Virtual Coins: The Application of Loss Aversion in Marketing

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Abstract

This paper investigates the impact of cash-equivalent reward points ("coins") on sales profit. By developing a theoretical model, we demonstrate how coins enhance sales profitability by leveraging consumers' loss aversion. Taking consumer heterogeneity into account, we classify individuals into high-type and low-type segments to explore optimal strategies for setting both coin allocation and pricing. This paper indicates that a small amount of coins should be distributed to all consumers as a means of try to increase profits. When a higher proportion of users exhibit relatively high loss aversion, it is preferable to adopt a screening strategy.

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1 Introduction

Consumer psychology is one of the key focal points of contemporary marketing. Shiv and Fedorikhin (1999), through two experiments, reveals that when the availability of cognitive processing resources is not high enough and there is a conflict between affect and cognition in consumer decision-making, affect exerts a stronger influence on the final choice, suggesting that marketing should not underestimate non-rational psychological drivers. The loss aversion discussed in this paper is identified by Kahneman (2011) as a negative emotional response that activates a fast and emotion-driven system.

In the field of marketing, loss aversion can serve as an effective theoretical foundation. Kahneman and Tversky (1979) and Kahneman and Tversky (1984) provide a specific concept about loss aversion that losses result in greater changes in utility compared to gains of the same magnitude. Under this theory, consumers will never derive positive utility from receiving a coupon only to let it expire unused, which provides a foundational guarantee for the effectiveness of many voucher-like promotional strategies.

One strategy that leverages loss aversion is the loyalty program. In such programs, consumers earn loyalty points through their purchases or other interactions with the firm, which they can then redeem. At some firms, points can be redeemed for everyday items such as mugs or toilet paper, while others allow points to be used for purchasing their own products. Additionally, another some companies permit customers to use points to offset cash payments on individual purchases. Points and status often expire, which triggers a feeling of loss and may cause excessive purchases or other behaviors aimed at maintaining status or redeeming points.

In this paper, we focus on such points ("coins") which can be redeemed for cash discounts and are handed out by the firm as opposed to being earned by past transactions. A representative example is Taobao, one of the largest e-commerce platforms in China. It distributes coins to users, which can be redeemed for cash at a fixed exchange rate of 100 coins to 1 CNY on selected products. There are three primary channels to obtain these coins: (i) daily check-ins, (ii) completing consumption-related tasks, and (iii) engaging in mini-game-like interactions with friends within the app. Since the first and third methods do not require any spending and involve little time costs, these coins can be approximately regarded as being distributed for free.

Compared to traditional promotion methods, we believe that coins deserve more study, as they offer several advantages that are more aligned with modern marketing practices.

i) The coin strategy leverages loss aversion. Compared to direct discounts, coins temporarily become our possessions, and when we lose them, we experience displeasure.

ii) The coin strategy directly influences the actual transaction price instead of the posted price. Frequent discounts may undermine brand image by drawing attention to price reductions, whereas coins do not directly alter the product's listed price, making it less likely for consumers to perceive pricing issues.

In this paper, we primarily investigate the impact of coin distribution on firm profitability under the assumption that consumers exhibit loss aversion toward coins, and explore how firms should design their coin distribution strategies accordingly. In section 2, we present the basic setup of the model based on loss aversion and briefly explains the choice of the utility functions. In section 3, we build the model under the simplest conditions, providing a theoretical foundation for how coin distribution can enlarge profit. In section 4, we develop the pooling model as a precursor to the screening model, with its results serving as a basis for comparative analysis with the outcomes of the screening model. In section 5, we present the screening model, which forms the core of the paper. It explores how firms should design coin distribution and pricing strategies when consumers are categorized into high-type and low-type across three parameters.

1.1 Related Work

Breugelmans and Liu-Thompkins (2017) constructed a regression model based on data from a US-based convenience store collected between December 2005 and February 2008. In addition, they conducted a lab experiment designed to simulate a multi-store shopping scenario. The final conclusion indicates that, for consumers without clear brand preferences, a finite loyalty program expiration policy does have a positive effect on purchasing, providing empirical support for the mechanism studied in this paper.

With reference to related literature, the main contribution of this paper lies in its analytical perspective and methodological framework.

Existing research on promotions focus on the effectiveness of promotional strategies. Bawa and Shoemaker (1987) constructed a coupon proneness index and found that coupon-sensitive households exhibit consistent coupon usage behavior across different types of products. In other words, if we observe that an agent tends to use coupons on certain product, it is likely that they will be happy to use coupons for all products. Gopalakrishnan and Park (2020) analyzed clickstream data and search behavior and demonstrated that coupons can help websites improve their profit by increasing the consumers' intention of visiting and that high-value customers with higher value coupons are more likely to purchase. Liu et al. (2021) worked with a small-to-medium-sized retailer in China and suggested that in the retail industry, retailers should place greater emphasis on distributing order coupons, which are used to offset in a total order, instead of price discounts. The concept of coins we focus on aligns to some extent with the order coupons. Besides, Lim et al. (2024) discussed consumers' strategies for utilizing loyalty points in situations involving multiple options. They examined how consumers choose between using points or money for purchases and how this choice is influenced by their point-earning behavior. That the variation in perceived value of loyalty points leads to different consumption choices reinforces the feasibility of the analytical perspective adopted in this paper.

Langen and Huber (2023) used casual machine learning to estimate the casual effect of coupon. In contrast, this paper adopts a theoretical approach and constructs a screening model, which is a relatively novel approach in marketing.

2 Setup

A monopolist produces a single good with price P and zero marginal cost. The consumer action is denoted as $x \in \{0, 1\}$. If $x = 1$, it means that the consumer buys this product;

if $x = 0$, it means that she does not buy the product. To promote sales, the monopolist distributes $\delta_s \geq 0$ coins to the consumer. The coins can be used at checkout to deduct an equivalent amount of currency. If the consumer does not make a purchase, the coins will expire and can no longer be used.

We assume that the utility associated with money is linear. If the consumer purchases the product, her utility equals the value of the product, minus the actual price she pays and the disutility from any unused coins. If the consumer does not purchase the product, she incurs disutility from losing all the coins.

$$u(x) = \begin{cases} v - (P - \delta_d) - \lambda u_\delta(\delta_s - \delta_d) & x = 1, \\ -\lambda u_\delta(\delta_s) & x = 0, \end{cases}$$

where $\lambda > 0$ is the parameter representing the degree of loss aversion and δ_d is the number of coins used. Here, $\delta_s - \delta_d \geq 0$.

Because of hedonic adaptation, we define u_δ as

$$u_\delta(\delta) = \begin{cases} \delta & \text{if } \delta \leq \bar{\delta}, \\ \bar{\delta} & \text{if } \delta > \bar{\delta}. \end{cases}$$

Since the coins can be used to offset monetary payment on a one-to-one basis, the initial utility function is also assumed to be linear.

For the monopolist, the profit from a purchase is

$$\pi = P - \delta_d.$$

3 Basic Model

In this section, we analyze the model with a single consumer and derive the optimal strategy for the monopolist seller. We add heterogeneity in the later sections.

First, note that in the optimal contract, the consumer can spend all her coins if she makes a purchase, which means $\delta_s = \delta_d = \delta$.

