(t) \cdot y(t) + \Pi(t). \tag{25} \]

Firms’ nominal profits, \( \Pi(t) \), are rebated to the household. Given \( \{ p(t), w(t), x(t), l(t), \Pi(t) \} \), the household’s problem is to choose \( \{ y(t), c(t), m(t) \} \) to maximize (23) subject to (24) and (25). Setting up the Hamiltonian of this optimal control problem, we can describe the solution with differential equations.

**Firms.** Firms maximize the discounted stream of real profits
\[ \int_{t=0}^{+\infty} e^{-\sigma t} \cdot [y(t) - w(t) \cdot l(t)] \, dt. \tag{26} \]

Firms employ \( n(t) \) producers and \( l(t) - n(t) \) recruiters. Each recruiter handles \( 1/\hat{\rho} \) vacancy so the numbers of producers and recruiters are related by
\[ l(t) = n(t) + \frac{\hat{\rho}}{\hat{q}(\theta(t))} \cdot [l(t) + \hat{s} \cdot l(t)]. \tag{27} \]

The firm sells output \( y(t) \) to customers. The amount of sales depend on product market tightness and the production of the firm:
\[ \dot{y}(t) = f(x(t)) \cdot (a \cdot n(t)^{\alpha} - y(t)) - s \cdot y(t). \tag{28} \]

Given \( \{ w(t), x(t), \theta(t) \} \), the firm’s problem is to choose \( \{ l(t), n(t), y(t) \} \) to maximize (26) subject to (27) and (28). Setting up the Hamiltonian of this optimal control problem, we can describe the solution with differential equations.
Steady-state equilibrium. The steady-state equilibrium is described by \( \{l, n, y, c, \theta, x\} \). These 6 variables satisfy the following 6 equations. Employment and output are related to tightnesses by

\[
\begin{align*}
l &= \frac{\hat{f}(\theta)}{\hat{s} + \hat{f}(\theta)} \cdot h, \\
y &= \frac{f(x)}{s + f(x)} \cdot a \cdot n^\alpha.
\end{align*}
\]

Employment of producers and consumption are related to employment and output by where

\[
\begin{align*}
n &= \left(1 - \hat{s} \cdot \frac{\hat{q}}{\hat{q}(\theta)}\right) \cdot l, \\
c &= \left(1 - s \cdot \frac{\rho}{q(x)}\right) \cdot y.
\end{align*}
\]

The optimal consumption decision of the household satisfies

\[
c = \left(\frac{\chi \cdot \sigma}{p}\right)^\epsilon \cdot \left[1 - (\sigma + s) \cdot \frac{\rho}{q(x)}\right]^\epsilon \cdot \mu.
\]

The optimal employment decision of the firm satisfies

\[
n = \left[\frac{a \cdot \alpha}{w} \cdot \frac{f(x)}{\sigma + s + f(x)} \cdot \left(1 - (\sigma + \hat{s}) \cdot \frac{\hat{\rho}}{\hat{q}(\theta)}\right)\right]^{-\frac{1}{\alpha}}.
\]

At the limit without discounting (\( \sigma \to 0 \)), the labor demand and supply curves and the aggregate demand and supply curves have exactly the same properties as in the static model. Therefore, the comparative steady states are exactly the same.

6 Empirical Evidence and Implications

In this section we construct new empirical series for labor utilization and recruiting costs. We combined these series with our model to re-examine the origin of employment fluctuations. We ask three questions: are price and real wage fully flexible? are employment fluctuations mostly driven by labor demand or labor supply shocks? and, are labor demand shocks mostly the consequence of aggregate demand or technology shocks?
6.1 Construction of a Labor Utilization Series

To assess the predictions of our model, we would like to estimate the response of labor utilization and other macroeconomic variables to aggregate demand and technology shocks. Since series for product market tightness, $x$, or labor utilization, $f(x)$, are not available, we first need to construct a series of labor utilization.

Our series of labor utilization is constructed starting from the available capacity utilization series. We use the measure of capacity utilization constructed by the Census Bureau from the Survey of Plant Capacity (SPC) from 1973 to 2013. Until 2006, the survey measures 4th-quarter capacity utilization rates. From 2008 to 2012, the survey measures the capacity utilization at quarterly frequency. To obtain a quarterly series for 1973–2007, we use a straight linear interpolation of the annual series into a quarterly series. We combine this interpolated series with the quarterly series for 2008–2012. Figure 12 plots the raw series of capacity utilization from the Census Bureau. It is clear that capacity utilization is subject to wide fluctuations over the business cycle.29

The concept of capacity utilization is based on capacity at full employment, whereas our con-

