

# PRODUCTION HETEROGENEITY UNDER SUDDEN STOPS: INSIGHTS FROM CHILE

Natalia Cárdenas Frías

Under the supervision of Xavier Ragot

Dual Master Degree in Economics and Quantitative Economics  
Sciences Po School of Research and Paris I Panthéon-Sorbonne

May 2023

## Abstract

Emerging market economies (EMEs) are more vulnerable to suddenly being excluded from financial markets in events of Sudden Stops. During these situations, access to liquidity for domestic agents is scarcer and more expensive for different economies in tandem even if the triggering event of the crisis is exogenous to their fundamentals. This shock in the financial market can have long-lasting effects on the real economy of an EME. I explore a heterogeneous firm model with *ad hoc* constraints on their access to external financing and build a framework on how in the absence of capital inflows from international markets, capital tends to be further misallocated provoking aggregate long-term losses. I do so building on data from Chile, a robust economy that suffered a Sudden Stop after the 1998 Russian Default.

## Acknowledgments

I would like to thank above all Xavier Ragot for his kind advice, guidance, availability and (lots) of patience over the last few months. Without his help and experience, I wouldn't have been able to begin to tackle an idea like this. I am also extremely grateful for Nicolas Coeurdacier's advice and his willingness to be a member of the defense jury. Finally, I am incredibly indebted to Alais Martin-Baillon who was willing to share with me her work in order to implement this type of model. Without her generosity, this paper would have been a bunch of equations only with little to no insight.

On a more personal note, I would like to thank my friends in this master notably Victor, Pol, Alexandre, Mathias, and Léa for all the conversations, mutual help and encouragement (even just for having coffee). Without them, this experience would have been certainly less enriching and more frustrating. To the Paris and Louvain QEMs, thank you for your generosity and for teaching me so much during the two years. Meeting you all is one of the great privileges of my life. I am certain that without your friendship, and kindness, I would have dropped out when I realized econ uses actual math. In particular, I thank Louis for his unconditional support, comfort and willingness to stick around even when I rant about macro. Lastly, I would like to thank my family for their support along the way -especially when I came home with the crazy idea of studying abroad- and friends from Bogota and Poitiers who consistently are there when I need a laugh.

All errors are my own.

# Contents

|   |           |
|---|-----------|
| <b>Introduction</b>   | <b>5</b>  |
| <b>1 Empirical motivation and related literature</b>                    | <b>7</b>  |
| 1.1 1998 Russian Default and a Latin American downturn . . . . .        | 7         |
| 1.2 Heterogeneous firms framework . . . . .                             | 10        |
| 1.3 Chilean firm heterogeneity . . . . .                                | 12        |
| <b>2 A basic model with production heterogeneity</b>                    | <b>15</b> |
| 2.1 Description of the economy . . . . .                                | 15        |
| 2.2 Agents' problems and policy functions . . . . .                     | 19        |
| 2.3 Competitive equilibrium characterization . . . . .                  | 22        |
| <b>3 Calibration for the steady state and stationary distributions</b>  | <b>24</b> |
| 3.1 Reduced heterogeneity representation: truncated histories . . . . . | 24        |
| 3.2 Parametization . . . . .  | 25        |
| 3.3 Stationary equilibrium and shock . . . . .                          | 26        |
| <b>4 Simulation and Dynamics</b>  | <b>30</b> |
| 4.1 Depreciation Shocks . . . . .                                       | 30        |
| 4.2 Aggregate TFP shocks . . . . .                                      | 33        |
| <b>Conclusion</b>   | <b>36</b> |
| <b>References</b>   | <b>37</b> |
| <b>Appendix A Analytical solutions</b>                                  | <b>41</b> |
| A.1 Exiting firms . . . . .   | 41        |
| A.2 Surviving firms . . . . .   | 42        |
| A.2.1 Unconstrained firms . . . . .                                     | 43        |
| A.2.2 Constrained firms . . . . .                                       | 44        |
| A.3 Representative Household . . . . .                                  | 44        |
| <b>Appendix B Data sources and management</b>                           | <b>46</b> |
| B.1 Plant-level data: the ENIA dataset . . . . .                        | 46        |
| B.2 Aggregate data . . . . .  | 49        |

## List of Figures

|   |  |    |
|---|--|----|
| 1 | 1998 Sudden Stop in Chile . . . . .                              | 9  |
| 2 | Kernel Distribution Estimation (1997-2000) . . . . .             | 13 |
| 3 | Stationary distribution on the capital grid . . . . .            | 28 |
| 4 | High Persistence shock on $\delta, \rho_\delta = 0.95$ . . . . . | 31 |
| 5 | Low Persistence shock on $\delta, \rho_\delta = 0.5$ . . . . .   | 31 |
| 6 | High Persistence shock on $Z, \rho_z = 0.95$ . . . . .           | 33 |
| 7 | Low Persistence shock on $Z, \rho_z = 0.5$ . . . . .             | 34 |

## List of Tables

|   |   |    |
|---|---|----|
| 1 | Baseline parameters description . . . . .   | 26 |
| 2 | Stationary equilibrium . . . . .  | 29 |
| 3 | Parameters aggregate shock to $\delta$ . . . . .  | 30 |
| 4 | Simulated moments of aggregates and equilibrium price following shock on the depreciation rate $\delta$ . . . . . | 32 |
| 5 | Parameters aggregate shock to $Z$ . . . . .   | 33 |
| 6 | Simulated moments of aggregates and equilibrium price following shock on the aggregate TFP level $Z$ . . . . .    | 34 |
| 7 | Comparison of before-after data cleaning for some important statistics . . . . .                                  | 47 |
| 8 | Descriptive Statistics, Full Panel . . . . .  | 48 |

## Introduction

Emerging markets economies (EMEs) are subject to more volatile and uncertain business cycles [[Aguiar and Gopinath, 2007](#), [Mendoza, 1991](#)]. They are also in a particular position concerning their access to capital markets: they tend to have less profound domestic financial markets than their wealthy counterparts [[Abiad et al., 2008](#)]<sup>1</sup> and they are more prone to be excluded from international markets. Indeed, not only EMEs are associated with higher risk -think for instance of sovereign risk<sup>2</sup>-, but they are especially vulnerable to dramatic global capital market corrections in the context of Sudden Stops. These are events where access to foreign liquidity dramatically dries out, and are characterized by sharp falls of capital inflows below trend, heavy devaluations of the local currency, interest rates shooting up, and asset prices crumbling resulting in abrupt adjustments [[Calvo and Talvi, 2008](#), [Calvo et al., 2006](#), [Mendoza, 2010](#)]. They are relatively common across EMEs and tend to affect different parts of the world in tandem as the distress in global capital markets works as a coordinating factor across them [[Eichengreen and Gupta, 2016](#)]. These adjustments are often accompanied by recessions and they can scar these economies moving forward as they often result in permanent losses in output and aggregate productivity [[Ates and Saffie, 2021](#), [Mendoza, 2010](#)].

Moreover, the last few decades have been extremely prolific in research about the role of financial frictions and incomplete markets in the macroeconomy<sup>3</sup>. In particular, considering how the existence of ad hoc credit constraints in stochastic settings can lead to the evaluation of models with uninsurable idiosyncratic risks. These constraints prevent at least some agents from acquiring a portfolio of assets that mimics Arrow-Debreu securities to fully hedge against future shocks with cross-sectional and aggregate consequences. Indeed, the existence of incomplete markets might generate distortions in the allocation of resources [[Hsieh and Klenow, 2009](#), [Moll, 2014](#)], of aggregate quantities, and in equilibrium prices that can widely differ from the conclusions of a representative agent model [see [Ahn et al., 2018](#), for an overview of these effects when considering heterogeneous consumers]. All in all, the absence of perfect insurance is at the heart of economies where risk becomes an obstacle to efficiency, and where even ex-ante identical agents are ex-post distinct between them and thus distributed over the state space.

This paper aims to use a standard model of heterogeneous firms to grasp how a Sudden Stop in an EME generates a long-standing dip in production and aggregate productivity. Building on [Martin-Baillon \[2021\]](#), this model illustrates the effects of the Sudden Stop following the 1998 Russian Default in the Chilean Economy. Indeed, the volatility that surrounded this default translated into a synchronized shutdown of international capital markets for other EMEs, in particular in Latin America. Chile, which had robust growth since the second half of the 1980s, with average growth surpassing 7% per annum, had a stiff slowdown in 1998 and a small recession in 1999 (−0.3% growth) [[Calvo and Talvi, 2008](#)]. Facing a Sudden Stop, the economy required to have a rapid adjustment on its current account, which went from running a deficit consistently since 1977 and

---

<sup>1</sup>This is a feature widely present in international finance models especially to describe global imbalances e.g [Caballero et al. \[2008\]](#), [Coourdacier et al. \[2015\]](#), [Mendoza et al. \[2009\]](#)

<sup>2</sup>See [Das et al. \[2012\]](#) for a survey on debt restructurings.

<sup>3</sup>See in particular the seminal work of [Kiyotaki and Moore \[1997\]](#), [Woodford \[1990\]](#) that is mainly interested in the consumer's consumption-saving decisions when they face uncertain income and incomplete markets.

surpassing 10% of the GDP in 1996 and 1997, to rapidly generating a surplus by 1999. This reaction can be explained by a diminished investment in the economy that allows to ship resources abroad and repay outstanding debt, which is verified by a slowdown in the gross domestic fixed investment. Finally, as is expected in these events it had a loss in productivity that can scar the economy well after the financial markets calm down [Mendoza, 2010]. In fact, Ates and Saffie [2021] estimates that this event provoked a permanent loss of productivity of 0.6% that will lead the Chilean economy to a new balanced growth path where consumption and output are 0.9% lower with respect to a counterfactual where this crisis did not propagate into its economy.

Using plant-level data for the period 1995-2007, I calibrate a discrete-time model with an exogenous exit risk, where atomistic producers face firm-specific productivity shocks and where they can only finance their investments using retained earnings. The economy is vulnerable to aggregate shocks, namely in aggregate TFP and in the depreciation rate which is common across firms. Producers seek to maximize the expected stream of dividends paid to a representative household, their sole shareholder. I leverage the transmission of the Russian Default 1998 crisis to other EMEs as the source of augmented aggregate uncertainty orthogonal to the fundamentals of the economy, but that reflects in how firms make investment decisions and how the economy allocates resources generating further capital misallocation and sustained losses in productivity and output.

The main intuition of the mechanism at play goes as follows. The shock on the international financial markets for emerging economies makes capital more expensive for all producers as it becomes more scarce. It requires the Chilean economy to devote a sizable part of its output to paying its current account deficit. As these resources are sent abroad they are essentially destroyed for domestic agents which tightens the firms' liquidity constraint. This worsens the capital misallocation of the economy and generates the persistent loss in TFP that is seen in the data. This allows testing this channel that goes from purely financial shocks to long-lasting effects in the real economy, that in the context of an EME can hurt its development path. This is important because as mentioned before EMEs tend to have more friction in their financial markets and they are prone to shocks like Sudden Stops. Further understanding their interaction and transmission is key for policy-makers that need to handle this sort of crisis not only in the short term to deal with an exacerbation of the business cycle but also in the long run.

The rest of the paper is organized as follows. Section 1 details the consequences of the 1998 Russian Default in the Chilean economy, how it reveals the features of a Sudden Stop and provides an overview of the literature review on the heterogeneous firm literature. Section 2 presents a basic heterogeneous firm model where producers can only finance their investments through retained earnings and its solution. Section 3 shows how the model presented in the previous section is calibrated for the steady state taken in 1997 and how we can model capital being more scarce as a Sudden Stop hits the economy. This worsens investment misallocation. Finally, section 4 simulates the Sudden Stop shock and displays the new steady state and the impulse response functions to aggregate shock. The last section concludes and provides hints to further analysis of this event in a production heterogeneity framework.

# 1 Empirical motivation and related literature

## 1.1 1998 Russian Default and a Latin American downturn

“Until then we were doing fine, but everything changed after Russia”

---

GUILLERMO ORTIZ, Mexico’s Central Bank governor during 1998’s IMF and World Bank annual meeting. Quoted by the NYT (1998).

On August 17th, 1998 the Russian Federation defaulted on part of its debt worth about 15 billion dollars, after months of uncertainty and action from the West and the IMF to try to stabilize their macroeconomic situation [Bohlen, 1998]. The year was marked by a profound recession, bank failure, high inflation and plunge in the value of the ruble [Kharas et al., 2001] that resulted in further interventions from international partners until the end of the decade [Das et al., 2012]. This event -accompanied by other sources of distress on the financial markets in the second half of 1998<sup>4</sup>- resulted in turmoil that propagate into other markets, in particular for EMEs that also saw their currencies devalue, an increased perception of risk and sinking value of their financial assets. Indeed, the higher volatility of asset prices, the reduced liquidity and the flight to safe assets made exposed other economies to financial instability [BIS, 1999].

Latin America was still recovering from the major hardship of the 1980s and benefited from unprecedented inflows of relatively inexpensive capital that financed growth [Bértola and Ocampo, 2012]. Unlike other major financial distress events in the 15 years prior -notably the 1994 Mexican Default that had focalized and temporary effects on capital inflows, and the 1997 Asian crisis that left largely unaffected the region-, the events of 1998 provoked a major, sudden and persistent shock to the region’s capital accounts. As Calvo and Talvi [2008] highlight, they profoundly altered the dynamic in capital markets as the region saw striking hikes in interest rates, doves in capital flows, and yet another slowdown in growth dynamics as the Russian crisis “precipitated a sudden, synchronized, large and persistent increase in the interest rates” (p.125). In short, the region saw the materialization of a Sudden Stop, a common feature in emerging markets where there is -a minima- a substantial decline in capital inflows when compared to the previous trend<sup>5</sup> needing abrupt adjustments. Across the region<sup>6</sup>, the effects of a reversal in access to liquidity manifested in different intensities emphasizing the role of domestic characteristics [Calvo et al., 2006, Mendoza, 2010] and policy responses [Eichengreen and Gupta, 2016, Valdés, 2007] in the aftermath of the initial shock.

Chile offers an interesting framework to assess the impacts of a Sudden Stop on the real economy. It is a small open economy, relatively well integrated to trade flows and international capital markets. It had a robust recovery after the 1982-1983 Latin American crisis and outperformed the rest of the region [Bergoeing et al., 2002].

---

<sup>4</sup>The 1999 BIS report cited above provides an interesting timeline of the issues on the international financial markets in Autumn 1998. It highlights the distress of the American financial provider LTCM in July and surveys the reaction by market participants in section two.

<sup>5</sup>There can be slight differences between in the definition of a Sudden Stop by different authors, in particular, to distinguish the effects of the capital markets drought. Cavallo et al. [2015] offers an interesting classification of these events to differentiate events according to their impact.

<sup>6</sup>An interesting feature of Sudden Stops is that -at least in the 1990s- they tend to impact whole regions at the same time, what Calvo et al. [2006] describe as ‘temporal bunching’. Eichengreen and Gupta [2016] note that more recent episodes tend to impact different regions in tandem.

During the second half of the 1980s and the 1990s, the country's implementation of structural reforms and eventual democratization led it to a prosperous and stable macroeconomic situation. Chile had steady and unprecedented growth levels (+7.6% on average for the period 1990-1997, well above the regional performance), stable and lower inflation, and high growth in productivity and employment [Jadresic and Zahler, 2000]. In this context of robust fundamentals, Chile was an important beneficiary of the abundant and relatively inexpensive capital inflows (despite the existence of capital controls) that came into Latin America which allowed it to fund investments. As Valdés [2007] describes in 1997, the net capital inflows accounted for more than 8% of Chile's GDP on average, excluding the accumulation of reserves by the Central Bank.

The effects of the 1998 crisis in Chile were certainly more subtle than in other countries of the region. Namely, Brazil received an IMF support package at the end of the year because of its precarious position [BIS, 1999], and Argentina's situation kept collapsing until its partial default in 2001 [Calvo and Talvi, 2008]. Nonetheless, it did translate into a crisis in the country (Figure 1) even if none of its fundamentals were directly impacted by the Russian collapse.