In order to ensure the purchase, the participation constraint $u(1) \geq u(0)$ must hold,

$$v - (P - \delta) + \lambda u_\delta(\delta) \geq 0.$$

Then we get

$$P \leq v + \delta + \lambda u_\delta(\delta).$$

Since $\partial\pi/\partial P = 1$, we choose $P = v + \delta + \lambda u_\delta(\delta)$. Given this pricing rule, we have two scenarios: either the firm provides coins below the threshold $\bar{\delta}$, or above it.

If $\delta \leq \bar{\delta}$, then $u_\delta(\delta) = \delta$. We have

$$P = v + (1 + \lambda)\delta,$$

then

$$\pi = v + \lambda\delta.$$

Proposition 3.1. *If $0 < \delta \leq \bar{\delta}$, π is increasing in δ.*

Therefore, we set $\delta^* = \bar{\delta}$ and, consequently, $P^* = v + (1+\lambda)\bar{\delta}$. In this case, we have

$$\pi^* = v + \lambda\bar{\delta},$$

where $\partial\pi^*/\partial\lambda = \bar{\delta} > 0$.

If $\delta > \bar{\delta}$, then $u_\delta(\delta) = \bar{\delta}$. We have

$$P = v + \delta + \lambda\bar{\delta}.$$

Consequently,

$$\pi = v + \lambda\bar{\delta}.$$

Since π is independent of δ, one could, in theory, raise both P and δ without bound. However, consumers will not be willing to pay more actual money for the product beyond $v + \lambda\bar{\delta}$. From the above process, we can find that the excess profit originates from the disutility of losing the coins. The maximum disutility therefore caps the profit.

To conclude, $\forall(P,\delta) \in \{(P^*,\delta^*)\}$, we have

$$\pi^* = v + \lambda\bar{\delta}.$$

Proposition 3.2. *π^* is increasing in v, λ and $\bar{\delta}$.*

Here, λ can be interpreted as the intensity of loss aversion—the greater the value of λ, the stronger the agent's reaction to losses. The term $\bar{\delta}$ can be understood as the sensitivity to loss aversion—the larger the value of $\bar{\delta}$, the less likely the agent is to become desensitized to losses. Therefore, when we consider an individual to be more loss-averse, this can manifest in three ways: an increase in λ, an increase in $\bar{\delta}$, or an increase in both. Undoubtedly, in all these cases, the profitability of sales is enhanced.

Propositions 3.1 and 3.2 together establish that in the presence of loss aversion, issuing coins has a positive effect on sales profit. This naturally leads us to explore the optimal coin allocation strategies under different conditions.

4 Pooling

In reality, consumers value the good differently and exhibit varying degrees of loss aversion. To promote more effectively, the monopolist may wish to tailor strategies to different groups.

There are three ways to divide the consumers into two types (high and low): v_H and v_L, λ_H and λ_L, and $\bar{\delta}_H$ and $\bar{\delta}_L$. The high-type accounts for a proportion $p \in [0,1]$; the low-type, $1-p$.

Before screening, we would like to use pooling as a baseline strategy. Since we do not practice price discrimination, as shown in section 3, we still allow the consumers to use all their coins. The profit in the pooling problem is

$$\max_{P,\delta} \; p(P-\delta)\mathbb{I}\{u_H(1) \geq u_H(0)\} + (1-p)(P-\delta)\mathbb{I}\{u_L(1) \geq u_L(0)\},$$

where u_H denotes the utility of high-type consumers and u_L denotes the utility of low-type consumers.

Case 4.1. *(different v) The individual rationality constraints are*

$$\begin{cases} v_H - (P - \delta) \geq -\lambda u_\delta(\delta), \\ v_L - (P - \delta) \geq -\lambda u_\delta(\delta). \end{cases}$$

We need $\delta \geq \bar{\delta}$ for an optimal equilibrium. If $v_H \leq v_L/p + (\lambda/p - \lambda)\bar{\delta}$, the optimal profit would be

$$\pi^* = v_L + \lambda\bar{\delta}.$$

Otherwise,

$$\pi^* = p(v_H + \lambda\bar{\delta}).$$

Observe that loss aversion implies that the seller is more likely (i.e., $(\lambda/p - \lambda)\bar{\delta} \geq 0$) to serve the whole market since the profit from serving either type is increased by a constant. The coin strategy therefore makes it more likely that the whole market is served.

Case 4.2. *(different λ) The individual rationality constraints are*

$$\begin{cases} v - (P - \delta) \geq -\lambda_H u_\delta(\delta), \\ v - (P - \delta) \geq -\lambda_L u_\delta(\delta). \end{cases}$$

We still need $\delta \geq \bar{\delta}$. If $\lambda_H \leq \lambda_L/p + (v/p - v)/\bar{\delta}$, the optimal profit would be

$$\pi^* = v + \lambda_L\bar{\delta}.$$

Otherwise,

$$\pi^* = p(v + \lambda_H\bar{\delta}).$$

Case 4.3. *(different $\bar{\delta}$) The individual rationality constraints are*

$$\begin{cases} v - (P - \delta) \geq -\lambda u_{\delta_H}(\delta), \\ v - (P - \delta) \geq -\lambda u_{\delta_L}(\delta). \end{cases}$$

If we care about all the consumers, we require $\delta \geq \bar{\delta}_L$ and

$$\pi^* = v + \lambda\bar{\delta}_L.$$

If we only care about the high-type consumers, we require $\delta \geq \bar{\delta}_H$ and

$$\pi^* = p(v + \lambda\bar{\delta}_H).$$

If $\bar{\delta}_H \geq \bar{\delta}_L/p + (v/p - v)\lambda$, we adopt the latter strategy.

In the cases with heterogeneous loss aversion, whether through λ or $\bar{\delta}$, by contrast, there is consumer heterogeneity only because of the usage of a coin strategy. Hence, the usage of a coin strategy might cause the monopolist to restrict supply and serve only the more profitable part of the market.

4.1 Comment

The application of pooling typically implies a baseline distribution of coins. As mentioned in the introduction, on platforms like Taobao, every user can receive an equal, limited amount of coins simply by logging into the app, checking in, and interacting with friends, without the need for any purchase. These coins can be used for purchases immediately. According to the results of the basic model and the pooling analysis, distributing a small number of coins to all users consistently generates a positive effect on sales.

5 Screening

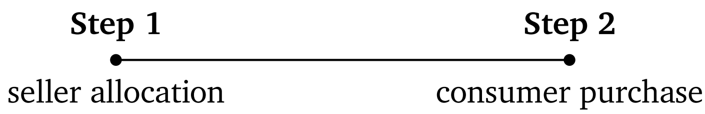
Now, the profit in the screening problem we want to optimize is

$$\max_{\{(P_k,\delta_k)\}_{k\in\{H,L\}}} p(P_H - \delta_H) + (1-p)(P_L - \delta_L).$$

We consider two ways to implement screening.

i) The first way is to adjust δ_s in Step 1. In this way, we allocate δ_{s_H} to the high-type and δ_{s_L} to the low-type. The remaining quantities of coins for the respective consumer types are represented by $\Delta\delta_{s_H} \geq 0$ and $\Delta\delta_{s_L} \geq 0$, which means that they are allowed to use $\delta_{s_H} - \Delta\delta_{s_H}$ and $\delta_{s_L} - \Delta\delta_{s_L}$ coins in Step 2.

ii) The second way is to adjust δ_d in Step 2. In this way, we allocate the same δ_s coins in Step 1, which means $\delta_{s_H} = \delta_{s_L} = \delta$, $\delta_{d_H} = \delta_H$ and $\delta_{d_L} = \delta_L$.