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29 A commonly used capacity utilization series is the monthly series constructed by the Federal Reserve Board (FRB) for the industrial sector since 1967. The series is constructed by combining monthly industrial production data with annual capacity data [Morin and Stevens, 2004]. Hence, there is a risk that fluctuations in capacity utilization are contaminated by fluctuations in industrial production, which would create a spurious correlation between output and utilization. For this reason, we prefer the series of the Census Bureau, even though it is available at higher frequency.
cept of labor utilization is based on capacity at current employment. We therefore correct the capacity utilization to obtain labor utilization. Let $g(a, n, \kappa) = a \cdot n^\alpha \cdot \kappa^{1-\alpha}$ be the production function of the representative firm under technology $a$, employment $n$, and capital stock $\kappa$. The capacity of the firm under current employment is $k = g(a, n, \kappa)$. The capacity of the firm under full employment is $K = g(a, N, \kappa)$, where $N$ is the level of full employment that respondents have in mind when they report capacity utilization, $cu$. Given the definition of capacity utilization in the SPC and the impact of labor utilization on output in the dynamic model, we can write

$$y = cu \cdot K = \frac{f}{s + f} \cdot k.$$ 

Manipulating these equation yields

$$\frac{f}{s + f} = cu \cdot \frac{g(a, N, \kappa)}{g(a, n, \kappa)} = cu \cdot \left( \frac{N}{n} \right)^\alpha.$$ 

This equation holds at any time $t$. We assume that $N$ is fixed over time. Taking log and first differences yields

$$\Delta \ln(f_t) = \left[ \frac{s + f}{s} \cdot [\Delta \ln(cu_t) - \alpha \cdot \Delta \ln(n_t)] \right].$$

To construct the labor utilization series, $\{f_t\}$, we set the average destruction rate of customer relationships to $s = 0.2$, the average labor utilization rate to $f = 0.8$, and the production function parameter to $\alpha = 0.66$. Since capacity utilization is measured for the industrial sector, we use employment in the industrial sector for $\{n_t\}$. Figure 80 plots our series of labor utilization for 1973–2013.

The capacity utilization series of the Census is the best series of capacity utilization that we found for the US, but it suffers from several limitations. One limitation is that it is only annual for many years. Another limitation is that capacity utilization is measured only for the manufacturing

30Morin and Stevens [2004] describe the capacity utilization as follows: “the capacity indexes are designed to embody the concept of sustainable practical capacity, defined as the greatest level of output each plant in a given industry can maintain within the framework of a realistic work schedule, taking account of normal downtime and assuming sufficient availability of inputs to operate machinery and equipment in place.”

31Employment in the industrial sector is the quarterly average of the seasonally adjusted monthly series for the number of employees in all goods-producing industries constructed by the BLS Current Employment Statistics (CES) program.
sector and not for the service sector. Figure 14 shows that this assumption is not unrealistic. For the 1999–2012 period, the Institute for Supply Management (ISM) constructed a series of capacity utilization for manufacturing and a comparable series for services. Figure 14 plots the two series. The two series have different levels, but once detrended, their cyclical fluctuations are highly correlated: the correlation between detrended capacity utilizations in services and manufacturing is 0.78. Figure 14 also plots the capacity utilization from the Census Bureau’s Survey of Plant Capacity for reference.

6.2 Construction of a Recruiting Wedge Series

To measure how the number of workers allocated to recruiting fluctuates over the business cycle, we study the fluctuations of the size of the recruiting industry. What we call the recruiting industry is the industry with North American Industry Classification System (NAICS) code 56131. The industry’s official name is “employment placement agencies and executive search services”. It is part of the super sector “professional and business services” (code 60) and of the sector “administrative and support and waste management and remediation service”. According to the official NAICS definition, this industry comprises establishments primarily engaged in one of the following: (1) listing employment vacancies and referring or placing applicants for employment; or (2) providing executive search, recruitment, and placement services. This industry includes employment
agencies, executive placement agencies, and executive search agencies. This industry does not include firms that provide personnel management services, human resource services, or temporary help services.

We concentrate on the number of employees in the industry. It is a seasonally adjusted monthly series computed by the BLS from the Current Employment Statistics (CES) survey. The series is available over the period 1990:M1–2013:M6. This industry is composed of 279,800 workers on average over the period. Employment placement agencies (NAICS code 561311) compose the majority of the industry (253,200 workers on average). Executive search services (NAICS code 561312) are only a small fraction of the industry (26,600 workers on average).