Following a textbook implication of Sudden Stops, Chile saw its currency heavily and speedily devalue<sup>7</sup>, spreads on its assets to shoot up and capital inflows to freeze. The Central Bank reacted with a stringent monetary policy to defend the peso, retain capital and contract demand. All in all, 1998 was a year of deceleration for the country as the growth rate went from 7.4% in 1997 to 4.2% in 1998 and culminated in a small recession in 1999 when GDP contracted by 0.3%. Importantly in my setting where I am interested in producers' decisions, gross domestic fixed investment dipped by 13.5%<sup>8</sup> in absolute value between 1998 and 1999, the equivalent of 5.3 points as a share of GDP. These adjustments also allowed the current account deficit to close rapidly, which is at the heart of the Sudden Stop literature and generated a surplus by 1999 of about 0.2% of the GDP.

Finally, and perhaps more interesting, this crisis is not without long-term consequences for the Chilean economy. Ates and Saffie [2021] use the same plant-level dataset as I do and leverage the initial shock of the Sudden Stop as an exogenous event unexpected for all agents of the economy<sup>9</sup> to assess the productivity and output implications. They find that during the crisis, as capital is more expensive, fewer firms enter the market but they have a larger likelihood to be highly productive due to selection. However, this event scarred the economy moving forward as it generates an estimated 0.6% loss of aggregate productivity and a 0.9% dip in aggregate output and consumption four years after the Russian default as the economy converges to a new steady state.

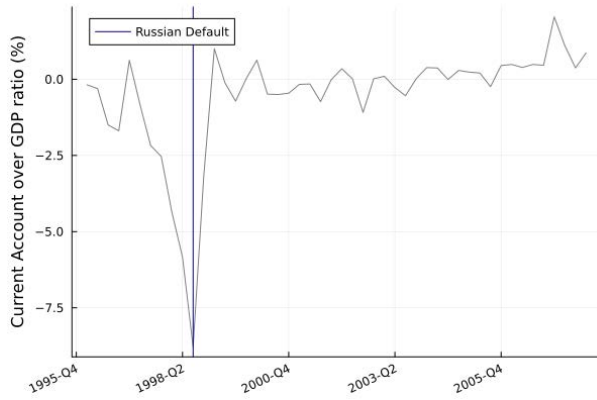
---

<sup>7</sup>Calvo and Talvi [2008], building on Calvo, Izquierdo and Talvi (2003) estimate that following the 1998 Sudden Stop, the Chilean peso devaluates 48% (in real terms) between June 1998 and December 2002 which allows the elimination of its current account deficit.

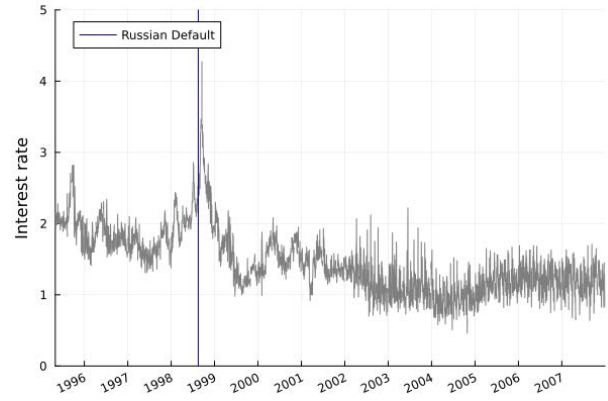
<sup>8</sup>The gross domestic investment -a broader measure- dipped by 15%.

<sup>9</sup>This build on Calvo et al. [2006] argument that the synchronized nature of the increase in spreads in assets across EMEs reflects a common triggering event. However, how it translates (if at all) in a major financial crisis and on distress in the real economy depends on country-specific factors (thus endogenous) in particular regarding the denomination of the country's liabilities





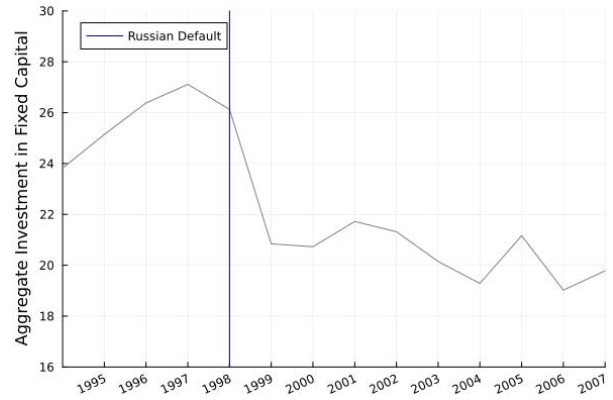
(a) Current Account adjustment



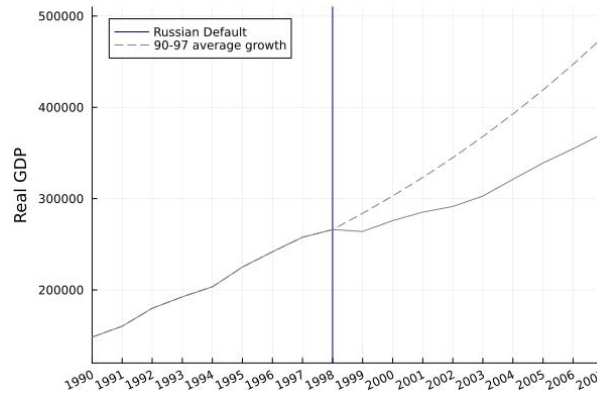
(b) Interest rate reaction



(c) Peso devaluation



(d) Gross domestic Fixed Investment (% of GDP)



(e) Real GDP

Figure 1: 1998 Sudden Stop in Chile

*Notes:* The blue line corresponds to August 17th, 1998, the official date of the Russian default. (a) Chile's quarterly current account balance data comes from the Central Bank and is seasonally adjusted. (b) Daily average nominal interest rate charged by commercial banks for lending at 90-day maturities. Data is retrieved from Chile's Central Bank. (c) Daily exchange rate with respect to the American dollar reported by Chile's Central Bank. (d) Share of gross domestic fixed investments on GDP (annual), taken from MOxLAD. (e) Annual data on Chile's real GDP (in pesos) from MOxLAD. The dashed line corresponds to the projection after 1998 of the GDP had the country sustained its average growth rate of the 1990-1997 period ( $\sim 7.6\%$ )

## 1.2 Heterogeneous firms framework

A rather recent strain of literature has focused its attention on how the presence of -binding- financial constraints for agents facing idiosyncratic risks can generate inefficiencies in the economy. This class of models, sometimes called *Standard Incomplete Models* or *Heterogeneous Agent Model*, first focused on consumer's saving behavior when facing uninsurable risks due to the existence of market frictions<sup>10</sup>. This approach provides powerful insights because despite agents being ex-ante identical, they are ex-post distributed over a grid on the state space as they face idiosyncratic risks and their decisions are affected by potentially binding ad hoc constraints. This endogenous distribution of agents is by itself a description of the heterogeneity of agents in the economy, a feature that is matched in the data and surpasses the representative agent assumption. At its core imperfect insurance against risk generates inefficiencies in the real economy that need to be taken into account to describe salient features of the world, from inequalities to the business cycles.

Heterogeneity among producers is not a new feature in macroeconomic studies. Namely, models in international trade leverage differences across firms to understand trade patterns, reallocation of resources and gains from trade [see [Melitz and Redding, 2014](#), for a survey of this matter]. Among them, in his extremely influential 2003 paper, [Melitz](#) presents an elegant model with monopolistic competition to rationalize that (i) that exporters firms are larger and more productive and (ii) that by opening up to trade there is a reallocation of resources towards the most efficient producers which ration the number of firms in the economy. In the case of Chile specifically and using a dataset equivalent to mine for previous decades<sup>11</sup>, [Pavcnik \[2002\]](#) shows that a sizable part of the country's improvement in productivity following the liberalization of the country between 1974 and 1979 is due to a reallocation of output towards more efficient plants.

More recently the tools of the broader heterogeneous agent literature have been applied to producers. [Hopenhayn \[1992\]](#) examine the stationary equilibrium of an economy with endogenous exit and no capital, where producers' productivity is idiosyncratic and follows a Markov process that influences the properties of their life cycle and of their industries. This is due to resources being reallocated from exiting firms to those that remain on the market. [Cooley and Quadrini \[2001\]](#) build a model with firms differing in their equity and with stochastic idiosyncratic productivity shocks where investment and default are endogenous and firms fund themselves with debt and retained earnings. They manage to generate prominent empirical observations, in particular, that smaller firms grow more because their higher marginal profit incentivizes them to invest more which translates into higher debt, lower dividends distributed and higher default risk.

Regarding the role in efficiency considerations of potential the misallocation of capital [Hsieh and Klenow \[2009\]](#) examine a monopolistic competition model augmented by ad hoc constraints -mainly institutional- can explain about half of the wedge in the aggregate TFP between the US and two big emerging economies, India and China. This is due to the distortions that these constraints, more acute in emerging economies (particularly in China), exert on firms' investment decisions and on the allocation of resources which hurt. More productive firms are

---

<sup>10</sup>The foundational work in this direction is [Aiyagari \[1994\]](#), [Deaton \[1991\]](#), [Huggett \[1993\]](#), [Krusell and Smith \[1998\]](#).

<sup>11</sup>She uses previous iterations of the ENIA survey (see Section 3.1) for the period 1979-1986. These previous data sets are not publicly available.

penalized as they cannot finance their need appropriately because they are constrained. Closer to my question, [Moll \[2014\]](#) explores how financial friction can generate distortions in aggregate TFP and cross-country income per capita disparities due to capital misallocation. He builds a tractable model where heterogeneous entrepreneurs face productivity shocks and credit constraints on external financing which produces misallocations. In his case, the possibility of generating funds to self-finance in scenarios where idiosyncratic productivity shocks are sufficiently persistent this second source of finance can compensate for the credit constraint and prevent TFP losses.

A more recent strand of literature focuses on how the business cycle interacts with the distortions that arise in cases of heterogeneous producers facing financing constraints. [Khan and Thomas \[2013\]](#) build a model where firms face persistent aggregate and individual TFP shocks, and can borrow to finance their investment in an environment with partial irreversibility of capital, and collateralized borrowing constraint in a DSGE setting. They find that these financial imperfections can amplify the business cycle generating long-lasting recessions, which match the effects of the 2008 crisis on the American economy. [Martin-Baillon \[2021\]](#) builds on to construct a quantitative apparatus to evaluate how to conduct optimal tax policy along business cycles in an environment where firms face financing constraints. Regarding the triggering of a crisis with heterogeneous producers, [Carvalho and Grassi \[2019\]](#) emphasize that when an economy is granular, idiosyncratic shocks to the largest producers alone can cause sizeable aggregate fluctuations.

Finally, the class of heterogeneous agent models needs to tackle non-trivial aggregation issues because having agents distributed over the steady state means that aggregate variables and prices depend on the (non-degenerate) distribution itself. Think about a simple case where agents differ only on their capital stock. Because the market interest rate depends is a function of capital, it will also be a function of the distribution of agents which formally is an infinite-dimensional object. Dealing with this issue complicates the analysis and can come with significant computational costs. To solve these problems, a growing literature has developed that aims to solve these models numerically, reducing the dimensionality of the problem while deriving insights from the heterogeneity among agents.

Most of it follows the lead of the approximate aggregation procedure developed by [Krusell and Smith \[1998\]](#) for heterogeneous households but that is also widely used for firms despite being very computationally intensive. It is a method of *projection within an approximate state space* that relies on the assumption that agents correctly forecast a sequence of moments of the distribution that are sufficient statistics to make optimal decisions. The other main class of solution techniques is based on *perturbation methods* and includes [Reiter \[2009\]](#), [Winberry \[2018\]](#). These methods solve for the steady state of the model ignoring any aggregate uncertainty to get a discrete description of the cross-sectional distribution of agents. The stationary equilibrium with rational expectations is then described as a system of linear equations (function of the grid found before) and allows for perturbations. [Terry \[2017\]](#) offers a useful survey and comparison of methods suitable to solve incomplete market models for producers with a discrete investment decision à la Khan and Thomas by discretizing the state space and includes the three methods mentioned above. Lastly, another approach to reduce the heterogeneity of a model is using *truncated histories*. A textbook description is found in [Ragot \[2018\]](#) and the method

is refined in [Le Grand and Ragot \[2022\]](#) to allow for truncations of different lengths. Its intuition is that a sufficiently rich distribution of agents can be described considering only the last  $N < \infty$  consecutive shocks in each individual's history so that two agents with the same past shocks will behave in the same way. This property allows aggregating all agents sharing a realization of the truncated idiosyncratic to recover a distribution defined over a finite state-space that can be simulated using Dynare [[Adjemian et al., 2022](#)]. This method is used in [Martin-Baillon \[2021\]](#) and allows to use of Ramsey problems to incorporate considerations of optimal policy.

### 1.3 Chilean firm heterogeneity

The implementation of this type of model requires micro-level data for the agents we are interested in so that we can get a cross-sectional view of the economy. The multiplication of such datasets is actually one of the drivers in the adoption of heterogeneous agent models in the literature. For this paper, I use plant-level data from the Chilean economy (1995-2007) to estimate the distribution of firms in a small open economy vulnerable to Sudden Stops.

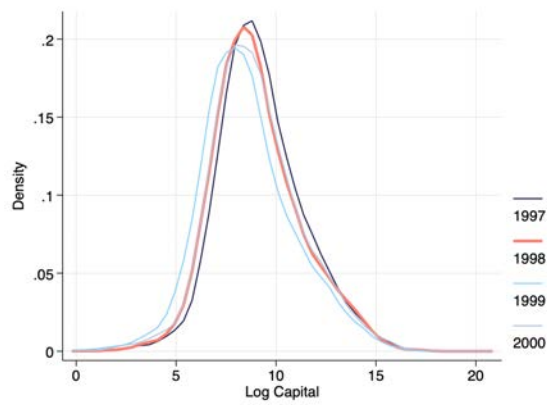
Since 1995, the Chilean Statistical Institute (*Instituto Nacional de Estadísticas* - INE) has conducted a yearly survey on national manufacturing establishments, the ENIA - *Encuesta Nacional Industrial Anual*. Its results are publicly available here, in a Microsoft Access format. The ENIA's unit of observation is industrial plants and it covers all the manufacturing establishments that:

- (i) Have at least 10 employees and have been in activity for at least 6 months in the year of the survey if among single-plant firms.
- (ii) If a plant is attached to a firm with multiple sites and has been operating for at least 6 months in the year of the survey, it is in the survey regardless of the number of employees.

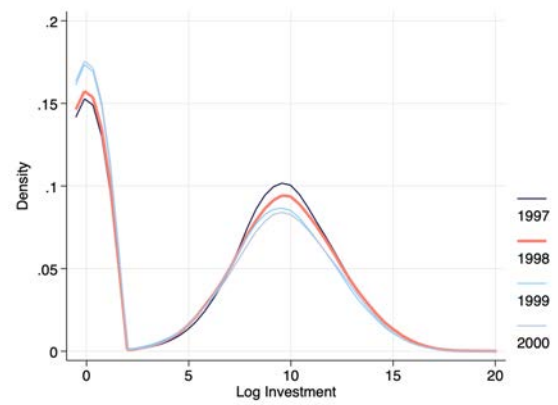
I use this source to build an unbalanced panel dataset for the period 1995-2007, with 69,311 observations for 11,093 plants. I describe the protocol used to build this dataset in Appendix B. I will assume, as [Ates and Saffie \[2021\]](#) do, that each unit of observation in the dataset, i.e. each plant, constitutes a firm as it is impossible to distinguish plants belonging to a larger firm to regroup them. This is also partially supported by [Pavcnik \[2002\]](#) observation that over 90% of the firms included in previous waves of ENIA are single-plant. Thus, 'plant' and 'firm' are used as synonyms hereafter.

The following kernel density estimations (Figure 2) allow us to have a first idea about how distinct firms are in the cross-section (more descriptive statistics are found in Table 8). Note that they are all performed in logs because the variables are extremely skewed to the left in levels making the distributions hard to read.