Proposition 5.1. *In the optimal screening equilibrium, it is impossible that* $\Delta\delta_{s_H} = \Delta\delta_{s_L} = 0$.

Proof. That $\Delta\delta_{s_H} = \Delta\delta_{s_L} = 0$ can be reflected in the incentive compatibility constraints as

$$\begin{cases} v - (P_H - \delta_H) \geq v - (P_L - \delta_L), \\ v - (P_L - \delta_L) \geq v - (P_H - \delta_H). \end{cases}$$

If both contracts spend all their coins, the two IC constrains read regardless of any differences in v. Combining both constraints, the profit is the same in both contracts, and it is without loss of generality to assume pooling. □

Assumption 1. *(sufficient allocation)* $\delta \geq \max\left\{\bar{\delta}\right\}$.

The only part can be decided by δ_s is still $u(0)$, which is the alternative option utility. The larger δ_s is, the larger domain is in participation constraints. There is no harm to have a δ_s as large as possible, so we make $\delta \geq \max\left\{\bar{\delta}\right\}$. In the following content, we think that assumption 1 holds by default.

Proposition 5.2. *The optimal solution obtained through the first method under assumption 1 can always be accomplished by the second method.*

Proof. We assume an acceptable set of solutions is $\left\{(\delta^*_{s_H}, \Delta\delta^*_{s_H}, P^*_H), (\delta^*_{s_L}, \Delta\delta^*_{s_L}, P^*_L)\right\}$.

We let

$$\Delta\delta_{s,inter} = \delta^*_{s_H} - \delta^*_{s_L}.$$

Then, we can always make ${\delta^*_{s_L}}' = \Delta\delta_{s,inter} + \delta^*_{s_L} = \delta^*_{s_H}$, $P^{*\prime}_L = P^*_L + \Delta\delta_{s,inter}$ and keep $\Delta\delta^*_{s_L} = \Delta\delta^*_{s_L}$. In this adjustment, the actual payment price remains unchanged at $P^*_L - \delta^*_{s_L} + \Delta\delta^*_{s_L}$, and the number of coins unused also remains at $\Delta\delta^*_{s_L}$. This implies that both the IR and IC conditions are unaffected, which means that $(\delta^*_{s_L}, \Delta\delta^*_{s_L}, P^*_L)$ is equivalent to $({\delta^*_{s_L}}', \Delta\delta^*_{s_L}, P^{*\prime}_L)$. We have now successfully expressed a solution obtained through the first method in the form of the second method. □

Based on proposition 5.2, we adopt the second method because it is easier to implement and requires less information.

5.1 Screening with different v

As we use the second method here, the participation constraints are

$$v_H - P_H + \delta_H - \lambda u_\delta(\delta - \delta_H) \geq -\lambda u_\delta(\delta), \tag{IR$_H$}$$

$$v_L - P_L + \delta_L - \lambda u_\delta(\delta - \delta_L) \geq -\lambda u_\delta(\delta), \tag{IR$_L$}$$

and the incentive compatibility constraints are

$$v_H - (P_H - \delta_H) - \lambda u_\delta(\delta - \delta_H) \geq v_H - (P_L - \delta_L) - \lambda u_\delta(\delta - \delta_L), \tag{IC$_H$}$$

$$v_L - (P_L - \delta_L) - \lambda u_\delta(\delta - \delta_L) \geq v_L - (P_H - \delta_H) - \lambda u_\delta(\delta - \delta_H). \tag{IC$_L$}$$

By combining eq. (IC$_H$) and eq. (IC$_L$), we obtain

$$(P_H - \delta_H) - (P_L - \delta_L) = \lambda \left[u_\delta(\delta - \delta_L) - u_\delta(\delta - \delta_H)\right].$$

Then, replacing $P - \delta$ with π, we obtain

$$\begin{cases} \pi_H \leq v_H + \lambda u_\delta(\delta) - \lambda u_\delta(\delta - \delta_H), \\ \pi_L \leq v_L + \lambda u_\delta(\delta) - \lambda u_\delta(\delta - \delta_L), \\ \pi_H - \pi_L = \lambda \left[u_\delta(\delta - \delta_L) - u_\delta(\delta - \delta_H)\right]. \end{cases}$$

Combine the first and third equations, we have

$$\pi_L \leq v_H + \lambda u_\delta(\delta) - \lambda u_\delta(\delta - \delta_L).$$

However,

$$v_H + \lambda u_\delta(\delta) - \lambda u_\delta(\delta - \delta_L) > v_L + \lambda u_\delta(\delta) - \lambda u_\delta(\delta - \delta_L).$$

So the true upper bound is

$$\pi_L = v_L + \lambda u_\delta(\delta) - \lambda u_\delta(\delta - \delta_L).$$

Then, according to the simplified IC constraint, we have

$$\pi_H = v_L + \lambda u_\delta(\delta) - \lambda u_\delta(\delta - \delta_H).$$

Substitute them into the profit function,

$$\pi = v_L + \lambda u_\delta(\delta) - \lambda p u_\delta(\delta - \delta_H) - \lambda(1-p)u_\delta(\delta - \delta_L).$$

Finally, we would like to have

$$u_\delta(\delta - \delta_H) = u_\delta(\delta - \delta_L) = 0,$$

which means $\delta_L^* = \delta_H^* = \delta$. In this case, the optimal strategy is

$$(P_H^*, \delta_H^*) = (P_L^*, \delta_L^*) = (v_L + \delta + \lambda\bar{\delta}, \delta),$$

and the best profit is

$$\pi_H^* = \pi_L^* = \pi^* = v_L + \lambda\bar{\delta}.$$

In this situation, we finally get a pooling strategy. Actually, we can foresee this result because the difference in v cannot prevent high-type consumers from deviating, just as what IC constraints show. Moreover, the existence of two IR constraints force us to choose the best low-type contract. Clearly, excluding low-type customers is also feasible, leading to a profit of $p(v_H + \lambda\bar{\delta})$ as in the pooling section.

5.2 Screening with different loss aversion

In this part, we finally consider different levels of loss aversion. As discussed above, loss aversion can be separated into two parts—λ and $\bar{\delta}$. In this section, we will state each part individually.