Another, more comprehensive, source of information for the recruiting costs faced by employers is the National Employer Survey (NES) conducted in 1997 by the Census Bureau. The goal of the survey was to gather employer data on employment practices, especially recruiting. 4,500 establishments took part in the 1997 survey and answered detailed questions about the methods

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32 Another approach would be to study the number of workers whose occupation is recruiting. The number of workers in an occupation across industries is computed by the BLS as part of the Occupational Employment Statistics (OES) survey. However, this number is available at most once a year, so it does not allow to study the business cycle properties of the number of workers in the occupation. The 2010 Standard Occupational Classification (SOC) code is 13-1071. The occupation’s official name is “human resources specialists”. According to the official SOC definition, the occupation includes workers who perform activities in the human resource area. It includes employment specialists who screen, recruit, interview, and place workers, but excludes compensation specialists (SOC code 13-1141), training specialists (SOC code 13-1151), or labor relations specialists (SOC code 13-1075). The number of workers with this occupation in May 2012 is 394,380. These workers are distributed evenly across industries.
used to recruit applicants and select candidates. Villena Roldan [2010] analyses this survey and finds that firms spend 2.5% of their total labor cost in recruiting activities. In other words, 2.5% of the workforce is devoted to recruiting.33

6.3 Other Observable Variables

All data are for the US. The sample period is 1974:Q1–2013:Q2, which is the longest period for which we could construct a series for labor utilization. All data are seasonally adjusted. All labor-force data are for persons 16 years of age and older.

We measure output with quarterly real gross domestic product (GDP) constructed by the Bureau of Economic Analysis (BEA) as part of the National Income and Product Accounts (NIPA).

In the dynamic model, labor market tightness is the ratio of vacancy to unemployment. We measure unemployment with the quarterly average of the monthly unemployment level constructed by the Bureau of Labor Statistics (BLS) from the Current Population Survey (CPS). We measure vacancy with the quarterly average of the monthly vacancy index constructed by Barnichon [2010]. This index combines the online and print help-wanted indices of the Conference Board.34 We

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33In monetary terms in 1997, firms spent on average $4200 per recruited worker. See Cappelli [2001] for background information and a detailed description of the NES survey.
34The Conference Board indices are a standard measure of vacancy. Another common measure is the vacancy index constructed by the BLS from the Job Opening and Turnover Survey (JOLTS). We cannot use this other measure here.
divide vacancy by unemployment to obtain a quarterly series for labor market tightness.

We measure employment as the quarterly average of the monthly civilian employment level
constructed by the BLS from the CPS. We measure unemployment rate with the quarterly average
of the monthly unemployment rate constructed by the BLS from the CPS.

### 6.4 Rigid or Flexible Prices?

Before assessing the predictions of the model, we evaluate whether flexible prices or rigid prices
offer a better description of the data. As highlighted in Table 2, there is one main difference
between the two assumptions. With rigid prices, labor market tightness and labor utilization (or
equivalently, labor market tightness) fluctuate in response to demand and supply shocks. With
flexible prices—either efficient or bargained prices—labor market tightness and labor utilization
do not fluctuate in response to demand and supply shocks.

Figure 16 displays the cyclical fluctuation in labor market tightness and in the two measures
of labor utilization. To isolate fluctuations at business-cycle frequency, we take the difference be-
tween log of the series and a standard low-frequency trend—a Hodrick and Prescott [1997] filter
with smoothing parameter 1600. The figure confirms the well-known fact that labor market tight-
ness is subject to large fluctuations over the business cycle.\(^35\) The standard deviation of detrended
log labor market tightness is roughly 25%. The figure also shows that the cyclical fluctuations of
labor utilization have smaller magnitude to that of labor market tightness. The standard deviation
of detrended log labor utilization is 14%.

The cyclical fluctuation in labor market tightness and labor utilization that we measure are
large. This result suggests that a model with rigid prices is more appropriate to describe the
business-cycle fluctuations of macroeconomic variables. In the empirical analysis that follows,
we test the predictions of the model with rigid prices (Panel A in Table 2). In the context of our
model, the result also suggests that business-cycle fluctuations are inefficient because efficiency
implies a constant labor market tightness and labor utilization.

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\(^{35}\)See for instance Blanchard and Diamond [1989] and Shimer [2005].
6.5 Labor Demand or Labor Supply Shocks?

Labor supply shocks are the only shock that generates a negative correlation between recruiting wedge and employment. Labor demand shocks produce a positive correlation. In Figure 17(a), we plot together the cyclical fluctuations in recruiting wedge and employment. Once again, to isolate fluctuations at business-cycle frequency, we take the difference between log of the series and its trend, produced by a HP filter with smoothing parameter $\lambda = 1600$. Fluctuations in recruiting wedge and employment appear strongly positively correlated.

Figure 17(b) formalizes this observation. It displays the cross-correlogram of recruiting wedge and employment. The recruiting wedge leads employment by one lag. At one lag, the correlation of recruiting wedge and employment is large, about 0.8. The instantaneous correlation is broadly the same.