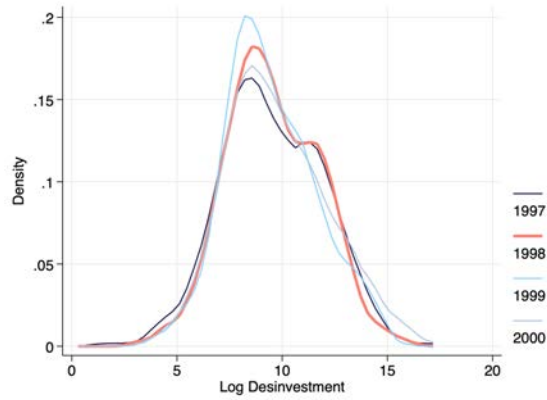
For the purposes of this work, we will note that most Chilean firms are small: in the three years before the crisis, over 65% of the plants in the survey had on average less than 20 employees (68.43% in 1998 and it surpasses 70% between 1999 and 2004). Importantly for my model where investment is made only via retained earnings, consistently along the period studied, over 90.5% of firms do not distribute dividends. Moreover, about 45% of them do not have debts before 1999. This access to external financing is important when considering firms' investment decisions but it is out of the scope of the model I will present in the following section. Concerning



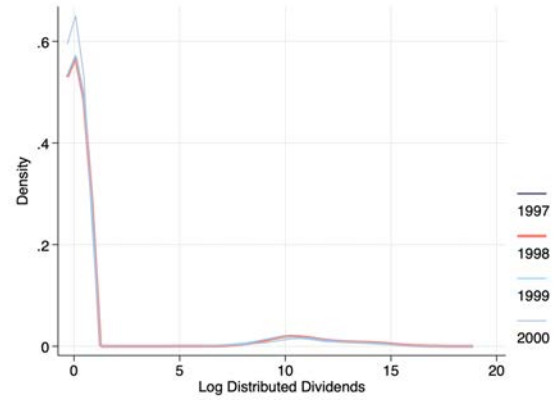
KDE on Capital



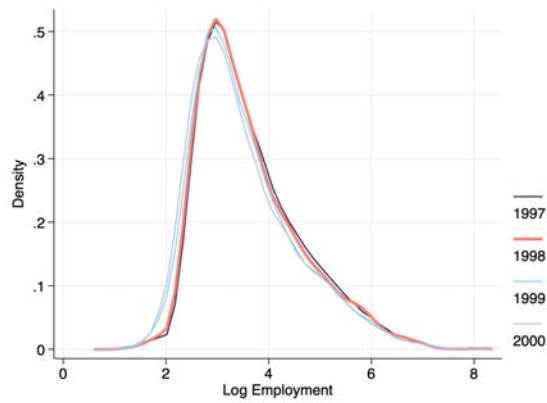
KDE on Investment



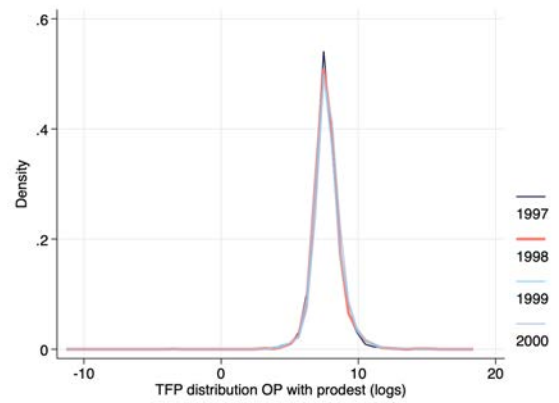
KDE on Desinvestment



KDE on Dividends



KDE on Employment



KDE on TFP

Figure 2: Kernel Distribution Estimation (1997-2000)

*Notes:* All estimations are performed on the ENIA dataset using Stata 17. I use the reported assets of the plants as a proxy for capital. Given that the values reported for gross domestic fixed investment take both positive and negative values, I considered all weakly positive values as investments, and all negative values as disinvestments before taking logs. Finally, TFP estimation is made using the Olley-Pakes method [Rovigatti and Mollisi, 2018].

the gross domestic fixed investment, a sizeable share of plants does not alter their capital stock (36% in 1997) and around half make positive investments (57.7% on average for 1995-1997, and 53.4% on average after 1999).

## 2 A basic model with production heterogeneity

I use an equivalent framework as in [Martin-Baillon \[2021\]](#) for a baseline quantitative model of heterogeneous. Unlike her, I ignore any policy intervention in order to focus on the assessment of the effects of another aggregate shock to mimic the effects of a Sudden Stop. Time is discrete, indexed by  $t = \{1, 2, \dots\}$ , and the economy is populated only by atomistic heterogeneous firms and identical infinitely-lived households.

### 2.1 Description of the economy

**Firms** There is a unit-measure continuum of firms indexed by  $j$ , ex-ante identical that behave as atomistic, price-taker producers. They are identified by their own productivity level and their stock of capital.

They seek to maximize the expected discounted value of the dividend plan  $(d_{jt})_{t \geq 0}$  paid to their shareholders, which is equivalent for them to maximize their inter-temporal value. Thus, their objective function is  $\forall j$ :

$$\mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \mathcal{B}_j^t d_{jt} \right] \quad (1)$$

Where  $\mathcal{B}_j < 1$  is the firm's discount factor.

At every period, the firms decide their production level  $y_{jt}$ , their labor demand  $l_{jt}$ , and the investment level  $i_{jt}$  which fixes the capital stock for the following period  $k_{jt+1}$ . They also choose the dividends  $d_{jt}$  -if any- that are distributed to their shareholders. We shall assume that local households are the sole owners of every firm in the economy.

**Production and capital accumulation** All firms produce a homogeneous final good<sup>12</sup> using a combination of labor and capital with a production function of the following form:

$$y_{jt} = Z_t \epsilon_{jt} F(k_{jt}, l_{jt})$$

where  $F : \mathbb{R}_+^2 \mapsto \mathbb{R}_+$ ,  $F'(\cdot) > 0$ ,  $F''(\cdot) < 0$  exhibits decreasing returns to scale and decreasing marginal returns in both inputs. We impose also that it satisfies Inada conditions, in particular,  $\lim_{k \rightarrow 0} \frac{\partial F}{\partial k} = +\infty$ ,  $\lim_{k \rightarrow +\infty} \frac{\partial F}{\partial k} = 0$ , and  $F(0, \cdot) = 0$ . This discourages firms to hit a null capital stock or wanting to invest ad infimum.

Capital accumulates according to the usual law of motion:

$$k_{jt+1} = (1 - \delta_t)k_{jt} + i_{jt}$$

where  $\delta_t \in (0, 1)$  is the depreciation rate of capital and  $i_{jt}$  the investment in capital made at  $t$ .

---

<sup>12</sup>For simplicity, we consider that this final good can also be used as input, as capital for production in the next period: it can be seen as a basket with all the goods in the economy.

**Risk structure** All producers face potentially four sources of uncertainty in this economy: two TFP shocks -both idiosyncratic and aggregate-, a death shock i.e. a risk of being forced out of the market, and changes in the depreciation rate common across producers.

*Idiosyncratic risks* At every period, all firms face an idiosyncratic productivity shock  $\epsilon_{jt} \forall t \geq 0$ , which is assumed to follow an AR(1) process:

$$\epsilon_{jt+1} = \rho_\epsilon \epsilon_{jt} + u_\epsilon \quad \text{where } \rho_\epsilon \in (0, 1), u_\epsilon \stackrel{\text{iid}}{\sim} \mathcal{N}(0, \sigma_\epsilon^2)$$

For every firm  $j$ , the collection of the first  $t$  realizations of the idiosyncratic risk, after its appearance on the market, is its history  $\epsilon_j^t = \{\epsilon_{j0}; \epsilon_{j1}; \dots; \epsilon_{jt}\}$ .

As usual in the literature, we will consider that this process for the TFP shock satisfies the Markov property so that  $\mathbb{P}(\epsilon_{jt+1} | \epsilon_j^t) = \mathbb{P}(\epsilon_{jt+1} | \epsilon_t) = \pi(\epsilon' | \epsilon)$  where  $\sum_{\epsilon'} \pi(\epsilon' | \epsilon) = 1, \forall \epsilon$ . This requires that (i) the stochastic process can be discretized in a grid  $E = \{\epsilon_1, \dots, \epsilon_n\}$ ,  $n < \infty$  using standard methods like [Tauchen \[1986\]](#) to find a finite state space, and that (ii)  $\forall j, u_\epsilon$  are independent and identically distributed. We can represent a well-behaved Markov Chain, meaning in particular that the transition probabilities do not change over time, with a transition matrix of the following form where all rows sum to one and all entries are non-negative.

$$\Pi(\epsilon) = \begin{bmatrix} \pi(\epsilon'_1 | \epsilon_1) & \dots & \pi(\epsilon'_n | \epsilon_1) \\ \vdots & \ddots & \vdots \\ \pi(\epsilon'_1 | \epsilon_n) & \dots & \pi(\epsilon'_n | \epsilon_n) \end{bmatrix}$$

Provided that a law of large numbers holds,  $\pi(\epsilon' | \epsilon)$  will represent the fraction of firms in the economy that will transition from a given  $\epsilon$  to an  $\epsilon'$ . This will also allow the existence of a unique stationary distribution of agents.

The other idiosyncratic source of risk for producers comes from a life-cycle component to the model whereby all firms face an exogenous exit probability  $\theta \in (0; 1)$  at every period which augments the uncertainty of the model. To maintain a distribution of firms over the unit interval, at every period  $\theta$  new firms enter the market with an initial level of capital  $k_0 > 0$ , a relatively small share of the mean capital at the stationary steady state. This follows from the observation that newly created firms tend to be small [see [Benavente and Külzer, 2008](#), for an empirical assessment of the evolution of firm creation and destruction in the Chilean economy between 1999 and 2006 using tax records]<sup>13</sup>. In the spirit of [Cooley and Quadrini \[2001\]](#) this feature incentivizes patterns where small young firms would like to invest heavily in their capital stock to attain their optimal size.

*Aggregate risk* The economy as a whole also faces aggregate productivity uncertainty. As discussed in the subsequent sections, I will not consider both shocks at the same time but it is easier to write the model

<sup>13</sup>This is broadly corroborated in my dataset: the capital of new entrants in 1996 is roughly half of the mean capital stock per firm. As we discuss later, the dataset excludes the smallest firms in the economy, those with less than 10 employees (and that appear on the tax records used by [Benavente and Külzer \[2008\]](#) if they belong to the formal economy), and therefore this share is very likely overestimated.



with both sources of uncertainty explicitly.

The aggregate level of productivity  $Z_t$  is stochastic and its moment over the business cycle the economy is in. We define  $Z_t = Z_0 e^{z_t}$  where  $z_t$  is a TFP shock that we assume follows an AR(1) process with persistence  $\rho_z$ .

$$z_{t+1} = \rho_z z_t + u_z \quad \text{where } \rho_z \in [0, 1], u_z \stackrel{\text{iid}}{\sim} \mathcal{N}(0, \sigma_z^2)$$

Also, the economy can be subject to changes in the depreciation rate  $\delta_t$  to mimic outflows of capital used to close the current account deficit. For simplicity, I will assume that it follows a isomorph stochastic process than  $Z_t$  so that  $\delta_t = \delta_0 e^{\hat{\delta}_t}$  where  $\hat{\delta}_t$  is an AR(1) process:

$$\hat{\delta}_{t+1} = \rho_\delta \hat{\delta}_t + u_\delta \quad \text{where } \rho_\delta \in [0, 1], u_\delta \stackrel{\text{iid}}{\sim} \mathcal{N}(0, \sigma_\delta^2)$$

**Distortions and misallocation** The most important distortion in the model is that firms can finance their operation and their investment through retained earnings only. This is reflected in the existence of an ad hoc positivity constraint attached to  $d_{jt}$  at every period. This constraint forbids producers cannot extract resources from the households through equity. Note that there is no debt market in the model which eliminates firms' ability to access external finance. This is a simplifying yet restrictive restriction that allows for an easier computational implementation as it avoids the expansion of the state space in more directions.

This restriction on the firms' finances can generate misallocations of resources in the economy in the spirit of [Hsieh and Klenow \[2009\]](#). Indeed, small productive firms would like to invest a lot to attain their optimal size and maximize (1). However, they might not have sufficient retained earnings to attain that size (precisely because of their small capital), and because of the constraint and the absence of capital markets, they will be obliged to remain at a suboptimal size for at least one more period. In other words, they cannot insure against their idiosyncratic risk to be able to have the liquidity to invest in case of a positive productivity shock that would require them to scale their size.

We shall assume that the labor and final good markets work under perfect competition.

**Distribution** Firms are identified by their current level of capital  $k_{jt}$ , and the realization of their idiosyncratic TFP shock  $\epsilon_{jt}$ . We can therefore, summarize their distribution over the state space as a probability measure<sup>14</sup>  $\Lambda_t$  defined over the Borel  $\sigma$ -algebra generated by the open subsets of the product of the capital and the idiosyncratic productivity spaces  $K \otimes E$ .

$\Lambda_t$  is a time-dependent object with the following law of motion  $\Lambda_{t+1} = \Gamma(Z_t, \delta_t, \Lambda_t)$  where  $\Gamma(\cdot)$  is a mapping that describes the distribution's law of motion.

---

<sup>14</sup>A probability measure is a mapping  $\mathbb{P} : \mathcal{A} \mapsto [0, 1]$  where  $\mathcal{A} \in \mathcal{P}(\Omega)$  that satisfies: (i)  $\mathbb{P}(\emptyset) = 0$ , (ii)  $\mathbb{P}(\Omega) = 1$ , and (iii) is  $\sigma$ -additive i.e. for every sequence  $(A_n)_{n \in \mathbb{N}}$  of pairwise disjoint sets of  $\mathcal{A}$ ,  $\mathbb{P}\left(\bigcup_{n \in \mathbb{N}} A_n\right) = \sum_{n \in \mathbb{N}} \mathbb{P}(A_n)$

Finally, given that we assumed that  $\Pi(\epsilon)$  is well-behaved and thus stationary, and that some law of large numbers applies,  $\Lambda$  describes a share of firms in a given region of the probability space.

**Representative household** There is a multitude of identical infinitely-lived households. They consume the firms' production, supply labor in a perfectly elastic manner, and are the sole owners of all the producers. Thus, they receive the entirety of dividends distributed which complements their labor income.

Provided that their preferences are homothetic, we can aggregate this side of the economy and solve their problem as for a representative household with the aggregate variables directly (in upper cases). It chooses consumption streams  $(C_t)_{t \geq 0}$  and the labor supply  $(L_t)_{t \geq 0}$  by maximizing its intertemporal expected utility that we assume is separable between consumption and labor  $u(C_t, L_t) = v_c(C_t) + v_l(L_t)$ .

$$\mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \right]$$

where  $u : \mathbb{R}_+ \rightarrow \mathbb{R}$ ,  $u(\cdot) \in \mathcal{C}^2$ , is increasing and concave in consumption with  $u'(0, \cdot) = +\infty$

Setting the consumption good as the numéraire, its budget constraint reads  $w_t(Z_t, \delta_t, \Lambda_t)L_t + D_t = C_t$  at every period.

Note that the ownership structure of the economy implies that we can define the firm's discount factor in terms of the household's utility only. Indeed,  $\mathcal{B}_j$  will be equivalent to the consumer's pricing kernel and thus to the market price of risk and this is for all firms in the economy.

$$\mathcal{B}_j = \mathcal{B} = \beta \underbrace{\frac{\partial U(\cdot)/\partial C'}{\partial U(\cdot)/\partial C}}_{m_{t+1}}$$

**Timing** We will assume that at each period the agents get information and make decisions in the following order. Since the economy is infinitely-lived, this sequence of events will occur at  $\forall t \geq 0$ .

Beginning  $t$

- i. The new  $\theta$  entrants access the market with the same initial capital endowment  $k_0$ . They also are assigned an initial realization of the productivity shock  $\epsilon_0$  drawn from their ergodic distribution.
- ii. The economy is hit by the aggregate TFP shock  $z$ .
- iii. All firms participate in the lottery and know if they will exit the market at the end of the period and the realization of the idiosyncratic TFP shock.
- iv. The agents optimally make their decisions following their policy rules. The firms demand labor, pay their dividends (if any), produce and invest to set their capital stock for the following period. In parallel, the representative household supplies labor and consumes the final

good produced by the firms. All markets must clear.

v. The firms that were chosen to exit the market in (iii) do so.

End  $t$ , beginning  $t + 1$

## 2.2 Agents' problems and policy functions

This type of dynamic optimization problem in discrete time can be solved using standard recursive methods [see [Ljungqvist and Sargent, 2012](#), for a textbook treatment]. The agents' problems are written in recursive form using Bellman equations, where the prime notation refers to the next-period variables. Their solution is a collection of policy functions that characterize the optimal behavior of agents. The detailed analytical solution is found in Appendix A.