5.2.1 Different λ

We assume that the consumers differ only in terms of λ and they have the same $\bar{\delta}$. Since the monopolist distributes everyone $\delta \geq \bar{\delta}$ coins, the participation constraints are

$$v - P_H + \delta_H - \lambda_H u_\delta(\delta - \delta_H) \geq -\lambda_H u_\delta(\delta), \tag{IR$_H$}$$
$$v - P_L + \delta_L - \lambda_L u_\delta(\delta - \delta_L) \geq -\lambda_L u_\delta(\delta). \tag{IR$_L$}$$

For any agent who intends to deviate, the change in utility is determined by the change in the actual payment price, combined with the change in the loss resulting from the difference in the number of coins they are allowed to use. So the incentive compatibility constraints are

$$v - (P_H - \delta_H) - \lambda_H u_\delta(\delta - \delta_H) \geq v - (P_L - \delta_L) - \lambda_H u_\delta(\delta - \delta_L), \tag{IC$_H$}$$
$$v - (P_L - \delta_L) - \lambda_L u_\delta(\delta - \delta_L) \geq v - (P_H - \delta_H) - \lambda_L u_\delta(\delta - \delta_H). \tag{IC$_L$}$$

which can be rewritten in a clearer way as

$$(P_H - \delta_H) - (P_L - \delta_L) \leq \lambda_H \left[u_\delta(\delta - \delta_L) - u_\delta(\delta - \delta_H)\right],$$
$$(P_H - \delta_H) - (P_L - \delta_L) \geq \lambda_L \left[u_\delta(\delta - \delta_L) - u_\delta(\delta - \delta_H)\right].$$

Lemma 5.3. *If $\lambda_H > \lambda_L$, $\delta_L \leq \delta_H$.*

Proof. Based on eq. (IC_H) and eq. (IC_L), we have

$$
\begin{aligned}
\lambda_H \left[u_\delta(\delta - \delta_L) - u_\delta(\delta - \delta_H)\right] &\geq \lambda_L \left[u_\delta(\delta - \delta_L) - u_\delta(\delta - \delta_H)\right] \\
(\lambda_H - \lambda_L)\left[u_\delta(\delta - \delta_L) - u_\delta(\delta - \delta_H)\right] &\geq 0.
\end{aligned}
$$

For that $\lambda_H - \lambda_L > 0$,

$$
\begin{aligned}
u_\delta(\delta - \delta_L) - u_\delta(\delta - \delta_H) &\geq 0 \\
u_\delta(\delta - \delta_L) &\geq u_\delta(\delta - \delta_H) \\
\delta - \delta_L &\geq \delta - \delta_H \\
\delta_L &\leq \delta_H.
\end{aligned}
$$

□

Lemma 5.4. *If $\lambda_H > \lambda_L$, $P_H - \delta_H \geq P_L - \delta_L$.*

Proof. Substituting lemma 5.3 into eq. (IC_L), we have

$$
\begin{aligned}
v - (P_L - \delta_L) &\geq v - (P_H - \delta_H) + \lambda_L \left[u_\delta(\delta - \delta_L) - u_\delta(\delta - \delta_H)\right] \\
&\geq v - (P_H - \delta_H) \\
P_L - \delta_L &\leq P_H - \delta_H
\end{aligned}
$$

□

Because of lemma 5.3, without loss of generality, we let $\delta = \delta_H$. Now, the four constraints can be simplified as

$$
\begin{cases}
\pi_H \leq v + \lambda_H \bar{\delta}, \\
\pi_L \leq v + \lambda_L \bar{\delta} - \lambda_L u_\delta(\delta_H - \delta_L), \\
\pi_H \leq \pi_L + \lambda_H u_\delta(\delta_H - \delta_L), \\
\pi_H \geq \pi_L + \lambda_L u_\delta(\delta_H - \delta_L).
\end{cases}
$$

where we also replace $P - \delta$ with π.

Proposition 5.5. *When only λ differs, constraint IR_L binds.*

Proof. Since $0 \leq u_\delta(\delta_H - \delta_L) \leq \bar{\delta}$, we always have

$$
v + \lambda_H \bar{\delta} \geq \pi_L + \lambda_H u_\delta(\delta_H - \delta_L)
$$

If $\pi_L^* < v + \lambda_L \bar{\delta} - \lambda_L u_\delta(\delta_H - \delta_L)$, we have $\pi_H^* = \pi_L^* + \lambda_H u_\delta(\delta_H - \delta_L)$. If there is a positive value ε which still satisfies that $\pi_L^* + \varepsilon := \pi_L^{**} \leq v + \lambda_L \bar{\delta} - \lambda_L u_\delta(\delta_H - \delta_L)$. Then we can also have $\pi_H^{**} = \pi_L^{**} + \lambda_H u_\delta(\delta_H - \delta_L)$. Then,

$$
\begin{aligned}
\pi^{**} &= p\pi_H^{**} + (1-p)\pi_L^{**} \\
&= p(\pi_H^* + \varepsilon) + (1-p)(\pi_L^* + \varepsilon) \\
&= p\pi_H^* + (1-p)\pi_L^* + \varepsilon \\
&> \pi^*.
\end{aligned}
$$

Finally, the eq. (IR_L) must bind. □

Because of proposition 5.5, we get

$$\pi_L^* = v + \lambda_L\bar{\delta} - \lambda_L u_\delta(\delta_H - \delta_L),\ \pi_H^* = v + \lambda_L\bar{\delta} + (\lambda_H - \lambda_L)u_\delta(\delta_H - \delta_L).$$

Then

$$\pi^* = v + \lambda_L\bar{\delta} + (p\lambda_H - \lambda_L)u_\delta(\delta_H - \delta_L).$$

If $\lambda_H \leq \lambda_L/p$, we let $\delta_H = \delta_L$, then

$$(P_H^*, \delta_H^*) = (P_L^*, \delta_L^*) = (v + \delta + \lambda_L\bar{\delta}, \delta)$$

where $\delta \geq \bar{\delta}$ and $\pi_H^* = \pi_L^* = v + \lambda_L\bar{\delta}$. It is a pooling strategy.

If $\lambda_H \geq \lambda_L/p$, we let $\delta_H - \delta_L \geq \bar{\delta}$, then

$$(P_H^*, \delta_H^*) = (v + \lambda_H\bar{\delta} + \delta, \delta),\ (P_L^*, \delta_L^*) = (v + \delta - \bar{\delta}, \delta - \bar{\delta}),$$

and we have

$$\pi_L^* = v,\ \pi_H^* = v + \lambda_H\bar{\delta},\ \pi^* = v + p\lambda_H\bar{\delta}.$$

5.2.2 Different $\bar{\delta}$

Here we assume that the consumers differ only in terms of $\bar{\delta}$ and they have the same λ. The monopolist always distributes enough coins and the four constraints are

$$v - P_H + \delta_H - \lambda u_{\delta_H}(\delta - \delta_H) \geq -\lambda u_{\delta_H}(\delta), \qquad (\text{IR}_H)$$
$$v - P_L + \delta_L - \lambda u_{\delta_L}(\delta - \delta_L) \geq -\lambda u_{\delta_L}(\delta), \qquad (\text{IR}_L)$$

and

$$v - (P_H - \delta_H) - \lambda u_{\delta_H}(\delta - \delta_H) \geq v - (P_L - \delta_L) - \lambda u_{\delta_H}(\delta - \delta_L), \qquad (\text{IC}_H)$$
$$v - (P_L - \delta_L) - \lambda u_{\delta_L}(\delta - \delta_L) \geq v - (P_H - \delta_H) - \lambda u_{\delta_L}(\delta - \delta_H). \qquad (\text{IC}_L)$$