Our results that most fluctuations on the labor market are driven by labor demand shocks, and that mismatch shocks and participation shocks are unimportant, confirm the results from the classical analysis of Blanchard and Diamond [1989] with more recent data (their analysis stopped in 1989 whereas our analysis finishes in 2013) and alternative empirical series (their analysis focused on vacancies whereas we focus on the recruiting wedge).
6.6 Labor Demand Shocks: Aggregate Demand or Technology Shocks?

An aggregate demand shock is the only shock that generates a positive correlation between labor utilization and output. A technology shock produces a negative correlation. In Figure 19(a), we plot together the cyclical fluctuations in labor utilization and output. Once again, to isolate fluctuations at business-cycle frequency, we take the difference between log of the series and its trend, produced by a HP filter with smoothing parameter 1600. It does seem that fluctuations in labor utilization and output are positively correlated.

Figure 19(b) formalizes this observation. It displays the cross-correlogram of labor utilization and real output. Labor utilization leads output by one lag. At one lag, the correlation of labor utilization and output is quite large, about 0.6. The instantaneous correlation of labor utilization and output is about 0.5.

An implication of the positive correlation between labor utilization and real GDP is that aggregate demand shocks but not technology shocks seem to be an important source of cyclical fluctuations in labor demand and unemployment. Our result is consistent with that of Galí [1999], Shea [1998], and Basu, Fernald and Kimball [2006], among others. Unlike our work that focuses on the response of labor utilization, these papers focus on the response of hours worked to technology shocks. Using different strategies to identify technology shocks, they all show that positive technology shocks lead to declines in hours worked and increases in output. However, hours worked...
and output are positively correlated on average. These papers conclude from these observations that technology shocks are unlikely to be the source of business cycles.

The result that technology may not matter much at business-cycle frequency—even though measured productivity is procyclical—is not new. Although it is based on a different empirical methodology, our finding confirms the work of Basu [1996], who showed that cyclical fluctuations in measured productivity are driven mostly by variable utilization of labor and capital utilization and not by technology shocks or increasing returns to scale. Using a matching model of the product market and sophisticated estimation techniques, Bai, Rios-Rull and Storesletten [2012] reach the same conclusion that procyclical fluctuations in measured productivity are mostly driven by aggregate demand shocks and not technology shocks.

7 Conclusion

This paper proposes a simple and flexible model of aggregate demand and unemployment. The main feature of the model is that aggregate demand has an impact on unemployment through labor utilization within the firm.

The paper combines the predictions of the model with new empirical series to explore the sources of unemployment fluctuations. The empirical results support the view of employment
fluctuations driven by aggregate demand shocks that influence the labor demand in the presence of price and real-wage rigidities. While these results are consonant with existing results in the literature, such as the seminal results obtained by Blanchard and Diamond [1989] and Galí [1999], they are based on different empirical series and different correlations. And so far as we know, these results are the first to be obtained in an integrated treatment: while Blanchard and Diamond [1989] did separate between labor demand and labor supply shocks, the absence of frictions on the product market did not allow them to separate between different types of labor demand shocks (technology or aggregate demand); and while Galí [1999] did separate between technology or aggregate demand shocks, the absence of frictions on the labor market did not allow him to assess the importance of labor supply shocks (mismatch or moral hazard from UI).

The model could be used to analyse the impact on unemployment of policies that affect simultaneously aggregate demand, labor demand, and labor supply, as a broad range of policies do. The model could address the following questions: Should unemployment insurance be more generous in recessions to stimulate aggregate demand, or should it be less generous to incentivize jobseekers to search more? Should payroll tax shift from employees to employers in recessions to stimulate aggregate demand, or should it shift from employers to employees to reduce labor cost and stimulate hiring? Should the minimum wage rise in recessions to increase the income of poorer workers with a high marginal propensity to consume and thus stimulate aggregate demand, or should it fall
to stimulate hiring of low-income workers? Should income tax be more progressive in recessions to stimulate aggregate demand, or should it be more regressive to encourage work? Addressing some of these questions necessitates to extend the model to introduce heterogeneity among workers and across workers and firm owners, but we expect the extension to be fairly straightforward in a static framework.

References


Appendix

A Proofs

We start by proving a lemma that we use repeatedly in the proofs. The lemma characterizes the matching probabilities, defined by $q(x) = (1 + x^{-\eta})^{-\frac{1}{\eta}}$ and $f(x) = (1 + x^{-\eta})^{-\frac{1}{\eta}}$, the matching wedge, defined by $\tau(x) = \rho/(q(x) - \rho)$, and the tightnesses $x^m$ and $x^*$, defined by $f'(x^*) = \rho$ and $q(x^m) = \rho$.