**Firms** All firms in the economy face the same problem but their solution will depend them being constrained or not. Note that all firms with the same capital stock  $k$  and realization of the idiosyncratic TFP shock  $\epsilon$  will take the same decisions conditional on their draw in the exit lottery. We can write their dynamic problem as follows:

$$\begin{aligned}
V(k, \epsilon, Z, \delta, \Lambda) &= \max_{k', l, d, i} d + \mathcal{B} \mathbb{E} V(k', \epsilon', Z', \delta', \Lambda') \\
s.t. \quad k' &= (1 - \delta)k + i \\
i &= Z\epsilon F(k, l) - lw(Z, \delta, \Lambda) - d \\
d &\geq 0 \\
k' &\geq 0 \\
\Lambda' &= \Gamma(Z, \delta, \Lambda)
\end{aligned} \tag{2}$$

We will consider that the last inequality constraint  $k' \geq 0$  always slacks for simplicity in the implementation of the model. Moreover, the first three constraints can be written trivially as one  $d = Z\epsilon F(k, l) - lw(\cdot) + (1 - \delta)k - k' \geq 0$ . The KKT multiplier attached to it is  $\mu_d$ . As mentioned in Appendix A the KKT system is well-behaved.

Given that labor is an adjustment variable and that firms decide their demand for labor after getting to know the realization of the exit lottery and the TFP shocks, they can all adjust optimally. As shown in Appendix A regardless of their type, all firms will demand the optimal quantity of labor and their policy function. Their choice of labor is determined by an efficiency condition where the marginal product of labor equates to the equilibrium wage and the individual labor demand reads:

$$\mathbf{g}_l^*(\mathbf{k}, \epsilon, \mathbf{Z}, \delta, \Lambda) : \quad l^* = \left( \frac{\partial F}{\partial l} \right)^{-1} \left( \frac{w(Z, \delta, \Lambda)}{Z\epsilon} \right) \tag{3}$$

The choices of future capital and distribution of dividends vary across the situation of each firm. We distinguish three statuses among the firms: (i) they will leave the market, (ii) they are unconstrained, or (iii) their retained earnings are not enough to invest in their optimal stock of capital.

**Exiting firms** These firms got a ‘bad’ draw in the exit lottery described in the previous subsection. They know it’s their last period to produce and thus  $V(k', \epsilon', Z', \delta', \Lambda') = 0$ . This makes their problem become a one-period, where they want to maximize the last dividend stream distributed to the household. A very natural terminal condition appears for them  $k'^e = 0$  which sets up their policy function for capital:

$$\mathbf{g}_k^e(\mathbf{k}, \epsilon, \mathbf{Z}, \delta, \Lambda) : \quad k'^e = 0 \quad (4)$$

Using the law of motion of capital, we find the expression for investment also very intuitive as the firm liquidates all its capital to satisfy the terminal condition:

$$\mathbf{g}_i^e(\mathbf{k}, \epsilon, \mathbf{Z}, \delta, \Lambda) : \quad i^e = -(1 - \delta)k \quad (5)$$

Finally, the policy function for the dividends distributed is given by the budget constraint:

$$\mathbf{g}_d^e(\mathbf{k}, \epsilon, \mathbf{Z}, \delta, \Lambda) : \quad d^e = Z\epsilon F(k, l^*) - l^*w(Z, \delta, \Lambda) + (1 - \delta)k \quad (6)$$

Note that this should be weakly positive by the structure of the problem and thus should satisfy automatically the inequality constraint on  $d$ . Consider the case of a firm  $j$  that is chosen to leave the market at the end of the period. Provided that it has a weakly positive capital stock (it does by assumption), it would have some assets that it can liquidate to recover some value. If the cost of production exceeds the proceedings of the sale of current capital stock, the firm can choose simply redistribute them in dividends and not to produce. Therefore to behave rationally, exiting firms will pay weakly positive dividends on their last period of operation. This holds if the price of capital is strictly positive, which is the case in all non-degenerate markets with positive demand.

**Unconstrained Firms** These firms can finance the investment needed to attain their optimal size with retained earnings only and thus can act efficiently (denoted with  $*$ ). Using the FOC of the problem and provided that the derivative of the production function with respect to capital is bijective, we find the following closed-form expressions for the policy function for the next-period capital.

$$\mathbf{g}_k^*(\mathbf{k}, \epsilon, \mathbf{Z}, \delta, \Lambda) : \quad k'^* = \left( \frac{\partial F}{\partial k} \right)^{-1} \left( \frac{1}{\mathcal{B} \mathbb{E}(1 + \mu'_d) Z' \epsilon'} - \frac{(1 - \delta')}{Z' \epsilon'} \right) \quad (7)$$

Note that the presence of the multiplier  $\mu'_d$  denotes the existence of precautionary saving motifs for the firm. If a firm expects that the liquidity constraint will bind in the following period,  $\mathbb{E}(1 + \mu'_d) > 1$ , the firms will accumulate excess capital (while distributing dividends). Indeed, because of the assumption of decreasing returns of scale,  $\left( \frac{\partial F}{\partial k} \right)^{-1}$  is a decreasing function and thus  $\frac{\partial g_k^*}{\partial \mathbb{E} \mu'_d} > 0$ .

The capital decision pins down the investment decision:

$$\mathbf{g}_i^*(\mathbf{k}, \epsilon, \mathbf{Z}, \delta, \Lambda) : \quad i^* = \left( \frac{\partial F}{\partial k} \right)^{-1} \left( \frac{1}{\mathcal{B} \mathbb{E}(1 + \mu'_d) Z' \epsilon'} - \frac{(1 - \delta')}{Z' \epsilon'} \right) - (1 - \delta)k \quad (8)$$

Finally, the policy function for the dividends these firms are going to pay their shareholders is given by the budget constraint:

$$\mathbf{g}_d^*(\mathbf{k}, \epsilon, \mathbf{Z}, \delta, \Lambda) : \quad d^* = (1 - \delta)k + Z \epsilon F(k, l^*) - l^* w(Z, \Lambda) - \left( \frac{\partial F}{\partial k} \right)^{-1} \left( \frac{1}{\mathcal{B} \mathbb{E}(1 + \mu'_d) Z' \epsilon'} - \frac{(1 - \delta')}{Z' \epsilon'} \right) \quad (9)$$

**Constrained Firms** This last type of firm does not manage to finance its optimal investment with retained earnings and therefore cannot distribute dividends to the household.

$$\mathbf{g}_d^c(\mathbf{k}, \epsilon, \mathbf{Z}, \delta, \Lambda) : \quad d^c = 0 \quad (10)$$

Similarly, the capital choice is given by the optimality conditions of the firms' problem and pins down the policy function of investment -as in the previous case.

$$\mathbf{g}_k^c(\mathbf{k}, \epsilon, \mathbf{Z}, \delta, \Lambda) : \quad k'^c = \left( \frac{\partial F}{\partial k} \right)^{-1} \left( \frac{1 + \mu_d}{\mathcal{B} \mathbb{E}(1 + \mu'_d) Z' \epsilon'} - \frac{(1 - \delta')}{Z' \epsilon'} \right) \quad (11)$$

$$\mathbf{g}_i^c(\mathbf{k}, \epsilon, \mathbf{Z}, \delta, \Lambda) : \quad i^c = \left( \frac{\partial F}{\partial k} \right)^{-1} \left( \frac{1}{\mathcal{B} \mathbb{E}(1 + \mu'_d) Z' \epsilon'} - \frac{(1 - \delta')}{Z' \epsilon'} \right) - (1 - \delta)k \quad (12)$$

As in the previous case, the expectation of a future liquidity constraint to bind increases the capital accumulation of the firm because of precautionary motifs:  $\frac{\partial g_k^u}{\partial \mathbb{E} \mu'_d} > 0$ . However, as one expects, the fact that the current liquidity constraint binds depresses the capital stock of these firms:  $\frac{\partial g_k}{\partial \mathbb{E} \mu_d} < 0$ .

**Representative Household** Because none of the choice variables of the household determine a state, they are chosen solely on an instantaneous gratification criterion. Indeed, her dynamic program can be written without loss of generality as a sequence of independent one-period consumption-leisure utility maximization problems where at all periods she solves:

$$\max_{C, L} u(C, L) \quad s.t. \quad w(Z, \delta, \Lambda)L + D = C \quad (13)$$

Its labor supply and demand of the final good are given respectively by the following policy functions:

$$\mathbf{g}_L^{\text{HH}}(\mathbf{Z}, \delta, \Lambda) : L^{\text{HH}} = \left( \frac{\partial u}{\partial L} \right)^{-1} \left( -w(Z, \delta, \Lambda) \frac{\partial u}{\partial C} \right) \quad (14)$$

$$\mathbf{g}_C^{\text{HH}}(\mathbf{Z}, \delta, \Lambda) : C^{\text{HH}} = w(Z, \delta, \Lambda)L^{\text{HH}} + D \quad (15)$$

Note that the household receives is the sole shareholder of all the firms, its portfolio is perfectly diversified with respect to the idiosyncratic shocks.

## 2.3 Competitive equilibrium characterization

In this economy, the recursive equilibrium is a collection of aggregate quantities  $\{Y(\cdot), C(\cdot), K(\cdot), L(\cdot), D(\cdot)\}$ , of prices  $\{w(Z, \Lambda)\}$ , of policy functions  $\{g_k^x(\cdot), g_i^x(\cdot), g_l^x(\cdot), g_d^x(\cdot), g_C^{HH}(\cdot), g_L^{HH}(\cdot)\}$  for  $x \in \{e, *, c\}$ , of value functions  $\{V(k, \epsilon, Z, \Lambda), V^{HH}(\Lambda)\}$ , and of the probability measure  $\Lambda$ 's law of motion such that all agents' programs are solved optimally and all markets clear. That is

- Firms take as given  $w(Z, \delta, \Lambda)$ ,  $Z'$ ,  $\delta'$  and  $\Lambda' = \Gamma(Z, \delta, \Lambda)$  to solve for their dynamic maximization problem (2) of their value function  $V(k, \epsilon, Z, \delta, \Lambda)$  under its constraints. They optimally choose their future capital stock, investment, labor demand, and distribution of dividends following the corresponding policy functions describes in the previous section  $\{g_k^x(k, \epsilon, Z, \delta, \Lambda), g_i^x(k, \epsilon, Z, \delta, \Lambda), g_l^x(k, \epsilon, Z, \delta, \Lambda), g_d^x(k, \epsilon, Z, \delta, \Lambda)\}$  for  $x \in \{e, *, c\}$ , where the superscript denotes respectively the last period behavior for exiting firms, the optimal behavior and the constrained behavior.
- Households solve their problem to maximize  $V^{HH}(Z, \delta, \Lambda)$  under her budget constraint, to set their consumption and labor supply in concordance with their policy functions  $g_C^{HH}(Z, \delta, \Lambda)$ , and  $g_L^{HH}(Z, \delta, \Lambda)$ . They take as exogenous the market wage rate  $w(Z, \delta, \Lambda)$  and the aggregate objects  $Z, \delta$  and  $\Lambda$ .
- The resource constraint of the economy holds at all dates meaning that total production across firms is met the consumption level demanded by the households.

$$\begin{aligned} C^{HH}(Z, \delta, \Lambda) &= \int \int \underbrace{y(k, \epsilon, Z, \delta, \Lambda) + (1 - \delta)k}_{\text{Available resources in the economy}} - \underbrace{(1 - \theta) \{ \mathbb{1}_c g_k^c(k, \epsilon, Z, \delta, \Lambda) + \mathbb{1}_* g_k^*(k, \epsilon, Z, \delta, \Lambda) \}}_{\text{From investment of firms that will stay on the market}} \Lambda(\epsilon, k) d\epsilon dk \\ &= \int \int Z\epsilon F(k, g_l^*(k, \epsilon, Z, \delta, \Lambda)) + (1 - \delta)k \\ &\quad - (1 - \theta) \{ \mathbb{1}_c g_k^c(k, \epsilon, Z, \delta, \Lambda) + \mathbb{1}_* g_k^*(k, \epsilon, Z, \delta, \Lambda) \} \Lambda(\epsilon, k) d\epsilon dk \end{aligned}$$

Where  $\mathbb{1}_c$  is an indicator function for constrained firms and  $\mathbb{1}_*$  is an indicator for unconstrained firms. Note that  $\mathbb{1}_* = 1 - \mathbb{1}_c$ .

- The labor market clears for a unique value of the wage determined in equilibrium  $w(Z, \delta, \Lambda)$ :

$$L^{HH}(Z, \delta, \Lambda) = \int \int g_l^*(k, \epsilon, Z, \delta, \Lambda) \Lambda(\epsilon, k) d\epsilon dk$$

- The probability measure  $\Lambda$  that describes the firms' distribution, follows a law of motion  $\Lambda' = \Gamma(Z, \delta, \Lambda)$  which is consistent with firms' optimal decisions and their life cycle. In particular, we have that the distribution of the firms over the state space is a sum of surviving firms and newcomers that behave according

to their policy functions and that are either constrained or unconstrained.

$$\begin{aligned}\Lambda'(k', \epsilon'; Z, \Lambda) = & (1 - \theta) \underbrace{\int \int \{\mathbb{1}_* g_k^*(k, \epsilon, Z, \delta, \Lambda) + \mathbb{1}_c g_k^c(k, \epsilon, Z, \delta, \Lambda)\} \pi(\epsilon' | \epsilon) d\epsilon d\Lambda(k, \epsilon)}_{\text{Incumbents}} \\ & + \theta \underbrace{\int \int \{\mathbb{1}_* g_k^*(k_0, \epsilon_0, Z, \delta, \Lambda) + \mathbb{1}_c g_k^c(k_0, \epsilon_0, Z, \delta, \Lambda)\} \pi(\epsilon' | \epsilon) d\epsilon d\Lambda(k, \epsilon)}_{\text{New entrants}}\end{aligned}$$

The interpretation of the indicator functions is the same as above.

### 3 Calibration for the steady state and stationary distributions

I calibrate the model for the steady state and stationary equilibrium on a capital grid with 100 nodes using Julia v.1.8. [Bezanson et al., 2017]. The code is adapted from Martin-Baillon [2021] who very generously shared it with me for the purposes of this work. It is meant to mimic the cross-sectional distribution of firms over the capital grid that we can see on the ENIA data set for 1997.

This simulation of the model relies on a uniform truncated histories method to get a reduced-heterogeneity representation of the economy that can be simulated. Indeed, as previously mentioned, the state space of the model described in section 2 includes a high-dimensional object,  $\Lambda$ , that needs to be represented in a more succinct manner in order to calibrate the stationary equilibrium and simulate the economy. In this section, we consider that the depreciation rate is fixed  $\delta$  and time-invariant at the steady state in order to get the stationary distributions. My goal is to calibrate the model with two different values of  $\delta$  to check its effects on the equilibrium and the cross-sectional distribution of producers over the capital grid.

#### 3.1 Reduced heterogeneity representation: truncated histories

To perform a quantitative assessment of the model, it is required to find a way to reduce heterogeneity among agents so the calibration is computationally feasible. To do so, this code uses uniform truncated histories as a way to do so [Le Grand and Ragot, 2022, Ragot, 2018].

A  $N$ -truncated history  $\tilde{e}^N$  is the collection of the last  $N < \infty$  idiosyncratic shocks of agent  $j$  before the realization of the current idiosyncratic state  $e \in \mathcal{E}$  is known. Since it iterates backward, it is useful to think about  $e_0$  as today's beginning-of-period idiosyncratic state. The  $N$ -truncated history is going to be  $\tilde{e}_j^N = \{\tilde{e}_{jN-1}; \dots; \tilde{e}_{j0}\}$ . After today's shock  $e_0$  occurs, the truncated history is going to be  $e^N = \{e_{jN-1}; \dots; e_{j0}\}$  where the absence of tildes signifies an end-of-period history. Importantly, we will consider that all agents sharing the same truncated history will behave in exactly the same way as -by construction- they will have the same beginning of period capital. The rationale is that the most recent shocks are those that carry the most information on the current situation for the agent. Indeed, the shock that hit the firm this year has more weight in how the producers make decisions today than the one that occurred, say, 100 years ago.