Combining eq. (IC_H) and eq. (IC_L), we get

$$u_{\delta_H}(\delta - \delta_L) - u_{\delta_H}(\delta - \delta_H) \geq u_{\delta_L}(\delta - \delta_L) - u_{\delta_L}(\delta - \delta_H)$$
$$\Rightarrow u_{\delta_H}(\delta - \delta_L) - u_{\delta_L}(\delta - \delta_L) \geq u_{\delta_H}(\delta - \delta_H) - u_{\delta_L}(\delta - \delta_H)$$

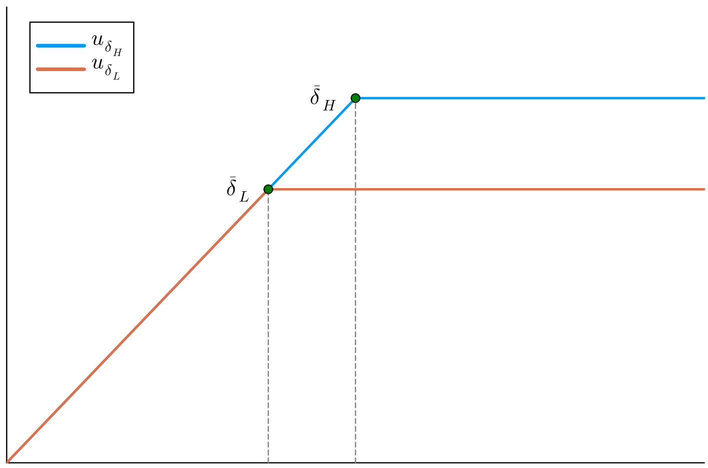


Figure 1: Diagram of u_{δ_H} and u_{δ_L}

Based on fig. 1, we conduct a case-by-case analysis by categorizing into three scenarios: $\delta - \delta_L \le \bar{\delta}_L$, $\bar{\delta}_L \le \delta - \delta_L \le \bar{\delta}_H$ and $\bar{\delta}_H \le \delta - \delta_L$.

Case 5.1. *$\delta - \delta_L \le \bar{\delta}_L$. Then, $\delta - \delta_H \le \bar{\delta}_L$. Moreover, $u_{\delta_H}(\delta - \delta_L) = u_{\delta_L}(\delta - \delta_L) = \delta - \delta_L$ and $u_{\delta_H}(\delta - \delta_H) = u_{\delta_L}(\delta - \delta_H) = \delta - \delta_H$.*

Under case 5.1, from the IC constraints, we still take the trick, replacing $P - \delta$ with π, and get

$$\pi_H = \pi_L + \lambda(\delta_H - \delta_L).$$

Then, from eq. (IR_H), we have

$$\begin{aligned} \pi_L &\le v + \lambda u_{\delta_H}(\delta) - \lambda u_{\delta_H}(\delta - \delta_H) - \lambda(\delta_H - \delta_L) \\ &= v + \lambda\bar{\delta}_H - \lambda(\delta - \delta_H) - \lambda(\delta_H - \delta_L) \\ &= v + \lambda\bar{\delta}_H - \lambda(\delta - \delta_L). \end{aligned}$$

Compare with eq. (IC_H),

$$\begin{aligned} \pi_L &\le v + \lambda u_{\delta_L}(\delta) - \lambda u_{\delta_L}(\delta - \delta_L) \\ &= v + \lambda\bar{\delta}_L - \lambda(\delta - \delta_L) \\ &< v + \lambda\bar{\delta}_H - \lambda(\delta - \delta_L). \end{aligned}$$

So, the total profit is

$$\begin{aligned} \pi &= p\left[v + \lambda\bar{\delta}_L - \lambda(\delta - \delta_H)\right] + (1-p)\left[v + \lambda\bar{\delta}_L - \lambda(\delta - \delta_L)\right] \\ &= v + \lambda\bar{\delta}_L + \lambda(p\delta_H + (1-p)\delta_L - \delta). \end{aligned}$$

So, we would like to make $\delta_H^* = \delta_L^* = \delta$, then

$$\pi_H^* = \pi_L^* = \pi^* = v + \lambda\bar{\delta}_L,$$

with $P_H^* = P_L^* = v + \lambda\bar{\delta}_L + \delta$.

Case 5.2. *$\bar{\delta}_L \le \delta - \delta_L \le \bar{\delta}_H$. Then, $\delta - \delta_H \le \delta - \delta_L$. Moreover, $u_{\delta_L}(\delta - \delta_L) = \bar{\delta}_L$, $u_{\delta_H}(\delta - \delta_L) = \delta - \delta_L$, $u_{\delta_H}(\delta - \delta_H) = \delta - \delta_H$ and $\delta_H \ge \delta_L$.*

Under case 5.2, eq. (IR_H) and eq. (IR_L) would be like

$$\begin{cases} \pi_H \le v + \lambda\bar{\delta}_H - \lambda(\delta - \delta_H), \\ \pi_L \le v, \end{cases}$$

and eq. (IC_H) and eq. (IC_L) would be like

$$\begin{cases} \pi_H - \pi_L \le \lambda(\delta_H - \delta_L), & \\ \pi_H - \pi_L \ge \lambda(\bar{\delta}_L - \bar{\delta}_L), & \text{if } \delta - \delta_H \ge \bar{\delta}_L, \\ \pi_H - \pi_L \ge \lambda(\bar{\delta}_L - \delta + \delta_H), & \text{if } \delta - \delta_H < \bar{\delta}_L. \end{cases}$$

For that $\delta_H \ge \delta_L$, the IC constraints hold when $\delta - \delta_H \ge \bar{\delta}_L$. Furthermore, due to $\delta - \delta_L \ge \bar{\delta}_L$, the IC constraints still hold when $\delta - \delta_H < \bar{\delta}_L$.

Because we set $\delta - \delta_L \leq \bar{\delta}_H$, from eq. ($\text{IC}_H$),

$$\begin{aligned}\pi_H &\leq \pi_L + \lambda(\delta_H - \delta_L), \\ &= v + \lambda(\delta_H - \delta_L) \\ &= v + \lambda\bar{\delta}_H - \lambda(\delta - \delta_H) - \lambda\left[\bar{\delta}_H - (\delta - \delta_L)\right], \\ &\leq v + \lambda\bar{\delta}_H - \lambda(\delta - \delta_H).\end{aligned}$$

Now, the profit is

$$\pi = p\left[v + \lambda(\delta_H - \delta_L)\right] + (1-p)v$$

We make $\delta_H - \delta_L = \bar{\delta}_H$, then we get the optimal equilibrium that

$$\pi_H^* = v + \lambda\bar{\delta}_H, \; \pi_L^* = v, \; \pi^* = v + p\lambda\bar{\delta}_H.$$

Case 5.3. *$\delta - \delta_L \geq \bar{\delta}_H$. Then, $u_{\delta_H}(\delta - \delta_L) = \bar{\delta}_H$ and $u_{\delta_L}(\delta - \delta_L) = \bar{\delta}_L$.*

Under case 5.3, we have two situations. First, if $\delta - \delta_H \geq \bar{\delta}_H$ too, then $u_{\delta_H}(\delta - \delta_H) = \bar{\delta}_H$ and $u_{\delta_L}(\delta - \delta_H) = \bar{\delta}_L$. According to the incentive compatibility constraints, we can see that

$$\pi_H = \pi_L.$$

Then, according to the individual rationality constraints, we know that

$$\pi_H^* = \pi_L^* = v.$$

Here, case 5.3 eliminates both the difference of the disutility of loss between the two strategies and the difference of the disutility of loss between buy or not. This result tells us that a strategy involving the generous issuance of coins but strict limitations on their usage is equivalent to a strategy in which no coins are issued at all.