**Lemma A1.** The functions $f$, $q$, and $\tau$ and the values $x^m$ and $x^*$ satisfy the following properties:

- $x^m = (\rho^{-\eta} - 1)^{\frac{1}{\eta}}$
- $f(x^m) = (1 - \rho^\eta)^{\frac{1}{\eta}}$.
- $\tau(x^m) = +\infty$ and $1/(1 + \tau(x^m)) = 0$.
- $x^* = \left(\rho^{-\frac{\eta}{1-\eta}} - 1\right)^{\frac{1}{\eta}}$
- $q(x^*) = \rho^{\frac{1}{1-\eta}}$.
- $f(x^*) = \left(1 - \rho^{\frac{\eta}{1-\eta}}\right)^{\frac{1}{\eta}}$.
- $\tau(x^*) = 1/(\rho^{-\frac{\eta}{1-\eta}} - 1)$ and $1/(1 + \tau(x^*)) = 1 - \rho^\eta$.
- $f(0) = 0$ and $\lim_{x \to +\infty} f(x) = 1$.
- $f$ is positive, smooth, and strictly increasing on $[0, +\infty)$.
- $q(0) = 1$ and $\lim_{x \to +\infty} q(x) = 0$.
- $q$ is positive, smooth, and strictly decreasing on $[0, +\infty)$.
- $\tau(0) = \rho/(1 - \rho)$ and $\lim_{x \to +\infty} \tau(x) = +\infty$.
- $\tau$ is positive, smooth, and strictly increasing on $[0, x^m]$.
- $f(x) = q(x) \cdot x$.
- $q'(x) = -q(x)^{1+\eta} \cdot x^{-\eta}$.
- $f'(x) = q(x)^{1+\eta}$.

**Proof.** The results follow from simple algebra. \qed
A.1 Proof of Proposition 2

In this section we propose an extension of Proposition 2 and prove it.

PROPOSITION A1. For any $p > 0$, there are two equilibria: $(x^m, 0)$ and $(x, c)$ where $x$ is implicitly defined by

$$
(1 + \tau(x))^{\varepsilon - 1} \cdot f(x) \cdot k = \chi^\varepsilon \cdot \frac{\mu}{p^\varepsilon}
$$

and $c = c^\varepsilon(x)$. In particular, $x \in (0, x^m)$ and $c > 0$.

Proof. We are looking for a tightness $x \in [0, x^m]$ that satisfies $c^\varepsilon(x) = c^\varepsilon(x, p)$ or

$$
\frac{1}{1 + \tau(x)} \left[ f(x) \cdot k - \chi^\varepsilon \cdot \frac{\mu}{p^\varepsilon} \cdot \frac{1}{(1 + \tau(x))^{\varepsilon - 1}} \right] = 0.
$$

$1/(1 + \tau(x^m)) = 0$ by definition of $x^m$; hence, $(x^m, c^\varepsilon(x^m)) = (x^m, 0)$ is always an equilibrium. Furthermore, $1/(1 + \tau(x)) > 0$ for $x < x^m$ so any equilibrium tightness $x < x^m$ must satisfy (A1). Since $\varepsilon > 1$, Lemma A1 implies that $x \mapsto (1 + \tau(x))^{\varepsilon - 1} \cdot f(x)$ is strictly increasing and $(1 + \tau(0))^{\varepsilon - 1} \cdot f(0) = 0$ and $\lim_{x \to x^m} (1 + \tau(x))^{\varepsilon - 1} \cdot f(x) = +\infty$. Thus, there is a unique $x \in (0, x^m)$ that satisfies (A1). \hfill \square

A.2 Proof of Proposition 1

An efficient allocation $(x, c)$ is such that $c = c^\varepsilon(x)$ and $x$ maximizes $c^\varepsilon(x)$. Lemma 1 implies that $x = x^*$ and $c = c^\varepsilon$. There is a unique price $p^*$ that implements the efficient allocation. This price satisfies

$$
(p^*)^\varepsilon = \chi^\varepsilon \cdot \frac{\mu}{k} \cdot \frac{1}{f(x^*) \cdot (1 + \tau(x^*))^{\varepsilon - 1}}.
$$

The reason is that with $p = p^*$, $x^*$ satisfies (A1).

A.3 Proof of Proposition 5

In this section we propose an extension of Proposition 5 and prove it.