With this notation, we can build a transition matrix for the histories. If an agent can move from  $\tilde{e}^N$  to  $e^N$  it means that he had  $e = \tilde{e}_0$  and  $e' = e_0$ . The transition between histories is thus governed by the Markov matrix described in Section 2. We say that two histories are compatible if this change can occur and we denote it with the indicator function  $\mathbb{1}_{\tilde{e}^N \succcurlyeq e^N}$  being equal to one. Then the probability matrix across truncated histories offer given by:

$$M(\mathbf{X}) = \mathbb{1}_{\tilde{e}^N \succcurlyeq e^N} \Pi_{e_0, \tilde{e}_0}(\mathbf{X}) \quad (16)$$

Where  $\mathbf{X} = \{K, S, L, D, w, Z, \delta, g_k^o, g_l^o\}$  designs the aggregate state vector,  $g_k^o, g_l^o$  design the discretized policy functions (for both the constrained and unconstrained firms), and  $\Pi$  is defined in Section 2.  $M(\mathbf{X})$  is thus a transition (sparse) matrix implied by decision rules.



This assumption on behavior allows aggregating agents so that the economy is represented by a finite number of ‘representative’ agents, each with a different realization of their truncated history. With this simplification in the aggregation, the steady state of the model can be calibrated with value function iterations and checking for the market clearing conditions. Indeed, the distribution of producers is discretized in a finite number of bins or ‘representative’ histories. This number of histories is given in the case of uniform truncations by all the combinations of shocks that can occur in the  $N$  periods we are considering in the truncated history, i.e. by  $n^N$  where  $n$  is the number of states of the idiosyncratic process. Here I’ll consider truncations of length  $N = 4$  and an idiosyncratic TFP shock with  $n_e = 6$  states, thus the cross-sectional distribution considers  $6^4 = 1296$  histories. Let  $S_{e^N}$  represent the number of agents with a given truncated history  $e^N$ . With the previous notation of  $M$ , we can easily find a law of motion for this discrete distribution of agents over the idiosyncratic space.

$$S'_{e^N} = S_{e^N} M(\mathbf{X}) \quad (17)$$

This allows us to find a stationary equilibrium using value function iterations.

Finally, this approach incorporates new parameters  $\xi > 0$  that will allow capturing any residual heterogeneity within the bin from shocks that occurred before the last period of the truncation. The choice of those  $\xi$  can be made to have a more relevant description of the distributions. In my case with producers, they are defined as the ratios of the per capita marginal production (in each bin) after the realization of today’s shock and the per capita marginal production that could have been executed with the beginning-of-period capital. It is also particularly well suited to incorporate a Ramsey problem and assess optimal policies. In my case, there is no instrument for policy and I fix an exogenous null tax rate to circumvent it.

### 3.2 Parametization

To calibrate the model and find a steady state I will make the following choices. I use a Julia v1.8 [Bezanson et al. \[2017\]](#) taken from [Martin-Baillon \[2021\]](#) whom I thank once again. This subsection aims to calibrate the quantitative model for the 1996 plant data for Chile, considering it as the steady state before the crisis following [Ates and Saffie \[2021\]](#). As usual, these calibrations omit further aggregate shocks.

**Production function** We will assume firms produce using Cobb-Douglas technology. Their production will therefore be determined by:

$$y_{j,t} = Z_t \epsilon_t k_{jt}^\alpha l_{jt}^\nu$$

Where  $\alpha, \nu \in (0; 1)$  represent respectively the capital and labor share in the production process. To satisfy our assumptions regarding decreasing returns, we will have  $\alpha + \nu < 1$ .

**Utility function** The representative household utility is the isoelastic:

$$U(C_t, L_t) = \frac{C_t^{1-\gamma} - 1}{1-\gamma} - \frac{1}{\chi} \frac{L_t^{1+(1/\phi)}}{1 + \frac{1}{\phi}}$$

Where  $\gamma \geq 0$  is the elasticity of substitution for consumption,  $\phi > 0$  is the Frisch elasticity of labor supply, and  $\chi > 0$  is a scaling parameter. By the properties of the isoelastic utility functions, in the special case where  $\gamma = 1$ , the first component of  $U(\cdot)$  becomes a log utility:  $U(C_t, L_t) = \log(C_t) + \frac{1}{\chi} \frac{L_t^{1/\phi}}{1/\phi}$ . The scaling parameter depends on equilibrium variables and it corresponds to  $\chi = \frac{w \partial U / \partial C(C)}{L^\phi}$  where  $w, C, L$  are the equilibrium values for the wage, the consumption level and labor supply.

**Parameters** I use the following parameters closely following the choices made in [Ates and Saffie \[2021\]](#) and in [Martin-Baillon \[2021\]](#) to calibrate the model on annual data:

| Parameter   | Description                                | Value  |   |
|-------------|--|--------|---|
| Preferences |  |        |   |
| $\beta$     | Discount Factor                            | 0.96   | Usual value                                 |
| $\gamma$    | Elasticity of substitution for consumption | 0.6    | <a href="#">Mendoza [1991]</a>              |
| $\phi$      | Frisch elasticity of labor supply          | 1.455  | <a href="#">Mendoza [1991]</a>              |
| Production  |  |        |   |
| $\alpha$    | Capital share                              | 0.25   | Author's choice                             |
| $\nu$       | Labor share                                | 0.55   | <a href="#">Guerriero [2019]</a>            |
| $\delta_0$  | Depreciation rate (baseline)               | 8%     | <a href="#">Bergoeing et al. [2002]</a>     |
| Shocks      |  |        |   |
| $\theta$    | Exit probability                           | 1.34%  | <a href="#">Benavente and Külzer [2008]</a> |
| $\rho_z$    | Persistence idiosyncratic TFP shock z      | 0.8    | <a href="#">Martin-Baillon [2021]</a>       |
| $\sigma_z$  | Variance of z's innovation $u_z$           | 29.14% | Author's choice                             |
| $n_z$       | Number of grid points for z                | 6      | Author's choice                             |
| $N$         | Lenght truncated histories                 | 4      | Author's choice                             |

Table 1: Baseline parameters description

Notes: [Mendoza \[1991\]](#) is a study on small open economies (Canada). The capital share is set to maintain decreasing marginal retruns at 80% as in [Martin-Baillon \[2021\]](#). The exit prbability corresponds to the average destruction rate of firms with more than 10 employees found in [Benavente and Külzer \[2008\]](#) for 1999-2006 with tax records. The variance of the innovation of the idiosyncratic risk is set so that the second moment of the stationary distribution on the capital grid corresponds to the data's for the distribution of plants' capital in 1997 (= 2.13).

### 3.3 Stationary equilibrium and shock

The stationary equilibrium satisfies the conditions described in section 2.3. The baseline steady state is found numerically as described above for 1997. Note that this calibration assumes rational expectations and abstracts from the aggregate shocks: we want to assess how the cross-section behaves in the presence of uninsurable idiosyncratic risk.

In my model, I do not model any access to external financing nor an international financial market where the materialization of a Sudden Stop shock would be straightforward. I did not do so for computational reasons and because little over 20% of the firms in my data set have any revenue from exports, only 6% had any foreign equity and 24.1% reported importing intermediate goods in 1997.

Hence, I model the distress in financial markets in a very ad hoc manner, as a shock on the depreciation rate of installed capital. The rationale goes as follows. With an intertemporal approach, the current account of a

country is simply defined by the difference between its aggregate savings and its investment  $CA = S - I$ . While the economy described in section 2 does not model explicitly any saving decision -the household has a fully diversified portfolio and firms are excluded from external financing- capital accumulation works as a sort of saving vehicle at least for firms.

As noted in section 2.2 the investment policy function for surviving firms has a positive derivative with respect to  $\mathbb{E} \mu'_d$  is positive highlighting precautionary motifs in the decision of accumulating (excess) capital. Indeed, firms that can only finance their investments through retained earnings know that in case of having a positive realization of  $\epsilon'$  they will like to scale their production as their marginal profit will be high.

Since there is no asset to save and insure against their productivity risk, firms have the incentive to accumulate excess capital to be able to use their retained earnings if needed in the following period. Take a closer look at their resource constraint:

$$\forall j, t \quad k' = \underbrace{(1 - \delta)k + Z\epsilon F(k, l)}_{\text{Positive effect of } k} - lw(\cdot) - d$$

By possessing more installed capital, the feasibility set for  $k'$  is trivially enlarged. This means that firms' optimal behavior might imply accumulating extra resources to avoid being constrained in the future making firms potentially 'big'. This might not correspond to the efficient allocation that might prefer that those resources finance the growth of smaller productive firms with high marginal profits. This mechanism is at the heart of the misallocation of capital that we want to explore.

To mimic capital being more expensive, I assume that the Sudden Stop increases the depreciation rate of capital  $\tilde{\delta} > \delta$ . By doing so, the economy is destroying more resources at every period. This can be associated with the effects of financial markets drying out in different manners. (i) The economy is collecting the extra resources as it needs to ship funds abroad in order to close the current account deficit, which is at the heart of these events. The wedge  $(\tilde{\delta} - \delta)k$  is in fact the value that is paid to the rest of the world to generate capital outflows. (ii) In a perspective that emphasizes the importance of asset pricing in the midst of a financial crisis in the spirit of Fisherian Deflation mechanisms [Bianchi and Mendoza, 2020] this increased depreciation represents in fine a loss in the valuation of the firms. Since the model excludes trading titles, this mechanism is not present here but it is widely described in the literature.

I calibrated the stationary equilibrium in the model, first with the baseline specification presented in the previous subsection taking  $\delta = 0.8$  as given. It reproduces the unimodal log distribution of firms' capital that is present in the dataset (see Figure 2) but it's not smooth enough, particularly on the right tail. The second calibration shows the stationary equilibrium if one assumes that the depreciation rate shifts by 1% and remains at  $\tilde{\delta} = 0.808$  forever after while all other parameters stay at their baseline values. It is a static analysis that works more as a proof of concept of comparative statics on how the cross-sectional distribution behaves. Interestingly, the distribution does shift to the left as in the data.

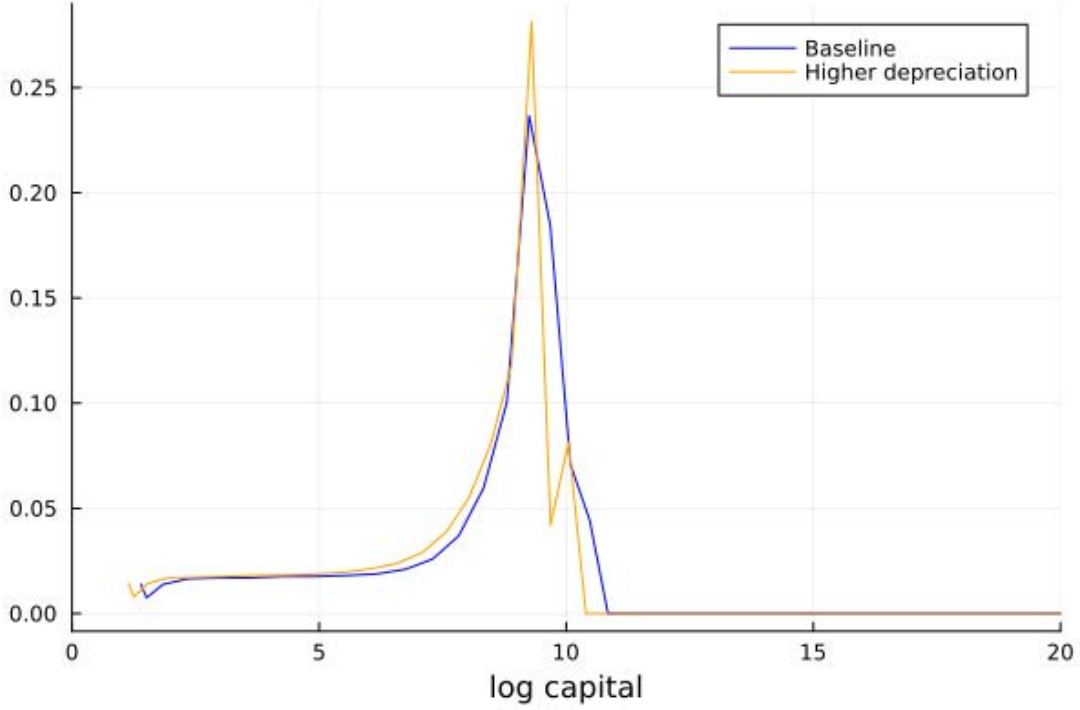


Figure 3: Stationary distribution on the capital grid

*Notes:* This represents the stationary distribution of firms on the capital grid with two different values for the depreciation rate  $\delta = 0.8$  in the baseline and  $\tilde{\delta} = 0.0808$ . As with the KDE, the values of the grid and capital stocks are augmented by one before taking logs.

Concerning capital misallocation, following [Martin-Baillon \[2021\]](#), it can be assessed as the ratio of optimal investment  $\hat{I}$ , computed as if all producers manage to attain their optimal size and  $\mu_d = 0 \forall j, t$ , and realized investment level in the economy  $I$ . Trivially, the closer to one this ratio, the better the economy is at allocating capital towards its more efficient use. This measure decreases by 0.018 with the increased depreciation which is in line with the theory.

$$I_{miss} = \frac{\hat{I}}{I}$$

Table 2 summarizes the aggregate variables at both steady states. Note that most of them increase with higher depreciation. Indeed, even to replace the installed capital, the economy will have to produce more which augments the demand for labor and the wage. These last two variations must be at the origin of the higher consumption level as dividends are diminished. Note that investment and net investment augment while the misallocation worsens. Combined with lower dividends on aggregate, these are signs that the unconstrained, bigger firms are the ones investing in capital, likely beyond their efficient level, while smaller firms remain to have a sub-optimal size given their idiosyncratic productivity.

|                  | <b>Baseline</b><br>$\delta = 8\%$ | <b>Higher depreciation</b><br>$\tilde{\delta} = 8.08\%$ | <b>Difference</b> |
|------------------|-----------------------------------|---|-------------------|
| Equilibrium wage | 1.24                              | 1.27  | 2.25%             |
| Capital          | 2.09                              | 2.37  | 11.84%            |
| Investment       | 0.17                              | 0.19  | 12.80%            |
| Net Investment   | 2.09                              | 2.37  | 11.85%            |
| Misallocation    | 0.58                              | 0.57  | -1.84%            |
| Dividends        | 0.16                              | 0.15  | -8.25%            |
| Production       | 0.72                              | 0.75  | 3.65%             |
| Labor            | 0.32                              | 0.32  | 1.44%             |
| Consumption      | 0.55                              | 0.56  | 0.53%             |

Table 2: Stationary equilibrium

*Notes:* All values are in aggregate levels. Note in particular that to plot the capital distribution before, the capital grid had to be normalized with respect to the data what explains the small values here. Net Investment corresponds to the investment net of depreciation and wage is the equilibrium price that clears the labor market.

## 4 Simulation and Dynamics

Now I run simulations of the stationary equilibrium found in the previous section. This is done by perturbation with Dynare for Matlab [Adjemian et al., 2022]. I chose to test the effects of an aggregate shock with each component independently. Instead of shocking the steady state with both a negative TFP shock and a positive shock on the depreciation rate at the same time, I assume one is fixed at the time. My rationale is to assess how a shock in the depreciation rate can impact the aggregate variables through the misallocation, and how this shock can differ from an aggregate TFP which is more common in the literature.

All the below is made assuming a horizon of 25 years and simulating the economy 10000 times.

### 4.1 Depreciation Shocks

As mentioned in a previous subsection, I argue that the excess capital destruction from a positive shock in  $\delta$  can be seen as a way to model capital being more expensive in cases of Sudden Stops. Similarly, it is a way to model capital outflows (or the lack of inflows) that characterize these events. Remind that in Section 2 I defined the depreciation rate as  $\delta = \delta_0 e^{\hat{\delta}}$  where  $\hat{\delta} \sim AR(1)$ . The parametrization for the rational expectation stationary equilibrium made before was made as if  $\delta = \delta_0 = 0.8$  (non-stochastic) which corresponds to the data before the crisis.