Second, if $\delta - \delta_H \leq \bar{\delta}_H$, we have $u_{\delta_H}(\delta - \delta_H) = \delta - \delta_H$ and $\delta_L \leq \delta_H$. Then, eq. (IR_H) and eq. (IR_L) would be like

$$\begin{cases} \pi_H \leq v + \lambda\bar{\delta}_H - \lambda(\delta - \delta_H), \\ \pi_L \leq v, \end{cases}$$

and eq. (IC_H) and eq. (IC_L) would be like

$$\begin{cases} \pi_H - \pi_L \leq \lambda(\bar{\delta}_H - \delta + \delta_H), \\ \pi_H - \pi_L \geq \lambda(\bar{\delta}_L - \bar{\delta}_L), & \text{if } \delta - \delta_H \geq \bar{\delta}_L, \\ \pi_H - \pi_L \geq \lambda(\bar{\delta}_L - \delta + \delta_H), & \text{if } \delta - \delta_H < \bar{\delta}_L. \end{cases}$$

For that $\bar{\delta}_H \geq \bar{\delta}_L$ and $\bar{\delta}_H \geq \delta - \delta_H$, these IC constraints always hold. We make $\pi_L = v$, then the profit is

$$\pi = p\left[v + \lambda\bar{\delta}_H - \lambda(\delta - \delta_H)\right] + (1-p)v.$$

We make $\delta = \delta_H$, which means that $\delta_H - \delta_L \geq \bar{\delta}_H$, then we get the optimal equilibrium that

$$\pi_H^* = v + \lambda\bar{\delta}_H, \; \pi_L^* = v, \; \pi^* = v + p\lambda\bar{\delta}_H.$$

This result, in conjunction with the findings under case 5.2, extends the domain of definition for the screening solution under varying $\bar{\delta}$.

Finally, we need to compare the strategies obtained respectively from the three cases. If $\bar{\delta}_H \leq \bar{\delta}_L/p$, $v + \lambda\bar{\delta}_L \geq v + p\lambda\bar{\delta}_H$ and then we would choose the strategy obtained from case 5.1. Vice versa.

5.2.3 Different λ and $\bar{\delta}$

As we discussed at the end of section 3, we consider λ and $\bar{\delta}$ to be two manifestations of loss aversion in our model. Therefore, a person who is more loss averse is expected to have both a higher λ and a higher $\bar{\delta}$, meaning they react more intensely to losses and are less likely to become desensitized to them.

Here we assume that the consumers differ both in terms of λ and $\bar{\delta}$. The participation constraints are

$$v - P_H + \delta_H - \lambda_H u_{\delta_H}(\delta - \delta_H) \geq -\lambda_H u_{\delta_H}(\delta), \tag{IR$_H$}$$
$$v - P_L + \delta_L - \lambda_L u_{\delta_L}(\delta - \delta_L) \geq -\lambda_L u_{\delta_L}(\delta), \tag{IR$_L$}$$

and the incentive compatibility constraints are

$$v - (P_H - \delta_H) - \lambda_H u_{\delta_H}(\delta - \delta_H) \geq v - (P_L - \delta_L) - \lambda_H u_{\delta_H}(\delta - \delta_L), \tag{IC$_H$}$$
$$v - (P_L - \delta_L) - \lambda_L u_{\delta_L}(\delta - \delta_L) \geq v - (P_H - \delta_H) - \lambda_L u_{\delta_L}(\delta - \delta_H). \tag{IC$_L$}$$

Combining eq. (IC$_H$) and eq. (IC$_L$), we get

$$\lambda_H \left[u_{\delta_H}(\delta - \delta_L) - u_{\delta_H}(\delta - \delta_H)\right] \geq \lambda_L \left[u_{\delta_L}(\delta - \delta_L) - u_{\delta_L}(\delta - \delta_H)\right]$$

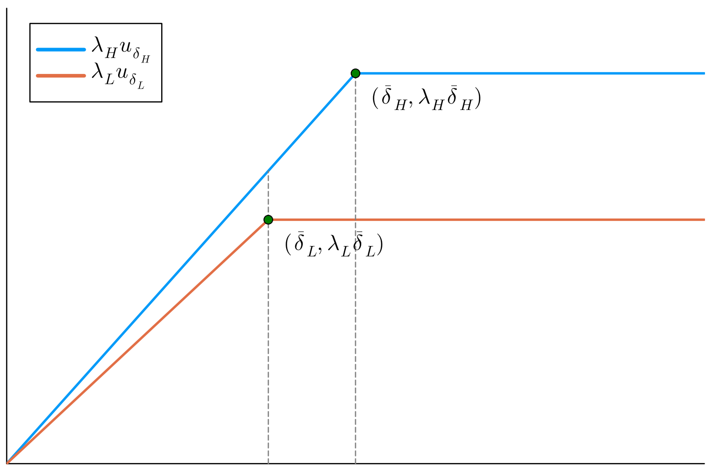


Figure 2: Diagram of $\lambda_H u_{\delta_H}$ and $\lambda_L u_{\delta_L}$

Based on fig. 2 and considering the properties of the combined IC function, we use $\delta - \delta_H$ as the basis for our case-by-case analysis.

Case 5.4. $\delta - \delta_H \leq \bar{\delta}_L$. *Then,* $u_{\delta_H}(\delta - \delta_H) = u_{\delta_L}(\delta - \delta_H) = \delta - \delta_H$.

If $\delta - \delta_L \leq \bar{\delta}_L$, we have $u_{\delta_H}(\delta - \delta_L) = u_{\delta_L}(\delta - \delta_L) = \delta - \delta_L$,

$$\begin{aligned} \lambda_H \left[(\delta - \delta_L) - (\delta - \delta_H)\right] &\geq \lambda_L \left[(\delta - \delta_L) - (\delta - \delta_H)\right] \\ \lambda_H(\delta_H - \delta_L) &\geq \lambda_L(\delta_H - \delta_L) \\ (\lambda_H - \lambda_L)(\delta_H - \delta_L) &\geq 0 \\ \delta_H &\geq \delta_L, \end{aligned}$$

and the four constraints would be like

$$\begin{cases} \pi_H \le v + \lambda_H \bar{\delta}_H - \lambda_H(\delta - \delta_H), \\ \pi_L \le v + \lambda_L \bar{\delta}_L - \lambda_L(\delta - \delta_L), \\ \pi_H - \pi_L \le \lambda_H(\delta_H - \delta_L), \\ \pi_H - \pi_L \ge \lambda_L(\delta_H - \delta_L). \end{cases}$$