PROPOSITION A2. For any $p > 0$ and $w > 0$, there are three general equilibria: $(x, \theta, n, c)$ where $x < x^m$ and $\theta < \theta^m$ solve the system

$$
h^{1 - \alpha} \cdot \hat{f}(\theta)^{1 - \alpha} \cdot (1 + \hat{\tau}(\theta))^{\alpha} = f(x) \cdot \frac{\alpha \cdot \mu}{w}
$$

and $n = n^\varepsilon(\theta) > 0$ and $c = c^\varepsilon(x, n) > 0$; $(x^m, \theta^m, 0, 0)$; and $(x^m, \theta, 0, n)$ where $\theta < \theta^m$ solves (A2) for $x = x^m$ and $n = n^\varepsilon(\theta) > 0$. 59
Proof. In general equilibrium, \((x, \theta)\) satisfies the following system of two equations:

\[
n^d(\theta) = n^d(\theta, x, w) \tag{A4}
\]
\[
c^d(x, n^d(\theta)) = c^d(x, p). \tag{A5}
\]

**First case: \(\theta < \theta^m\) and \(x < x^m\).** 1/(1 + \(\tau(x)\)) \(\in (0, +\infty)\) and 1/(1 + \(\tilde{\tau}(\theta)\)) \(\in (0, +\infty)\) so we can rewrite the system (A4)–(A5) as (A2)–(A3). Equation (A4) is equivalent to

\[
\left(\frac{1}{1 + \tilde{\tau}(\theta)}\right)^{1-\alpha} \left[ f(\theta)^{1-\alpha} \cdot h^{1-\alpha} - \frac{a \cdot \alpha}{w} \cdot f(x) \cdot \frac{1}{(1 + \tilde{\tau}(\theta))^{\alpha}} \right] = 0. \tag{A6}
\]

Since the first factor is positive, this equation implies that the second factor must be zero. Multiplying the second factor by \((1 + \tilde{\tau}(\theta))^{\alpha}\) yields (A2). Following the proof of Proposition A1, we modify (A5) to obtain

\[
f(x) \cdot \alpha \cdot \left(\frac{f(\theta}{1 + \tilde{\tau}(\theta)} \cdot h\right)^{\alpha} \cdot (1 + \tau(x))^{\varepsilon - 1} = \chi^{\varepsilon} \cdot \frac{\mu}{\rho^{\varepsilon}}. \tag{A7}
\]

Multiplying both sides of the equation by \(\alpha/w\) and substituting (A2) into this equation yields (A3).

We show that for any \(p > 0\) and \(w > 0\), the system (A2)–(A3) admits a unique solution. Since \(\alpha < 1\), Lemma A1 implies that \(\theta \mapsto f(\theta)^{1-\alpha} \cdot (1 + \tilde{\tau}(\theta))^{\alpha}\) is strictly increasing from \(0\) to \(+\infty\) for \(\theta \in [0, \theta^m]\). Hence, equation (A2) implicitly defines \(\theta\) as a function of \(x \in [0, \theta^m]\). Lemma A1 shows that \(f\) is strictly increasing from 0 to 1 on \((0, +\infty)\); thus, \(\Theta^L\) is strictly increasing on \((0, +\infty), \Theta^L(0) = 0, \text{ and } \lim_{x \to +\infty} \Theta^L(x) = \theta^L > 0\) where \(\theta^L \in (0, \theta^m)\) is implicitly defined by

\[
h^{1-\alpha} \cdot f(\theta^L)^{1-\alpha} \cdot (1 + \tilde{\tau}(\theta^L))^{\alpha} = a \cdot \alpha/w.
\]

If \([\alpha/(w \cdot h)] \cdot \chi^\varepsilon \cdot (\mu/p^{\varepsilon}) \geq 1\), define \(x^p(p, w)\) by

\[
(1 + \tau(x^p))^{\varepsilon - 1} = \frac{\alpha}{w \cdot h} \chi^\varepsilon \cdot \frac{\mu}{p^{\varepsilon}}.
\]

If \([\alpha/(w \cdot h)] \cdot \chi^\varepsilon \cdot (\mu/p^{\varepsilon}) < 1, x^p(p, w) \equiv 0\). Since \(\varepsilon > 1\), Lemma A1 implies that \(x \mapsto (1 + \tau(x))^{\varepsilon - 1}\) is strictly increasing from 1 to \(+\infty\) for \(x \in [0, x^m]\); therefore, \(x^p\) is well defined and \(x^p(p, w) \in (0, x^m)\). Lemma A1 shows that \(f\) is strictly increasing from 0 to 1 on \((0, +\infty), \text{ which implies that equation (A3) implicitly defines } \theta \text{ as a function of } x \in (x^p(p, w), x^m): \theta = \Theta^P(x)\). Moreover, \(\Theta^P\) is strictly decreasing on \((x^p(p, w), x^m), \lim_{x \to x^p(p, w)} \Theta^P(x) = +\infty, \text{ and } \lim_{x \to x^m} \Theta^P(x) = 0\).