I use Dynare to shock that stationary equilibrium with a positive shock in  $\hat{\delta}$ . I assess the effect of two shocks with medium and high persistence. I avoid very low values of  $\rho_\delta$ , say 0.1 because the Chilean economy never got similar inflows of capital as in the 1990's before the crisis. The parameters for the aggregate shock are summarized in Table 3.

| Parameter       | Description                            | Low persistence | High persistence |
|-----------------|--|-----------------|------------------|
| $Z$             | Level aggregate TFP                    | 1               | 1                |
| $\delta_0$      | Baseline depreciation rate             | 0.08            | 0.08             |
| $\rho_\delta$   | Persistence depreciation shock         | 0.5             | 0.95             |
| $\sigma_\delta$ | Variance innovation depreciation shock | 0.0001          | 0.00001          |

Table 3: Parameters aggregate shock to  $\delta$

Below, I show the impulse response functions of the economy to the two specifications of the economy

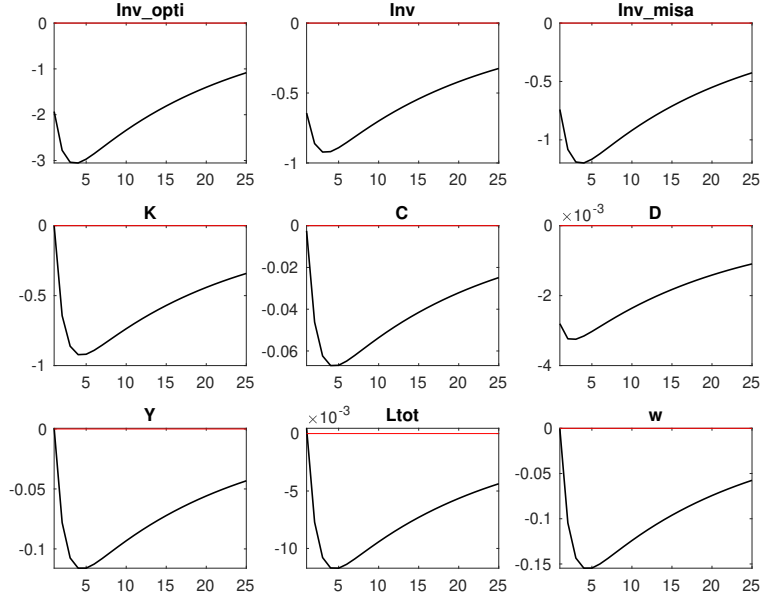


Figure 4: High Persistence shock on  $\delta$ ,  $\rho_\delta = 0.95$

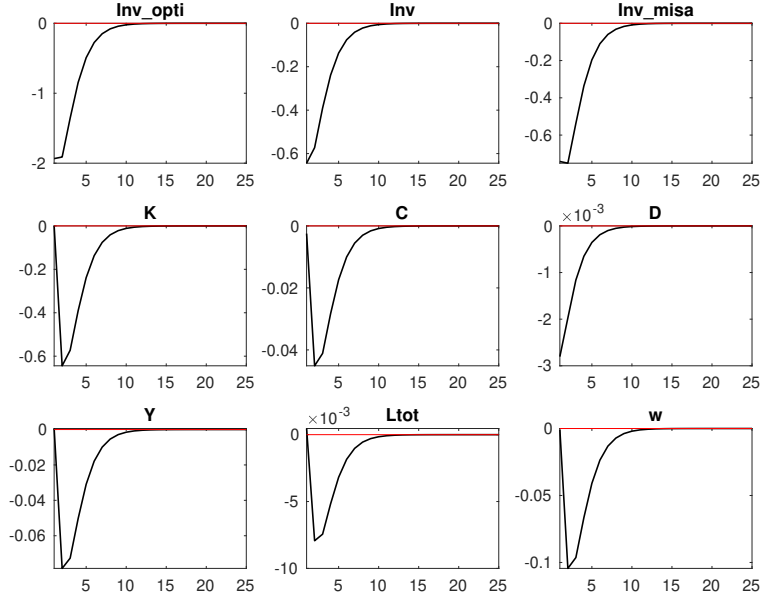


Figure 5: Low Persistence shock on  $\delta$ ,  $\rho_\delta = 0.5$

The simulated moments for aggregate values (and equilibrium wage) are summarized in Table 6

| Aggregate variable              |                  | Mean  | Std. Dev |
|---------------------------------|------------------|-------|----------|
| <b>Optimal Investment</b>       | High persistence | 1.267 | 10.896   |
|                                 | Low persistence  | 1.236 | 3.247    |
| <b>Realized investment</b>      | High persistence | 2.105 | 3.287    |
|                                 | Low persistence  | 2.096 | 1.000    |
| <b>Investment misallocation</b> | High persistence | 0.590 | 4.274    |
|                                 | Low persistence  | 0.602 | 1.268    |
| <b>Consumption</b>              | High persistence | 0.561 | 0.239    |
|                                 | Low persistence  | 0.561 | 0.071    |
| <b>Dividends</b>                | High persistence | 0.161 | 0.011    |
|                                 | Low persistence  | 0.161 | 0.004    |
| <b>Output</b>                   | High persistence | 0.728 | 0.415    |
|                                 | Low persistence  | 0.727 | 0.125    |
| <b>Capital</b>                  | High persistence | 2.107 | 3.286    |
|                                 | Low persistence  | 2.097 | 1.000    |
| <b>Labor</b>                    | High persistence | 0.318 | 0.042    |
|                                 | Low persistence  | 0.317 | 0.013    |
| <b>Wage</b>                     | High persistence | 1.261 | 0.552    |
|                                 | Low persistence  | 1.260 | 0.166    |

Table 4: Simulated moments of aggregates and equilibrium price following shock on the depreciation rate  $\delta$

In the previous section, we already realized that higher depreciation comes at the cost of increased investment misallocation. This exercise allows us to see that this misallocation is higher in cases of highly persistent shocks, which is pretty intuitive.

Importantly, we can see that these shocks, even with moderate persistence have a significant effect on the dynamics of the economy in the middle run. While the aggregate variables end up recovering within the horizon considered for the lower persistent shock, they remain below the mean for almost 10 years. In the case of a highly persistent shock, they do not in over 25 years. In the case of output in the high persistent shock, it is still 0.91% below trend 5 years after the shock which is pretty consistent with the results in [Ates and Saffie \[2021\]](#).

Finally, in the spirit of [Hsieh and Klenow \[2009\]](#) with these simulated moments of aggregate variables we can construct a first approximation of a productivity index of these two economies. Consider that the aggregate economy produces using a Cobb-Douglas technology such that  $Y = AK^\alpha L^\nu$  where A is a measure of aggregate TFP. Then using our parameters and the values found just above for aggregate quantities we can deduce the “physical productivity” of the economy as:

$$TFPQ = \frac{Y}{K^\alpha (wL)^\nu} = \frac{Y}{K^{0.25} (wL)^{0.55}} \quad (18)$$

This is not a rigorous depiction of the model as it neglects the aggregation issues at the heart of the heterogeneous agent model. Nonetheless, it can provide a first indication of how efficiently the overall economy produces. Note that since we set exogenously a deterministic  $Z = 1$ , one should respect these values to be in



that neighborhood.

Computing this we find that in the case of low persistent shocks this value is slightly higher than one 1.0003, while in the high persistent shock, it is slightly lower, 0.9999. Indeed, the difference is small but it gives a hint of diminished TFP consistent with [Ates and Saffie \[2021\]](#). It is straightforward that a lower TFP decreases the economy's steady state, and this is somewhat confirmed in the next subsection that treats an exogenous negative shock to the economy's technology.

## 4.2 Aggregate TFP shocks

As an experiment, I simulate the economy in an analogous fashion as before but now the aggregate shock relies on  $Z$  holding  $\delta$  constant and equal to  $\delta_0 = 8\%$ . I report their IRFs and simulated moments.

| Parameter  | Description                            | Low persistence | High persistence |
|------------|--|-----------------|------------------|
| $\delta$   | Baseline depreciation rate             | 0.8             | 0.8              |
| $z_0$      | Baseline depreciation rate             | 0.08            | 0.08             |
| $\rho_z$   | Persistence depreciation shock         | 0.5             | 0.95             |
| $\sigma_z$ | Variance innovation depreciation shock | 0.0001          | 0.00001          |

Table 5: Parameters aggregate shock to  $Z$

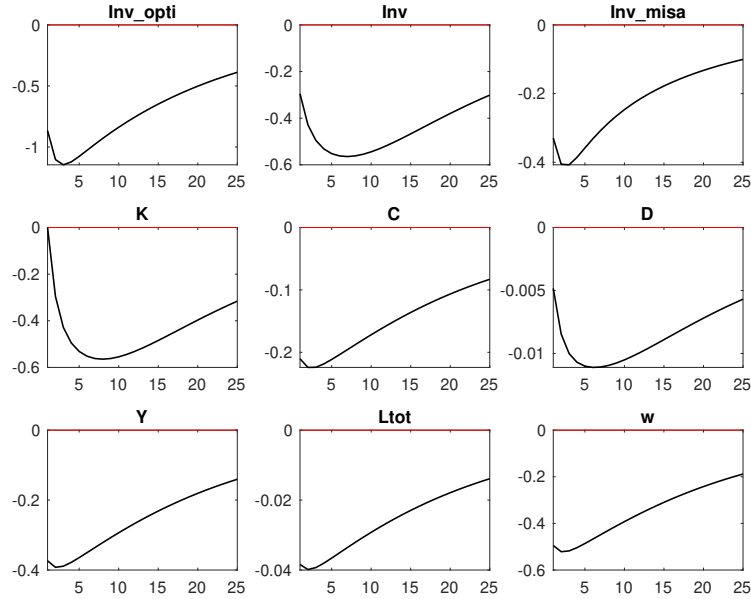


Figure 6: High Persistence shock on  $Z$ ,  $\rho_z = 0.95$

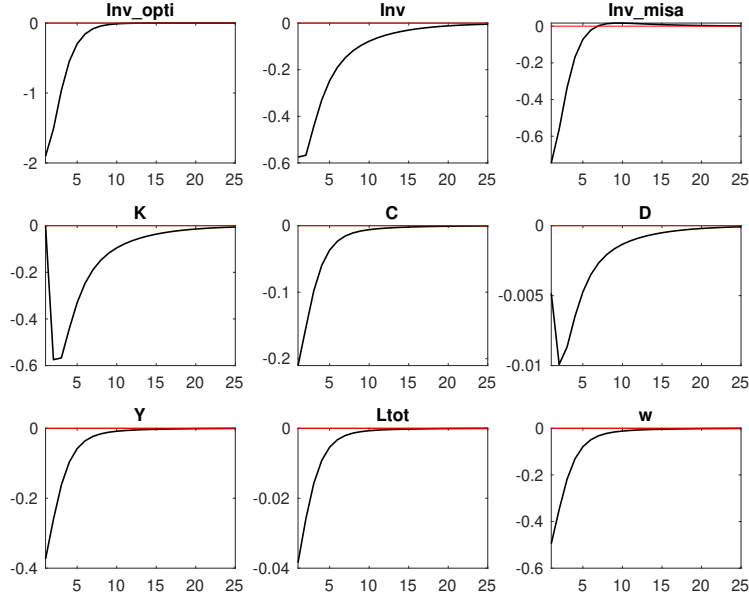


Figure 7: Low Persistence shock on  $Z$ ,  $\rho_z = 0.5$

| Aggregate variable       |                  | Mean  | Std. Dev |
|--------------------------|------------------|-------|----------|
| Optimal Investment       | Low persistence  | 1.248 | 3.996    |
|                          | High persistence | 1.236 | 2.719    |
| Realized investment      | Low persistence  | 2.104 | 2.428    |
|                          | High persistence | 2.096 | 1.065    |
| Investment misallocation | Low persistence  | 0.593 | 1.251    |
|                          | High persistence | 0.590 | 1.020    |
| Utility                  | Low persistence  | 2.106 | 2.427    |
|                          | High persistence | 2.098 | 1.065    |
| Consumption              | Low persistence  | 0.563 | 0.823    |
|                          | High persistence | 0.561 | 0.291    |
| Dividends                | Low persistence  | 0.161 | 0.047    |
|                          | High persistence | 0.161 | 0.017    |
| Output                   | Low persistence  | 0.731 | 1.413    |
|                          | High persistence | 0.727 | 0.502    |
| Labor                    | Low persistence  | 0.318 | 0.141    |
|                          | High persistence | 0.317 | 0.051    |
| Wage                     | Low persistence  | 1.265 | 1.887    |
|                          | High persistence | 1.260 | 0.670    |

Table 6: Simulated moments of aggregates and equilibrium price following shock on the aggregate TFP level  $Z$

We should highlight that in this case that aggregate variables tend to ‘jump’ as the shock hits which was not really the path of the crisis in Chile. Recall that negative growth only hit in 1999. Moreover, the scales are

smaller than for the shocks on  $\delta$  suggesting more contained effects for the same persistence.

Once again we have a worse situation in terms of capital misallocation when the shock is persistent. Trivially, unlike the previous case, here we cannot attribute changes in technology to the misallocation induced by the constraints as it is purely exogenous while it still has effects on the other aggregates along the way.

## Conclusion

This paper aimed at exploring how distress on financial markets, combined with financial frictions that prevent full insurance can have long-lasting effects on an EME. Indeed, movements on the financial markets can exacerbate the role of binding constraints on resource allocation, harming the economy moving forward as it lags behind efficiency. In doing so, I develop a model with heterogeneous production that generates capital misallocations that scars the efficiency of the economy. This is an interesting approximation to this issue but the model, while emphasizing the role of uninsurable risk and inequality, remains too simplistic. A rigorous analysis requires to explicitly modeling at least access to credit for producers surely under the form of costly access to external financing. Another assumption worth discussing is the unique good consumed in autarky. Indeed, a Sudden Stop usually implies heavy devaluation as Chile lived. While this might be a problem for importers of intermediate goods, it will make exporters more competitive and therefore the effects of the augmented cost of capital might differ substantially depending on the ability of the domestic firms to export. Finally, incorporating policy instruments, in particular monetary policy, might give some light on why neighbors had very distinct experiences in the aftermath of the Russian Default. In fact, the HANK literature has been very prolific in showing that the underlying heterogeneity of an economy is key to understanding how monetary policy affects agents' decisions.

## References

- Abdul Abiad, Enrica Detragiache, and Thierry Tresselt. A New Database of Financial Reforms. *IMF Working Paper*, WP/08/266:30, December 2008.
- Stéphane Adjemian, Houtan Bastani, Michel Juillard, Frédéric Karamé, Ferhat Mihoubi, Willi Mutschler, Johannes Pfeifer, Marco Ratto, Normann Rion, and Sébastien Villemot. Dynare: Reference Manual Version 5. Dynare Working Papers 72, CEPREMAP, January 2022.
- Mark Aguiar and Gita Gopinath. Emerging Market Business Cycles: The Cycle Is the Trend. *Journal of Political Economy*, 1(115):69–103, 2007. doi: 10.1086/511283.
- SeHyoun Ahn, Greg Kaplan, Benjamin Moll, Thomas Winberry, and Christian Wolf. When Inequality Matters for Macro and Macro Matters for Inequality. *NBER Macroeconomics Annual*, 32:1–75, April 2018. ISSN 0889-3365, 1537-2642. doi: 10.1086/696046.
- S. Rao Aiyagari. Uninsured Idiosyncratic Risk and Aggregate Saving. *The Quarterly Journal of Economics*, 109(3):659–684, August 1994. doi: 10.2307/2118417.
- Sina T. Ates and Felipe E. Saffie. Fewer but Better: Sudden Stops, Firm Entry, and Financial Selection. *American Economic Journal: Macroeconomics*, 13(3):304–356, 2021. ISSN 1945-7707, 1945-7715. doi: 10.1257/mac.20180014.
- José Miguel Benavente and Cintia Külzer. CREACIÓN Y DESTRUCCIÓN DE EMPRESAS EN CHILE. *Estudios de economía*, 35(2), December 2008. ISSN 0718-5286. doi: 10.4067/S0718-52862008000200006.
- Raphael Bergoeing, Patrick J Kehoe, Timothy J Kehoe, and Raimundo Soto. A decade lost and found: Mexico and Chile in the 1980s. *Review of Economic Dynamics*, 5(1):166–205, 2002. ISSN 1094-2025. doi: 10.1006/redy.2001.0150.
- Luis Bértola and José Antonio Ocampo. *The Economic Development of Latin America since Independence*. Oxford University Press, Incorporated, Oxford, UNITED KINGDOM, 2012. ISBN 978-0-19-163824-4.
- Luis Bértola and María Rey. The Montevideo-Oxford Latin American Economic History Database (MOxLAD): Origins, Contents and Sources. *Economic History of Developing Regions*, 33(3):209–224, September 2018. ISSN 2078-0389. doi: 10.1080/20780389.2018.1532286.
- Jeff Bezanson, Alan Edelman, Stefan Karpinski, and Viral B Shah. Julia: A fresh approach to numerical computing. *SIAM review*, 59(1):65–98, 2017.
- Javier Bianchi and Enrique G. Mendoza. A Fisherian approach to financial crises: Lessons from the Sudden Stops literature. *Review of Economic Dynamics*, 37:S254–S283, August 2020. ISSN 10942025. doi: 10.1016/j.red.2020.06.001.

