Propping up the Job Ladder? The Dynamic Effects of Subsidising Labour Hoarding in Recessions

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I examine the effects of subsidising labour hoarding in recessions in a dynamic search and matching model with heterogeneous workers, on-the-job search, and aggregate shocks. I show that while subsidising labour hoarding has negative welfare effects for all with flexible wages, in the presence of a minimum wage it may be welfare-improving for workers while also causing wage compression, with benefits accruing mainly to those just above the bottom of the skill distribution.

I would like to give thanks to my advisor Jean-Marc Robin for his guidance and advice, without which this project would have turned out far less interesting. I am also grateful to Pierre Cahuc for his generous discussions while I was beginning to frame the project, and to Florian Oswald for his excellent instruction in Julia. This thesis is also dedicated to Shristi, my family, and all the friends I found on this masters and in Paris, without whom this would have been truly impossible.

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1. Introduction

The pandemic recession of 2020 posed an unprecedented challenge to social insurance systems around the globe. Policy-makers had to reckon with an unprecedentedly deep recession, with severe restrictions on the work people could do. The response many of them found differed significantly from previous recessions: rather than simply providing unemployed workers with unemployment insurance (UI) - policy-makers went to lengths to try and either keep these workers in their job, or ensure that when the pandemic was over that their job would be waiting for them. In short, they aimed to subsidise labour hoarding during the recession. This element of the response took different forms in different countries, in most of Europe it took the form of furlough or short-time-work schemes, which subsidised hours reductions with the aim of incentivising employers to make employment adjustments on the intensive rather than extensive margin (Giupponi, Landais, and Lapeyre 2022). In the US it instead took the form of the Paycheck Protection Program, which provided temporary loans to alleviate firms' liquidity constraints in order to both keep workers in their jobs, and to allow firms to survive in order to recall their workers later (Gertler, Huckfeldt, and Trigari 2022). Meanwhile the recovery from the great recession, particularly in the labour market - has stood in sharp contrast to the recovery from the Great Recession. Rather a long era of stagnant wages and high unemployment, labour markets generally rebounded quickly, with unemployment soon dropping to it's pre-pandemic level or below, and with increasing wages at the bottom end of the distribution driving wage compression (Autor, Dube, and McGrew 2023).

There is some evidence that the preservation of worker firm matches, and the policies which encouraged them, may have helped labour markets strong recovery. In particular, Hall and Kudlyak (2022) show that of those made unemployed in the US, a far larger portion than in previous recessions were "unemployed with jobs", i.e. made unemployed, but with the expectation that they would soon return to their previous jobs. Furthermore, Hall and Kudlyak (2022) show that the "unemployed with jobs" generally did far better in the recoveries from both recessions than the "jobless" unemployed. Meanwhile, Gertler, Huckfeldt, and Trigari (2022) show in a structural model that the US Paycheck Protection Program may have played a key role in preventing the spread of more harmful "jobless" unemployment.

The Job Ladder. In this article I aim to examine one mechanism through which such policies may have significant effects: the job-ladder. The job-ladder refers to the mechanism by which workers improve their wage by moving from worse jobs to better jobs throughout their career via on-the-job search. Recently increasing attention has been paid to the cyclicality of the job-ladder - noting that this process is generally faster in good times than in bad and emphasising it as an important amplification mechanism for shocks

in the job market (Moscarini and Postel-Vinay 2018b). Meanwhile, evidence that increased employer competition and job-to-job transitions at the bottom of the income distribution may have been an important force in explaining the post-pandemic wage compression in the US, demonstrates the potential importance of such policies effect on the job ladder in explaining both wages, and trends in inequality (Autor, Dube, and McGrew 2023). The importance of the job ladder therefore makes it a highly relevant consideration when considering social insurance policy in recessions. Indeed, subsidising labour hoarding has potentially important effects on the job ladder - in particular preventing unemployment in the low state may help preserve worker's position on the ladder, allowing them to continue to carry-out on-the-job search from their current position, rather than having to start again from the bottom after becoming unemployed.

To examine the effects of subsidising labour hoardings on the job ladder, I use a dynamic search-and-matching model with heterogeneous workers, aggregate shocks, and on-the-job search - namely the Robin (2011) model, which I modify to include a legal minimum wage. This model includes an easily understandable job-ladder mechanism via the sequential auctions framework, and allows me to generate rich results as to the effects of subsidising labour hoarding, on unemployment, wages, and inequality. Including the minimum wage allows me to examine how the effects of such policy change in the presence of wage rigidity at the bottom of the income distribution, which may be particularly binding in recessions.