The system (A2)–(A3) is equivalent to

\[
\begin{align*}
\Theta^L(x) &= \Theta^P(x) \\
\theta &= \Theta^P(x)
\end{align*}
\]

Given the properties of the functions \(\Theta^L\) and \(\Theta^P\), we conclude that this system admits a unique solution \((x, \theta)\) with \(x \in (x^p(p, w), x^m)\) and \(\theta \in (0, \theta^L)\).

**Second case: \(x = x^m\).** \(c^d(x^m, n^d(\theta)) = 0 = c^d(x^m, p)\) so (A5) is necessarily satisfied. When (A4) is rewritten as (A6), it is clear that it admits exactly two solutions for \(x = x^m: \theta^m\) and \(\theta^L(x^m) < \theta^m\).
To summarize, there are exactly two general equilibria when $x = x^m$: $(x^m, \theta^m, 0, 0)$ and $(x^m, \theta, 0, n)$ where $\theta < \theta^m$ solves (A2) for $x = x^m$ and $n = n^i(\theta) > 0$.

**Third case:** $\theta = \theta^m$. $n^i(\theta^m) = 0 = n^d(\theta^m, x, w)$ so (A4) is necessarily satisfied. Then, $n^i(\theta^m) = 0$ so (A5) becomes $c^i(x, 0) = c^d(x, p)$. Since $c^i(x, 0) = 0$, $x$ solves $c^d(x, p) = 0$, which imposes $x = x^m$. Thus, we are back to the second case. 

\[ A.4 \text{ Proof of Proposition 3} \]

An efficient allocation $(x, \theta, c, n)$ is such that $n = n^i(\theta)$ and $\theta$ maximizes $n^i(\theta)$. Lemma 2 implies that $\theta = \theta^*$. An efficient allocation is also such that $c = c^i(x, n^*)$ and $x$ maximizes $c^i(x, n^*)$. Lemma 1 implies that $x = x^s$ and $c = c^* = (f(x^s) - \rho \cdot x^s) \cdot n^s)^\alpha$.

There exists a unique pair $(p^*, w^*)$ that implements the efficient allocation. The pair satisfies

\[
w^* = a \cdot \alpha \cdot f(x^s) \cdot h^{\alpha - 1} \cdot \hat{f}(\theta^*)^{\alpha - 1} \cdot (1 + \hat{\tau}(\theta^*))^{-\alpha}
\]

\[
(p^*)^\varepsilon = \frac{\alpha}{w^*} \cdot \chi^\varepsilon \cdot \mu \cdot \frac{1}{h \cdot \hat{f}(\theta^*) \cdot (1 + \tau(x^s))^{1 - \varepsilon}}.
\]

The reason is that with $p = p^*$ and $w = w^*$, $(x^s, \theta^*)$ satisfies the system (A2)-(A3). We can simplify the expression for the efficient prices using the facts that $n^* = h \cdot \hat{f}(\theta^*) / (1 + \hat{\tau}(\theta^*))$ and $c^* = f(x^s) \cdot k^s / (1 + \tau(x^s))$, where $k^s = a \cdot (n^s)^\alpha$ is efficient capacity.

\[ A.5 \text{ Proof of Proposition 4} \]

We build on the proof of Proposition 5 and use the same notations. The function $\Theta^L : [0, +\infty) \times (0, +\infty) \rightarrow (0, +\infty)$ is defined such that $\theta = \Theta^L(x, w)$ solves (A2). The function $\Theta^L$ is strictly increasing in $x$ and strictly decreasing in $w$. The function $\Theta^d : \{(x, p, w) | p > 0, w > 0, x^d(p, w) < x < x^m \} \rightarrow (0, +\infty)$ is defined such that $\theta = \Theta^d(x, p, w)$ solves (A3). The function $\Theta^d$ is strictly decreasing in $x$, strictly decreasing in $p$, and strictly decreasing in $w$. The first part of the proof is illustrated in Figure 1(a). The second part of the proof is illustrated in Figure 1(b).