- BIS. A review of financial markets events in Autumn 1998. Technical report, Bank of International Settlements, Basel, Switzerland, October 1999.
- Celestine Bohlen. Russia acts to fix sinking finances. *The New York Times*, August 1998. ISSN 0362-4331.
- Ricardo J Caballero, Emmanuel Farhi, and Pierre-Olivier Gourinchas. An Equilibrium Model of “Global Imbalances” and Low Interest Rates. *American Economic Review*, 98(1):358–393, February 2008. ISSN 0002-8282. doi: 10.1257/aer.98.1.358.
- Guillermo A. Calvo and Ernesto Talvi. Sudden Stop, Financial Factors, and Economic Collapse in Latin America: Learning from Argentina and Chile. In Narcís Serra and Joseph E. Stiglitz, editors, *The Washington Consensus Reconsidered*, pages 119–149. Oxford University Press Oxford, first edition, April 2008. ISBN 978-0-19-953408-1 978-0-19-171465-8. doi: 10.1093/acprof:oso/9780199534081.003.0008.
- Guillermo A. Calvo, Alejandro Izquierdo, and Ernesto Talvi. Sudden Stops and Phoenix Miracles in Emerging Markets. *The American Economic Review*, 96(2):405–410, 2006. ISSN 0002-8282.
- Vasco M. Carvalho and Basile Grassi. Large Firm Dynamics and the Business Cycle. *American Economic Review*, 109(4):1375–1425, April 2019. ISSN 0002-8282. doi: 10.1257/aer.20151317.
- Eduardo Cavallo, Andrew Powell, Mathieu Pedemonte, and Pilar Tavella. A new taxonomy of Sudden Stops: Which Sudden Stops should countries be most concerned about? *Journal of International Money and Finance*, 51:47–70, March 2015. ISSN 0261-5606. doi: 10.1016/j.jimonfin.2014.10.001.
- Nicolas Coeurdacier, Stéphane Guibaud, and Keyu Jin. Credit Constraints and Growth in a Global Economy. *American Economic Review*, 105(9):2838–2881, September 2015. ISSN 0002-8282. doi: 10.1257/aer.20130549.
- Thomas F. Cooley and Vincenzo Quadrini. Financial Markets and Firm Dynamics. *American Economic Review*, 91(5):1286–1310, December 2001. ISSN 0002-8282. doi: 10.1257/aer.91.5.1286.
- Udaibir S Das, Michael G Papaioannou, and Christoph Trebesch. Sovereign Debt Restructurings 1950–2010: Literature Survey, Data, and Stylized Facts. *IMF Working Paper*, WP/12/203, August 2012.
- Angus Deaton. Saving and Liquidity Constraints. *Econometrica*, 59(5):1221–1248, September 1991. doi: 10.2307/2938366.
- Barry Eichengreen and Poonam Gupta. Managing Sudden Stops. *World Bank Policy Research Working Paper*, WPS7639, April 2016. doi: 10.1596/1813-9450-7639.
- Marta Guerriero. The Labor Share of Income around the World: Evidence from a Panel Dataset. *Asian Development Bank Institute - ADBI Working Paper*, 920, February 2019.
- Hugo A. Hopenhayn. Entry, Exit, and firm Dynamics in Long Run Equilibrium. *Econometrica*, 60(5):1127–1150, 1992. ISSN 0012-9682. doi: 10.2307/2951541.
- Chang-Tai Hsieh and Peter J. Klenow. Misallocation and Manufacturing TFP in China and India. *The Quarterly Journal of Economics*, 124(4):1403–1448, 2009. ISSN 0033-5533.

- Mark Huggett. The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of Economic Dynamics and Control*, 17(5):953–969, 1993. ISSN 0165-1889. doi: 10.1016/0165-1889(93)90024-M.
- Esteban Jadresic and Roberto Zahler. Chile’s Rapid Growth in the 1990s: Good Policies, Good Luck, or Political Change. *IMF Working Paper*, WP/00/153:36, October 2000.
- Aubhik Khan and Julia K. Thomas. Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity. *Journal of Political Economy*, 121(6):1055–1107, December 2013. doi: 10.1086/674142.
- Homi J. Kharas, Brian Pinto, and Sergei Ulatov. An Analysis of Russia’s 1998 Meltdown: Fundamentals and Market Signals. *Brookings Papers on Economic Activity*, 2001(1):1–68, 2001. ISSN 1533-4465. doi: 10.1353/eca.2001.0012.
- Nobuhiro Kiyotaki and John Moore. Credit Cycles. *The Journal of Political Economy*, 105(2):211–248, 1997.
- Per Krusell and Anthony A. Smith, Jr. Income and Wealth Heterogeneity in the Macroeconomy. *Journal of Political Economy*, 106(5):867–896, October 1998. ISSN 0022-3808, 1537-534X. doi: 10.1086/250034.
- François Le Grand and Xavier Ragot. Refining the Truncation Method to Solve Heterogeneous-Agent Models. *Annals of Economics and Statistics*, (146):65–92, 2022. ISSN 2115-4430. doi: 10.2307/48674139.
- Paul Lewis. Latin Americans Say Russian Bailout Leaves Too Little for Them. *The New York Times*, October 1998. ISSN 0362-4331.
- Lili Liu. Entry-exit, learning, and productivity change Evidence from Chile. *Journal of Development Economics*, 42(2):217–242, December 1993. ISSN 0304-3878. doi: 10.1016/0304-3878(93)90019-J.
- Lars Ljungqvist and Thomas J. Sargent. *Recursive Macroeconomic Theory*. The MIT Press, 2012. ISBN 978-0-262-01874-6.
- Alais Martin-Baillon. When should we tax firms? Optimal corporate taxation with firm heterogeneity. <https://bibnum.sciencespo.fr/s/catalogue/ark:/46513/sc171n7d>, 2021.
- Marc J. Melitz. The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity. *Econometrica*, 71(6):1695–1725, 2003. ISSN 0012-9682.
- Marc J. Melitz and Stephen J. Redding. Chapter 1 - Heterogeneous Firms and Trade. In Gita Gopinath, Elhanan Helpman, and Kenneth Rogoff, editors, *Handbook of International Economics*, volume 4 of *Handbook of International Economics*, pages 1–54. Elsevier, January 2014. doi: 10.1016/B978-0-444-54314-1.00001-X.
- Enrique G. Mendoza. Real Business Cycles in a Small Open Economy. *The American Economic Review*, 81(4):797–818, 1991. ISSN 00028282.
- Enrique G. Mendoza. Sudden Stops, Financial Crises, and Leverage. *The American Economic Review*, 100(5):1941–1966, 2010. ISSN 0002-8282.

- Enrique G. Mendoza, Vincenzo Quadrani, and José-Víctor Ríos-Rull. Financial Integration, Financial Development, and Global Imbalances. *Journal of Political Economy*, 117(3):371–590, June 2009. ISSN 0022-3808. doi: 10.1086/599706open\_in\_new.
- Benjamin Moll. Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation? *American Economic Review*, 104(10):3186–3221, October 2014. ISSN 0002-8282. doi: 10.1257/aer.104.10.3186.
- Jenny Núñez Hormazábal, Brenda Rain Garrido, and Isaac Castillo Basaure. Base de datos de seguimiento de establecimientos de la Encuesta Nacional Industrial Anual (ENIA), June 2018.
- Nina Pavcnik. Trade Liberalization, Exit, and Productivity Improvements: Evidence from Chilean Plants. *The Review of Economic Studies*, 69(1):245–276, 2002. ISSN 0034-6527.
- Xavier Ragot. Chapter 4 - Heterogeneous Agents in the Macroeconomy: Reduced-Heterogeneity Representations. In Cars Hommes and Blake LeBaron, editors, *Handbook of Computational Economics*, volume 4 of *Handbook of Computational Economics*, pages 215–253. Elsevier, 2018. doi: 10.1016/bs.hescom.2018.02.001.
- Michael Reiter. Solving heterogeneous-agent models by projection and perturbation. *Journal of Economic Dynamics and Control*, 33(3):March 2009, March 2009. doi: 10.1016/j.jedc.2008.08.010.
- Gabriele Rovigatti and Vincenzo Mollisi. Theory and Practice of Total-Factor Productivity Estimation: The Control Function Approach using Stata. *The Stata Journal*, 18(3):618–662, September 2018. ISSN 1536-867X. doi: 10.1177/1536867X1801800307.
- George Tauchen. Finite state markov-chain approximations to univariate and vector autoregressions. *Economics Letters*, 20(2):177–181, 1986. ISSN 0165-1765. doi: 10.1016/0165-1765(86)90168-0.
- Stephen J. Terry. Alternative Methods for Solving Heterogeneous Firm Models. *Journal of Money, Credit and Banking*, 49(6):1081–1111, 2017. ISSN 0022-2879.
- Rodrigo Valdés. Policy responses to Sudden Stops in capital flows: The case of Chile in 1998. *Central Bank of Chile - Working Papers*, 430:71, October 2007.
- Thomas Winberry. A method for solving and estimating heterogeneous agent macro models. *Quantitative Economics*, 9(3):1123–1151, 2018. ISSN 1759-7323. doi: 10.3982/QE740.
- Michael Woodford. Public Debt as Private Liquidity. *The American Economic Review*, 80(2):382–388, 1990. ISSN 0002-8282.



## Appendix A Analytical solutions

To simplify the notation below, I drop the  $j$  index corresponding to an individual firm. They are implicit and still correspond to those found in equation(2). In another abuse of notation, below I write the wage  $w$  as a parameter. However, it is pinned down in equilibrium and therefore it is a function of the state space. The correct notation is  $w(Z, \Lambda)$ .

### A.1 Exiting firms

Their problem boils down to a one-period maximization of the last dividend payment they are going to make to their shareholders. Because it is their last period in the economy, the terminal condition  $k'^e = 0$  is natural for them and they will simply maximize their instantaneous gratification.

By the law of motion of capital of the firms' problem (2) and given this terminal condition, we find  $i^e = -(1 - \delta)k$ , meaning that the firm will liquidate all its assets before its closure. It will run its final operation and give the biggest dividend possible to its shareholders. Their problem boils down to:

$$\max_{l,d} \quad d \quad s.t. \quad -(1 - \delta)k = Z\epsilon F(k, l) - lw - d$$

Replacing  $d$  from the constraint and taking FOC (necessary and sufficient given our assumptions on the production function being concave) we find that the condition on labor demand is given by equality between the marginal and marginal product of labor. It corresponds to an efficiency condition.

$$w = Z\epsilon \frac{\partial F(\cdot)}{\partial l}$$

Provided that the derivative of the production function with respect to labor is bijective, we can invert it to recover the labor demand of these firms.

$$l^* = \left( \frac{\partial F}{\partial l} \right)^{-1} \left( \frac{w}{Z\epsilon} \right)$$

Replacing this value in the constraint we find indeed:

$$d^e = Z\epsilon F(k, l) - l^*w + (1 - \delta)k \geq 0$$

## A.2 Surviving firms

The firms that know they will have at least one more period to produce in the economy face the following problem.

$$\begin{aligned}
V(k, \epsilon, Z, \Lambda) &= \max_{k', l, d, i} d + \mathcal{B} \mathbb{E} V(k', \epsilon', Z', \Lambda') \\
s.t. \quad k' &= (1 - \delta)k + i \\
i &= Z\epsilon F(k, l) - lw - d \\
d &\geq 0 \\
k' &\geq 0
\end{aligned}$$

After the replacement of the constraints into the objective function, we can write the following Karush-Kuhn-Tucker system to account for the inequality constraints:

$$\begin{aligned}
V(k, \epsilon, Z, \Lambda) &= \max_{k', l} (1 - \delta)k + Z\epsilon F(k, l) - lw - k' + \mathcal{B} \mathbb{E} V(k', \epsilon', Z', \Lambda') \\
&\quad + \mu_d [(1 - \delta)k + Z\epsilon F(k, l) - lw - k'] + \mu_k (k') \\
s.t. \quad d &= (1 - \delta)k + Z\epsilon F(k, l) - lw - k' \\
-d &\leq 0 \\
-k' &\leq 0 \\
\mu_d(-d) &= 0 \\
\mu_k(-k) &= 0 \\
\mu_d, \mu_k &\geq 0
\end{aligned}$$

Remark that given that both the equality and inequality constraints are linear on the choice variables, this KKT system automatically satisfies the required regularity conditions, here because of the Linear Constraint Qualification (LCQ). Since this is the strongest constraint qualification condition, we can be sure that the solutions found below satisfy KKT.

The optimality conditions of the KKT system are given by:

$$\begin{aligned}
\text{Capital: } \quad & \frac{\partial V(k, \epsilon, Z, \Lambda)}{\partial k'} = 0 \iff 1 + \mu_d - \mu_k = \mathcal{B} \mathbb{E} \frac{\partial V(k', \epsilon', Z', \Lambda')}{\partial k'} && \text{FOC} \\
& \frac{\partial V(k, \epsilon, Z, \Lambda)}{\partial k} = (1 + \mu_d) \left[ (1 - \delta) + Z\epsilon \frac{\partial F}{\partial k} \right] && \text{Envelop condition } \forall t \\
& \therefore 1 + \mu_d - \mu_k = \mathcal{B} \mathbb{E}(1 + \mu'_d) \left[ (1 - \delta) + Z'\epsilon' \frac{\partial F}{\partial k'} \right] \\
\\ 
\text{Labor: } \quad & \frac{\partial V(k, z, \vartheta)}{\partial l} = 0 \iff \left[ Z\epsilon \frac{\partial F(\cdot)}{\partial l} - w \right] (1 + \mu_d) = 0 && \text{FOC} \\
& \therefore Z\epsilon \frac{\partial F}{\partial l} = w
\end{aligned}$$

Note that the condition of labor does not depend on any of the multipliers and corresponds to an efficiency condition  $MPL = w$ . Therefore, all firms will demand the optimal amount of labor at every period, which is consistent with intuition as it is an adjustment variable.

As in the previous section, provided that the derivative of production with respect to labor is invertible, we find the same explicit expression for labor demand  $l^* = \left( \frac{\partial F}{\partial l} \right)^{-1} \left( \frac{w}{Z\epsilon} \right)$ .

In what follows, we will assume that the constraint on capital always slacks so that  $\mu_k = 0$  for all firms so we focus on the distortions coming from the constraint on access to capital only.

I will segment the surviving firms between constrained and unconstrained producers to find the levels of investment and dividends distributed to the household.

### A.2.1 Unconstrained firms

For them, the KKT multiplier on dividends is null  $\mu_d = 0$  and thus they are in a position where they can finance the investment needed to stay at their optimal size with retained earnings only and can distribute the rest of them. The optimality condition on capital in this case reads:

$$\frac{1}{\mathcal{B}} = \mathbb{E}(1 + \mu'_d) \left[ (1 - \delta) + Z'\epsilon' \frac{\partial F}{\partial k'} \right]$$

Provided that the derivative of the production function with respect to capital is bijective and thus invertible, we find an expression for the capital stock at  $t + 1$ .

$$k'^* = \left( \frac{\partial F}{\partial k} \right)^{-1} \left( \frac{1}{\mathcal{B} \mathbb{E}(1 + \mu'_d) Z'\epsilon'} - \frac{(1 - \delta)}{Z'\epsilon'} \right)$$

This fully pins down the optimal level of investment:

$$i^* = k'^* - (1 - \delta)k$$

Finally, using the budget constraint of the firm, we find that it distributes dividends equal to:

$$d^* = (1 - \delta)k + Z\epsilon F(k, l^*) - l^*w - k'^* > 0$$

### A.2.2 Constrained firms

These are the firms for which the liquidity constraint binds i.e.  $\mu_d > 0$ . By the complementary slackness conditions, it must be that  $d^c = 0$ .