Then

$$\begin{aligned} \pi_H &\le \pi_L + \lambda(\delta_H - \delta_L) \\ &\le v + \lambda_L \bar{\delta}_L - \lambda_L(\delta - \delta_L) + \lambda_H(\delta_H - \delta_L) \\ &= v + \lambda_H \bar{\delta}_H - \lambda_H(\delta - \delta_H) - \lambda_H(\bar{\delta}_H - \delta + \delta_L) + \lambda_L(\bar{\delta}_L - \delta + \delta_L), \\ &\le v + \lambda_H \bar{\delta}_H - \lambda_H(\delta - \delta_H), \end{aligned}$$

which means that eq. (IR_L) binds. Therefore, the profit is

$$\pi = v + \lambda_L \bar{\delta}_L - \lambda_L(\delta - \delta_L) + p\lambda_H(\delta_H - \delta_L).$$

Without loss of generality, we think $\delta = \delta_H$. Then, if $\lambda_H \ge \lambda_L/p$, we make $\delta_L = \delta_H - \bar{\delta}_L$ and the optimal solution is

$$\pi_H^* = v + \lambda_H \bar{\delta}_L,\ \pi_L^* = v,\ \pi^* = v + p\lambda_H \bar{\delta}_L.$$

Otherwise, we let $\delta = \delta_H = \delta_L$, the optimal solution is

$$\pi_H^* = \pi_L^* = \pi^* = v + \lambda_L \bar{\delta}_L.$$

If $\delta - \delta_L \ge \bar{\delta}_L$, we have $u_{\delta_L}(\delta - \delta_L) = \bar{\delta}_L$ and $\delta_H \ge \delta_L$. The two IR constraints are

$$\begin{cases} \pi_H \le v + \lambda_H \bar{\delta}_H - \lambda_H(\delta - \delta_H), \\ \pi_L \le v, \end{cases}$$

and the two IC constraints are

$$\begin{cases} \pi_H - \pi_L \le \lambda_H(\delta_H - \delta_L), & \text{if } \delta - \delta_L \le \bar{\delta}_H, \\ \pi_H - \pi_L \le \lambda_H(\bar{\delta}_H - \delta + \delta_H), & \text{if } \delta - \delta_L \ge \bar{\delta}_H, \\ \pi_H - \pi_L \ge \lambda_L(\bar{\delta}_L - \delta + \delta_H). \end{cases}$$

For that $\bar{\delta}_H \ge \bar{\delta}_L$ and $\delta - \delta_L \ge \bar{\delta}_L$, these IC constraints always hold.

If $\bar{\delta}_L \le \delta - \delta_L \le \bar{\delta}_H$,

$$\begin{aligned} \pi_H &\le \pi_L + \lambda_H(\delta_H - \delta_L), \\ &\le v + \lambda_H(\delta_H - \delta_L) \\ &= v + \lambda_H \bar{\delta}_H - \lambda_H(\delta - \delta_L) - \lambda_H(\bar{\delta}_H - \delta + \delta_L), \\ &\le v + \lambda_H \bar{\delta}_H - \lambda_H(\delta_H - \delta_L). \end{aligned}$$

So, the profit is

$$\pi = v + p\lambda_H(\delta_H - \delta_L).$$

Then we make $\delta = \delta_H$ and $\delta_H - \delta_L = \bar{\delta}_H$, the optimal solution is

$$\pi_H^* = v + \lambda_H\bar{\delta}_H,\ \pi_L^* = v,\ \pi^* = v + p\lambda_H\bar{\delta}_H.$$

If $\delta - \delta_L \geq \bar{\delta}_H$, the profit is

$$\pi = v + p\lambda_H(\bar{\delta}_H - \delta + \delta_H).$$

The only thing we need to do is to let $\delta = \delta_H$ and the optimal solution is

$$\pi_H^* = v + \lambda_H\bar{\delta}_H,\ \pi_L^* = v,\ \pi^* = v + p\lambda_H\bar{\delta}_H$$

Case 5.5. $\bar{\delta}_L \leq \delta - \delta_H \leq \bar{\delta}_H$. *Then* $u_{\delta_H}(\delta - \delta_H) = \delta - \delta_H$ *and* $u_{\delta_L}(\delta - \delta_H) = \bar{\delta}_L$.

In the combined IC constraint,

$$\lambda_H[u_{\delta_H}(\delta - \delta_L) - \delta + \delta_H] \geq \lambda_L[u_{\delta_L}(\delta - \delta_L) - \bar{\delta}_L].$$

If $\bar{\delta}_L \leq \delta - \delta_L < \delta - \delta_H$, then $u_{\delta_L}(\delta - \delta_L) - \bar{\delta}_L = 0$ and $u_{\delta_H}(\delta - \delta_L) - \delta + \delta_H < 0$, which does not satisfy the IC constraint.

If $\delta - \delta_L < \bar{\delta}_L$, then $u_{\delta_H}(\delta - \delta_L) - \delta + \delta_H \leq u_{\delta_L}(\delta - \delta_L) - \bar{\delta}_L < 0$, which does not satisfy the IC constraints.

Therefore, we only care about $\delta - \delta_L \geq \delta - \delta_H$, which means that $\delta_H \geq \delta_L$. Then, the IR constraints are

$$\begin{cases} \pi_H \leq v + \lambda_H\bar{\delta}_H - \lambda_H(\delta - \delta_H), \\ \pi_L \leq v, \end{cases}$$

If $\delta - \delta_H \leq \delta - \delta_L \leq \bar{\delta}_H$, the IC constraints are

$$\begin{cases} \pi_H - \pi_L \leq \lambda_H(\delta_H - \delta_L), \\ \pi_H - \pi_L \geq 0. \end{cases}$$

Then,

$$\begin{aligned} \pi_H &\leq \pi_L + \lambda_H(\delta_H - \delta_L), \\ &\leq v + \lambda_H(\delta_H - \delta_L) \\ &= v + \lambda_H\bar{\delta}_H - \lambda_H(\delta - \delta_H) - \lambda_H(\bar{\delta}_H - \delta + \delta_L), \\ &\leq v + \lambda_H\bar{\delta}_H - \lambda_H(\delta - \delta_H). \end{aligned}$$

The profit is

$$\pi = v + p\lambda_H(\delta_H - \delta_L).$$

We make $\delta_H = \delta - \bar{\delta}_L$, $\delta_L = \delta - \bar{\delta}_H$ and $\delta_H - \delta_L = \bar{\delta}_H - \bar{\delta}_L$. The optimal solution is

$$\pi_H^* = v + \lambda_H(\bar{\delta}_H - \bar{\delta}_L),\ \pi_L^* = v,\ \pi^* = v + p\lambda_H(\bar{\delta}_H - \bar{\delta}_L).$$

It is not the best solution in global domain.

If $\delta - \delta_L \geq \bar{\delta}_H$, the IC constraints are

$$\begin{cases} \pi_H - \pi_L \leq \lambda_H(\bar{\delta}_H - \delta + \delta_H), \\ \pi_H - \pi_L \geq 0. \end{cases}$$

The profit is

$$\pi = v + p\lambda_H(\bar{\delta}_H - \delta + \delta_H).$$

We let $\delta - \delta_H = \bar{\delta}_H$, the optimal solution here is

$$\pi_H^* = \pi_L^* = \pi^* = v,$$

which can not be the best solution in global domain either.

Case 5.6. $\bar{\delta}_H \leq \delta - \delta_H$. *Then* $u_{\delta_H}(\delta - \delta_H) = \bar{\delta}_H$ *and* $u_{\delta_L}(\delta - \delta_H) = \bar{\delta}_L$.