**First part:** condition such that $\theta < \theta^*$. Let $w^L$ be defined by $\Theta^L(x^m, w^L) = \theta^*$. For all $w > w^L$ and for all $x \in [0, x^m]$, $\Theta^L(x, w) < \theta^*$. For all $w \leq w^L$, there exists a unique $x \in [0, x^m]$ such that $\Theta^L(x, w) = \theta^*$. We implicitly define the function $x^L : (0, w^L] \rightarrow [0, x^m]$ by $\Theta^L(x^L(w), w) = \theta^*$. In particular, $\lim_{w \rightarrow 0} x^L(w) = 0$, $x^L(w^*) = x^*$ and $x^L(w^L) = x^m$ and $x^L$ is strictly increasing. We define the function $p^\theta : (0, w^L) \rightarrow (0, +\infty)$ by

\[
p^\theta(w) = \chi \cdot \left[ \frac{(1 + \tau(x^L(w)))^{1 - \varepsilon}}{h \cdot \hat{f}(\theta^*)} \cdot \frac{\alpha \cdot \mu}{w} \right]^{\frac{1}{\varepsilon}}.
\]

The function $p^\theta$ is strictly decreasing from $+\infty$ to 0 for $w \in (0, w^L)$ and $p^\theta(w^*) = p^*$. By definition, $\Theta^d(x^L(w), p^\theta(w), w) = \theta^*$. Let $\theta$ denote equilibrium labor market tightness and $x$ denote equilibrium product market tightness. For any $w > w^L$, $\theta < \theta^*$ because $\Theta^d(x, w) < \theta^*$ for all $x \in [0, x^m]$ and because $\theta =
Figure A1: Illustration of the proof of Proposition 4
\(\Theta^L(x, w)\). Consider \(w \leq w^L\). For any \(p > p^\theta(w)\), \(\Theta^P(x^L(w), p, w) < \Theta^P(x^L(w), p^\theta(w), w) = \theta^* = \Theta^L(x^L(w), w)\). Given that \(\Theta^L\) is strictly increasing in \(x\) and \(\Theta^P\) is strictly decreasing in \(x\) and \(\Theta^P\) cross only once in a \((x, \theta)\) plane, we conclude that \(x < x^L(w)\) for any \(p > p^\theta(w)\). Thus, \(\theta = \Theta^L(x, w) < \Theta^L(x^L(w), w) = \theta^*\) for any \(p > p^\theta(w)\). To simplify the exposition, we extend the definition of \(p^\theta\) by \(p^\theta(w) = 0\) for all \(w \geq w^L\). To summarize, \(\theta < \theta^*\) if and only if \(p > p^\theta(w)\) for any \(w > 0\).

**Second part: condition such that \(x < x^*\).** We define the function \(p^x : (0, +\infty) \rightarrow (0, +\infty)\) by

\[
p^x(w) = \chi \cdot \left[ \frac{(1 + \tau(x^*))^{1-\varepsilon}}{h \cdot f(\Theta^L(x^*, w))} \cdot \frac{\alpha \cdot \mu}{w} \right]^{\frac{1}{2}}.
\]

By definition, \(\Theta^P(x^*, p^x(w), w) = \Theta^L(x^*, w)\); thus, \(x^*\) is the equilibrium product market tightness when the value of the real wage is \(w\) and the value of the price is \(p^x(w)\). We define the auxiliary function \(Z : (0, +\infty) \rightarrow (0, +\infty)\) by

\[
Z(w) = f(x^*) \cdot a \cdot \alpha \cdot n^x(\Theta^L(x^*, w)).
\]

Given that \(\Theta^L(x^*, w^*) = \theta^*\) and \(\Theta^L\) is strictly increasing in \(w\), \(\Theta^L(x^*, w) > \theta^*\) if and only if \(w > w^*\). Using Lemma 2 and the fact that \(\Theta^L\) is strictly increasing in \(w\), we infer that \(Z\) is strictly increasing for \(w \in (0, w^*)\) and strictly decreasing for \(w \in (w^*, +\infty)\). Given the definition of \(\Theta^L\), we infer that

\[
Z(w) = h \cdot w \cdot \hat{f}(\Theta^L(x^*, w))
\]

and therefore that

\[
p^x(w) = \chi \cdot \left[ \frac{(1 + \tau(x^*))^{1-\varepsilon}}{Z(w)} \cdot \frac{\alpha \cdot \mu}{w} \right]^{\frac{1}{2}}.
\]

The properties of \(Z\) imply that the function \(p^x\) strictly decreasing for \(w \in (0, w^*)\) and strictly increasing for \(w \in (w^*, +\infty)\) and \(p^x(w^*) = p^*\).

Let \(\theta\) denote equilibrium labor market tightness and \(x\) denote equilibrium product market tightness. Consider \(w \in (0, +\infty)\). For any \(p > p^x(w)\), \(\Theta^P(x^*, p, w) < \Theta^P(x^*, p^x(w), w) = \Theta^L(x^*, w)\). Given that \(\Theta^L\) is strictly increasing in \(x\) and \(\Theta^P\) is strictly decreasing in \(x\) and \(\Theta^P\) and \(\Theta^L\) cross only once in a \((x, \theta)\) plane, we conclude that \(x < x^*\) for any \(p > p^x(w)\). To summarize, \(x < x^*\) if and only if \(p > p^x(w)\) for any \(w > 0\).