We recover the level of capital that these firms will choose for the subsequent period with the optimality condition on capital as for the unconstrained case.

$$\begin{aligned} \frac{1 + \mu_d}{\mathcal{B}} &= \mathbb{E}(1 + \mu'_d) \left[ (1 - \delta) + Z'\epsilon' \frac{\partial F}{\partial k'} \right] \\ \therefore k'^c &= \left( \frac{\partial F}{\partial k} \right)^{-1} \left( \frac{1 + \mu_d}{\mathcal{B} \mathbb{E}(1 + \mu'_d) Z'\epsilon'} - \frac{(1 - \delta)}{Z'\epsilon'} \right) \end{aligned}$$

Once again, this fixes the investment choice:

$$i^c = k'^c - (1 - \delta)k$$

## A.3 Representative Household

Her UMP can indeed be written in a recursive form as:

$$\begin{aligned} V^{HH}(Z, \delta, \Lambda) &= \max_{C, L} u(C, L) + \beta \mathbb{E} V(Z', \delta', \Lambda') \quad s.t. \quad wL + D = C \\ \iff V^{HH}(Z, \delta, \Lambda) &= \max_L u(wL + D, L) + \beta \mathbb{E} V^{HH}(\Lambda) \end{aligned}$$

However, given that none of her choice variables generate intertemporal stocks, solving for this intertemporal UMP is perfectly equivalent to solving for the sequence of (independent) one-period labor-leisure problems.

$$\begin{aligned} \max_{C, L} u(C, L) \quad s.t. \quad wL + D &= C \\ \iff \max_L u(wL + D, L) \end{aligned}$$

Taking FOC, necessary and sufficient provided that  $u'(\cdot) > 0$ ,  $u''(\cdot) < 0$ , which is assumed, we find the usual efficiency condition.

$$\underbrace{w \frac{\partial u}{\partial C}}_{\text{Marginal benefit}} = \underbrace{-\frac{\partial u}{\partial L}}_{\text{Marginal cost}}$$

Assuming that the derivative of utility with respect to labor is bijective and thus can be inverted we find an

expression for labor supply.

$$\forall t \quad w \frac{\partial u}{\partial C} = - \frac{\partial u}{\partial L} \implies L^{HH} = \left( \frac{\partial u}{\partial L} \right)^{-1} \left( -w \frac{\partial u}{\partial C} \right)$$

Using the budget constraint, we easily find that:

$$C^{HH} = wL^{HH} + D$$

## Appendix B Data sources and management

### B.1 Plant-level data: the ENIA dataset

As presented in section ??, the data used to calibrate the model comes from an annual survey on Chilean manufacturing plants from 1995, the ENIA (*Encuesta Nacional Industrial Anual*) with results publicly available. Importantly, to maintain statistical secret the INE has manipulated the answers of the survey according to international standards to avoid particular identification of the plants [Núñez Hormazábal et al. \[2018\]](#). A change in this manipulation and in the way the plants are identified in 2007 makes it impossible to match the data from 1997-2007 and from 2008 onwards. I use the first time frame as it covers the 1998 shock and several years afterward.

This same data set is used by [Ates and Saffie \[2021\]](#) which also studies the effects of the 1998 shock<sup>15</sup>. I follow a similar strategy to clean the datasets to avoid the inclusion of observations with obvious measurement errors. In particular, I eliminated observations that presented any of the following anomalies:

- If the electricity consumption (*ELECONS*) is strictly negative.
- If the number of worked days (*DIATRA*) is weakly negative.
- If the gross value of production (*VBP*) is inferior to the value added (*VA*).
- If the value added (*VA*) is strictly negative.
- If the wages paid (*REMPAG*) is weakly negative.
- If the revenue from exports (*INGEXP*) is larger than the total revenue (*INGTOT*).

I summarize the effects of said manipulation on the total dataset in Table 7. As we can see, most of the entries remain in the dataset and represent minimal changes in the aggregate statistics considered. In particular, I keep over 97% of the observations and workforce per year.

Table 8 summarizes the descriptive statistics of the main variables of interest for the whole 1995-2007 period.

---

<sup>15</sup>Other studies build on previous versions of the ENIA, corresponding to years before 1995 and include [Pavcnik \[2002\]](#) and [Liu \[1993\]](#). The results for these previous iterations of the survey are not publicly available.

|             |      | <b>Number<br/>observations</b> | <b>Total<br/>employees</b> | <b>Gross Value<br/>of production</b> | <b>Total<br/>Revenue</b> | <b>Value<br/>Added</b> | <b>Total wage<br/>compensation</b> |
|-------------|------|--------------------------------|----------------------------|--------------------------------------|--------------------------|------------------------|------------------------------------|
| <b>1995</b> | pre  | 5512                           | 435483                     | 1.67E+10                             | 1.64E+10                 | 7.31E+09               | 1.43E+09                           |
|             | post | 5491                           | 435062                     | 1.67E+10                             | 1.64E+10                 | 7.31E+09               | 1.43E+09                           |
|             |      | 99.62%                         | 99.90%                     | 100.00%                              | 100.00%                  | 100.00%                | 100.00%                            |
| <b>1996</b> | pre  | 5854                           | 440909                     | 1.88E+10                             | 1.84E+10                 | 8.30E+09               | 1.60E+09                           |
|             | post | 5813                           | 439417                     | 1.88E+10                             | 1.84E+10                 | 8.30E+09               | 1.60E+09                           |
|             |      | 99.30%                         | 99.66%                     | 99.95%                               | 99.95%                   | 99.98%                 | 99.81%                             |
| <b>1997</b> | pre  | 5635                           | 427345                     | 2.02E+10                             | 1.96E+10                 | 8.69E+09               | 1.73E+09                           |
|             | post | 5540                           | 421959                     | 1.96E+10                             | 1.90E+10                 | 8.75E+09               | 1.71E+09                           |
|             |      | 98.31%                         | 98.74%                     | 97.12%                               | 97.04%                   | 100.71%                | 98.67%                             |
| <b>1998</b> | pre  | 5440                           | 407266                     | 2.18E+10                             | 2.12E+10                 | 1.03E+10               | 1.75E+09                           |
|             | post | 5372                           | 402275                     | 2.11E+10                             | 2.05E+10                 | 1.03E+10               | 1.72E+09                           |
|             |      | 98.75%                         | 98.77%                     | 97.01%                               | 96.69%                   | 99.90%                 | 98.17%                             |
| <b>1999</b> | pre  | 5308                           | 370145                     | 2.21E+10                             | 2.21E+10                 | 1.09E+10               | 1.70E+09                           |
|             | post | 5202                           | 364945                     | 2.14E+10                             | 2.14E+10                 | 1.06E+10               | 1.67E+09                           |
|             |      | 98.00%                         | 98.60%                     | 96.61%                               | 96.74%                   | 97.88%                 | 98.29%                             |
| <b>2000</b> | pre  | 5161                           | 364123                     | 2.60E+10                             | 2.56E+10                 | 1.24E+10               | 1.77E+09                           |
|             | post | 5034                           | 359173                     | 2.49E+10                             | 2.44E+10                 | 1.15E+10               | 1.74E+09                           |
|             |      | 97.54%                         | 98.64%                     | 95.66%                               | 95.50%                   | 92.89%                 | 98.48%                             |
| <b>2001</b> | pre  | 5088                           | 361581                     | 2.77E+10                             | 2.69E+10                 | 1.12E+10               | 1.85E+09                           |
|             | post | 5027                           | 357385                     | 2.65E+10                             | 2.57E+10                 | 1.14E+10               | 1.81E+09                           |
|             |      | 98.80%                         | 98.84%                     | 95.78%                               | 95.64%                   | 101.52%                | 98.11%                             |
| <b>2002</b> | pre  | 5416                           | 379522                     | 3.03E+10                             | 2.96E+10                 | 1.22E+10               | 2.03E+09                           |
|             | post | 5361                           | 375236                     | 2.90E+10                             | 2.83E+10                 | 1.23E+10               | 1.97E+09                           |
|             |      | 98.98%                         | 98.87%                     | 95.61%                               | 95.47%                   | 101.15%                | 97.14%                             |
| <b>2003</b> | pre  | 5377                           | 387426                     | 3.39E+10                             | 3.30E+10                 | 1.36E+10               | 2.49E+09                           |
|             | post | 5312                           | 381471                     | 3.22E+10                             | 3.13E+10                 | 1.37E+10               | 2.43E+09                           |
|             |      | 98.79%                         | 98.46%                     | 94.92%                               | 94.85%                   | 100.74%                | 97.55%                             |
| <b>2004</b> | pre  | 5600                           | 410793                     | 4.46E+10                             | 4.35E+10                 | 1.99E+10               | 2.48E+09                           |
|             | post | 5530                           | 404415                     | 4.30E+10                             | 4.19E+10                 | 2.00E+10               | 2.43E+09                           |
|             |      | 98.75%                         | 98.45%                     | 96.32%                               | 96.37%                   | 100.86%                | 97.91%                             |
| <b>2005</b> | pre  | 5516                           | 443558                     | 5.31E+10                             | 5.19E+10                 | 2.44E+10               | 2.69E+09                           |
|             | post | 5465                           | 436798                     | 5.23E+10                             | 5.11E+10                 | 2.45E+10               | 2.64E+09                           |
|             |      | 99.08%                         | 98.48%                     | 98.46%                               | 98.50%                   | 100.41%                | 98.32%                             |
| <b>2006</b> | pre  | 5275                           | 436069                     | 5.85E+10                             | 5.68E+10                 | 2.83E+10               | 2.74E+09                           |
|             | post | 5194                           | 428680                     | 5.81E+10                             | 5.64E+10                 | 2.84E+10               | 2.71E+09                           |
|             |      | 98.46%                         | 98.31%                     | 99.37%                               | 99.33%                   | 100.21%                | 98.94%                             |
| <b>2007</b> | pre  | 5037                           | 445074                     | 6.48E+10                             | 6.31E+10                 | 3.23E+10               | 2.85E+09                           |
|             | post | 4970                           | 436028                     | 6.17E+10                             | 6.00E+10                 | 3.25E+10               | 2.75E+09                           |
|             |      | 98.67%                         | 97.97%                     | 95.14%                               | 95.04%                   | 100.68%                | 96.45%                             |

Table 7: Comparison of before-after data cleaning for some important statistics

|      |               | Capital stock | Labor    | TFP      | Dividends | Investment | Desinvestment |
|------|---------------|---------------|----------|----------|-----------|------------|---------------|
| 1995 | Mean          | 9.406807      | 3.705281 | 7.419077 | 1.055841  | 5.989698   | 9.378529      |
|      | Std Deviation | 2.121638      | .9923017 | .8572163 | 3.335478  | 5.058197   | 2.061969      |
| 1996 | Mean          | 9.198254      | 3.659062 | 7.500331 | 1.023827  | 5.973169   | 9.450922      |
|      | Std Deviation | 2.116544      | .9733022 | .8604732 | 3.289957  | 5.149407   | 2.343056      |
| 1997 | Mean          | 9.279476      | 3.651884 | 7.582992 | 1.002151  | 6.016817   | 9.532272      |
|      | Std Deviation | 2.132907      | .9866814 | .8611793 | 3.26469   | 5.135712   | 2.366188      |
| 1998 | Mean          | 9.043665      | 3.615155 | 7.61564  | .9937935  | 5.825019   | 9.551346      |
|      | Std Deviation | 2.19897       | .9942857 | .8662748 | 3.249226  | 5.169454   | 2.156029      |
| 1999 | Mean          | 8.552355      | 3.522352 | 7.628845 | .9855846  | 5.330542   | 9.464169      |
|      | Std Deviation | 2.322675      | .9976149 | .8965816 | 3.191563  | 5.12733    | 2.115697      |
| 2000 | Mean          | 9.002944      | 3.493016 | 7.743723 | .7536397  | 5.265149   | 9.75445       |
|      | Std Deviation | 2.263393      | 1.014016 | .891577  | 2.86745   | 5.1747     | 2.389633      |
| 2001 | Mean          | 8.64178       | 3.413775 | 7.744264 | .836118   | 5.419206   | 9.38567       |
|      | Std Deviation | 2.325058      | 1.049362 | .9071409 | 3.02095   | 5.137558   | 2.2001        |
| 2002 | Mean          | 8.611305      | 3.397581 | 7.80828  | .859465   | 5.353573   | 9.449868      |
|      | Std Deviation | 2.292814      | 1.030868 | .8994559 | 3.046159  | 5.130801   | 1.987462      |
| 2003 | Mean          | 7.78482       | 3.435655 | 7.857717 | .9551817  | 5.546761   | 9.476911      |
|      | Std Deviation | 2.538903      | 1.032531 | .9148917 | 3.178745  | 5.186135   | 2.122955      |
| 2004 | Mean          | 8.531292      | 3.44566  | 7.945057 | .9456924  | 5.640956   | 9.707677      |
|      | Std Deviation | 2.266841      | 1.020684 | .9013947 | 3.181969  | 5.249903   | 2.306332      |
| 2005 | Mean          | 8.903901      | 3.498971 | 7.938524 | 1.005467  | 5.848626   | 9.819942      |
|      | Std Deviation | 2.347464      | 1.07677  | .8854242 | 3.285479  | 5.260696   | 2.173069      |
| 2006 | Mean          | 8.542727      | 3.51034  | 7.9977   | 1.022174  | 6.004605   | 9.758513      |
|      | Std Deviation | 2.433825      | 1.084801 | .9372467 | 3.309171  | 5.261311   | 2.327273      |
| 2007 | Mean          | 9.795246      | 3.548834 | 8.095714 | 1.139918  | 6.287454   | 10.13276      |
|      | Std Deviation | 2.358019      | 1.096898 | .9365606 | 3.492372  | 5.255746   | 2.263648      |

Notes: All the variables are in logs to be able to have an interpretable graphs as the actual distributions are havily skewed to the left. To avoid that accurate values equal to 0 in levels are dropped because of the log transformation, we increment all the observations above by one before making the transform. This manipulation only shifts the distribution in logs but maintains the ranks and its shape and therefore does not modify the results of interest. the TFP estimation is made using Olley-Pakes method for Stata [Rovigatti and Mollisi, 2018]

Table 8: Descriptive Statistics, Full Panel



## B.2 Aggregate data

I use two other main sources of data. Chile's Central Bank has a wide range of statistics publicly available [online](#). In particular, I used:

- The time series of the average interest rate that commercial banks charged for 90-day maturity loans for the 1995 - 2007 period. This data point is reported at a daily frequency.
- The daily observed exchange rate with respect to the American dollar (1995-2007). It is reported as the number of Chilean pesos needed to buy an American dollar on the foreign exchange market.
- The nominal current account quarterly data (1996 - 2007). I used a moving average technique implemented in Python's *statsmodels* package to decompose the series (*statsmodels.tsa.seasonal*) and seasonally adjust it. I also used the yearly data without adjustment.
- The nominal GDP series for all quarters in the 1996 - 2007 period. I seasonally adjusted it in the same way as the current account.
- The series on gross formation of gross domestic fixed investment in levels.
- The inflation series (1995-2007) used by the Central Bank as a deflator.

I also used some series from the MOxLAD (Montevideo-Oxford Latin American Economic History Data Base) [[Bértola and Rey, 2018](#)] that compiles other series from trusted sources in one unique database with homogeneous normalization [publicly available](#). While they have data for a much longer time frame, I used data from the 1980s and 1990s only, namely:

- The annual series for the aggregate value added in the manufacturing sector (in million of constant Chilean Pesos of 1970). The data is taken from the ECLAC SYLA - Statistical Yearbook for Latin America.
- The annual series of real GDP (in millions of constant Chilean Pesos of 1970) which is derived from [Bértola and Ocampo \[2012\]](#).
- The Unit Value of Import Index 1970=100.
- The annual series for the Gross Domestic Fixed Investment (% GDP), taken from the IMF IFS - International Financial Statistics.
- The series of Gross National Savings as a percentage of GDP. The original data comes from the World Bank.