In the combined IC constraint,

$$\lambda_H[u_{\delta_H}(\delta - \delta_L) - \bar{\delta}_H] \geq \lambda_L[u_{\delta_L}(\delta - \delta_L) - \bar{\delta}_L].$$

For any $\delta - \delta_L < \bar{\delta}_H$, we have $u_{\delta_H}(\delta - \delta_L) - \bar{\delta}_H < u_{\delta_L}(\delta - \delta_L) - \bar{\delta}_L \leq 0$, which doesn't satisfy the IC constraint. So, we only care about $\delta - \delta_L \geq \bar{\delta}_H$. In this situation, we have $\pi_H = \pi_L \leq v$, which cannot be the best solution.

Finally, we need to compare the strategies obtained from the three cases. The solutions in the last two cases are worse than that in the first case. So, if $\lambda_H \bar{\delta}_H \geq \lambda_L \bar{\delta}_L / p$, we choose the strategy that $(\delta_H^*, \delta_L^*) = (\delta, \delta - \bar{\delta}_H)$. Otherwise, we choose that $(\delta_H^*, \delta_L^*) = (\delta, \delta)$.

5.3 Comment

Table 1: Summary of screening results

	δ_H^*	δ_L^*	P_H^*	P_L^*	π_H^*	π_L^*
Different v	δ	δ	$v_L + \lambda\bar{\delta} + \delta$	$v_L + \lambda\bar{\delta} + \delta$	$v_L + \lambda\bar{\delta}$	$v_L + \lambda\bar{\delta}$
Different λ ($\lambda_H \leq \lambda_L/p$)	δ	δ	$v + \lambda_L\bar{\delta} + \delta$	$v + \lambda_L\bar{\delta} + \delta$	$v + \lambda_L\bar{\delta}$	$v + \lambda_L\bar{\delta}$
Different λ ($\lambda_H \geq \lambda_L/p$)	δ	$\delta - \bar{\delta}$	$v + \lambda_H\bar{\delta} + \delta$	$v + \delta - \bar{\delta}$	$v + \lambda_H\bar{\delta}$	v
Different $\bar{\delta}$ ($\bar{\delta}_H \leq \bar{\delta}_L/p$)	δ	δ	$v + \lambda\bar{\delta}_L + \delta$	$v + \lambda\bar{\delta}_L + \delta$	$v + \lambda\bar{\delta}_L$	$v + \lambda\bar{\delta}_L$
Different $\bar{\delta}$ ($\bar{\delta}_H \geq \bar{\delta}_L/p$)	δ	$\delta - \bar{\delta}_H$	$v + \lambda\bar{\delta}_H + \delta$	$v + \delta - \bar{\delta}_H$	$v + \lambda\bar{\delta}_H$	v
Both ($\lambda_H\bar{\delta}_H \leq \lambda_L\bar{\delta}_L/p$)	δ	δ	$v + \lambda_L\bar{\delta}_L + \delta$	$v + \lambda_L\bar{\delta}_L + \delta$	$v + \lambda_L\bar{\delta}_L$	$v + \lambda_L\bar{\delta}_L$
Both ($\lambda_H\bar{\delta}_H \geq \lambda_L\bar{\delta}_L/p$)	δ	$\delta - \bar{\delta}_H$	$v + \lambda_H\bar{\delta}_H + \delta$	$v + \delta - \bar{\delta}_H$	$v + \lambda_H\bar{\delta}_H$	v

where δ satisfies assumption 1.

Proposition 5.6. *In the optimal equilibrium,* $\delta_H^* \geq \delta_L^*$.

This proposition can be concluded from table 1. It is clear that we would like to gain more profit from high-type consumers since we have shown that greater loss aversion yields higher profit. Now, we confirm that the only way to achieve the higher profit is to allocate more coins and use a higher price.

From our investigation about different v, we can see that it is possible to separate high-type and low-type consumers. However, the customized strategy wouldn't be the optimal solution. In such cases, it is supposed to revisit the solution of pooling.

In the cases of heterogeneous loss aversion, the pooling strategy remains an important component. However, as the number of varying parameters in loss aversion increases, the likelihood of employing a pooling strategy decreases.

In the screening strategies, we always fully utilize the loss aversion of the high type, which means that our optimal solution is always achieved at the boundary. This suggests that our model may also perform well in screening involving multiple types.

The main advantage of screening over pooling in this section lies in the fact that the screening strategy, while aiming to serve the high type, does not exclude the low-type consumers from the market. This also increases the likelihood that the coin strategy chooses to serve the high type.

6 Conclusion

This paper is based on the theory of loss aversion and develops a formal model demonstrating the positive role of coins in enhancing profitability. It further proposes alternative coin distribution schemes using a screening model. When consumer heterogeneity is relatively low, applying a uniform strategy across all consumers can still yield substantial profits. However, as heterogeneity increases, it becomes necessary to evaluate whether to concentrate efforts on serving the high-type agents or to continue addressing the broader consumer base. Additionally, linking coins consumption to other value-added services may amplify the disutility consumers experience from losing coins, thereby further enhancing sales profitability.

This study begins with the fundamental nature of discount-based marketing strategies and emphasizes consumer heterogeneity, potentially offering a distinctive perspective within the current marketing landscape that increasingly values data analytics and machine learning.

We believe that the coin distribution method may prove more effective in markets characterized by price fluctuations or the presence of similar products at varying price levels, such as the airline industry or retail supermarkets.

6.1 Extra Discussion

The screening approach we use does not require too much collection of users' personal information, which may make it more acceptable to users and less costly to implement, especially in the future when concerns over cybersecurity and data privacy become increasingly prominent.

Here, we then address several contentious issues surrounding loss aversion and the endowment effect. If we refer only to Kahneman et al. (1990), these two concepts are closely connected, allowing us to blur the distinction in this discussion. However, due to the findings presented in Chapman et al. (2024), which demonstrated the endowment effect is not correlated to loss aversion with respect to risk prospects and took cautious utility (Cerreia-Vioglio et al., 2024) as an alternative. So, we find it necessary to treat them separately here. If the reader is interested in cautious utility, it could be explored as an extension. However, we put more weight on loss-aversion here since we focus on the disutility from loss. Anyway, even if there is a potential theoretical tension between the two concepts, neither paper denied the phenomena and it does not compromise the validity of the results presented in this paper.

The screening model in this paper only provides solutions at the vertices, which could be a consequence of setting the utility function as linear. Exploring alternative utility functions might lead to different conclusions which may care about the high-type and the low-type at the same time. However, I believe such changes would not meaningfully alter the structural insights of the results.

In addition, several open questions remain. For example, how should our strategies adapt when we consider more realistic settings? If we incorporate more aspects of loss aversion, consumers may have reference prices. In such cases, we have to consider the psychological impact of the listed prices on consumers. Similarly, if we account for more aspects of hedonic adaptation, it is worth asking whether consumers, after prolonged exposure to coin-allocation, would eventually come to systematically reject purchases when no coin is offered. Moreover, it is important to consider the potential for consumer pushback. If consumers become aware of the coins strategy, they may opt to disengage, as seen in past instances of price discrimination enabled by big data, where users altered their behavior once they perceived they were being targeted differently.

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