Results. I find that without a minimum wage, the model predicts a significant reduction in unemployment - achieved primarily by shifting low skill workers into low wage employment. Meanwhile the increase in firm reservation values when the economy is in a low aggregate state benefits those higher up the income distribution, increasing wage inequality. In the case without a minimum wage the effects on social welfare, whether examined by a utilitarian social planner or one focussed on worker's welfare specifically, are unambiguously negative except at very low or very high levels of the policy - where workers see slight benefits.

My results in the case where there is a binding minimum wage are rather different. In this case the reduction in unemployment is far less certain: the reduction in unemployment in the low state is more modest overall and it is largest for those just above the bottom of the skill distribution. Indeed over the whole business cycle the policy has a negative overall effect on the employment of the bottom ventile of workers. The wage results are also rather different, finding wage compression rather than an increase in wage inequality, suggesting that the job ladder effects of the policy may benefit those lower down the skill distribution. Overall I find that the policy causes an increase in the labour share and an increase in wage income for the middle three quintiles of the worker

skill distributions, with only the top and bottom quintiles seeing their wage income fall. I also find that the policy, while still welfare negative from the utilitarian social planner's point of you - is now overall welfare improving for workers.

Related Literature. I primarily contribute to the literature on the effect of labour hoarding policies, including but not limited to short-time-work (STW). My key contribution is to explicitly examine the dynamic effects of subsidising labour hoarding in the context of a job-ladder mechanism, to my knowledge the present paper is the first to do this.

On the empirical side, there is a recent quasi-experimental literature that generally suggests that short-time-word does indeed save jobs. This finding is common across Cahuc and Carcillo (2011), who use proximity of French firms to other firms that have previously used STW as an instrument for its use in the great recession; Kopp and Siegenthaler (2021), who compare Swiss firms who were accepted for STW to similar firms who were rejected; as well as Giupponi and Landais (2023) who exploit exogenous variation in Italian STW eligibility rules, with the latter two finding that not only does STW save jobs, but it does so amongst firms that would otherwise undertake large layoffs. On the structural side, Tilly and Niedermayer (2016) combine a model with German administrative data to examine the determinants of STW take-up as well as its effects on welfare, and find that while STW does prevent a significant amount of job loss, the resulting welfare gains are modest since it primarily protects those for whom the drop in earnings from unemployment is smallest. Finally, Giupponi, Landais, and Lapeyre (2022), provide a theoretical framework of "insuring jobs", in contrast to "insuring workers", with which to think about the continuum of social insurance policies from STW to UI.

There is also literature examining the effects of labour-hoarding policies on reallocation. In particular Cooper, Meyer, and Schott (2017) find, in a search-and-matching model with both firm and aggregate shocks but without on-the-job-search, that STW has significant harmful effects on allocative efficiency by reducing the vacancy-filling rate. Meanwhile, García-Cabo, Lipińska, and Navarro (2023) examine the effects of a wage subsidy policy in the context of sectoral shocks and reallocation, finding that such a policy may be preferable to UI in an economy with lower baseline inter-sector reallocation. Finally, Giupponi and Landais (2023) also find empirical evidence that if a negative shock is persistent, STW may harm reallocation by keeping workers in low-productivity firms. I contribute to this literature by not only considering reallocation via. unemployment, but also reallocation via on-the-job search.

Also related is the literature mentioned above on temporary unemployment - the most relevant empirical work here is Hall and Kudlyak (2022), mentioned above. As also mentioned above Gertler, Huckfeldt, and Trigari (2022) then put Hall and Kudlyak (2022)'s empirical framework into a structural model, building off of Fujita and Moscarini (2017)'s

seminal structural paper on recall, and use it to estimate the benefits of the US Paycheck Protection Program.

Finally, closely related is the business-cycle search-and-matching literature. This includes closely related sequential-auction models such as Lise and Robin (2017), which develops the Robin (2011) model to include ex ante heterogeneous firms as well as workers, and thus including a richer analysis of labour productivity via sorting. This issue with this model however is that while it is possible to include wages within it (such as in Pascal (2020), and Lise, Robin, and Pascal (2018)), this comes at the cost of some of the model's tractability which, like Robin (2011), relies on the joint job surplus depending only time via only the aggregate state of the economy. Another dynamic sequential-auction model is Moscarini and Postel-Vinay (2018a), which includes the wage distribution in the state space but achieves tractability through making heterogeneity ex-post - which makes it less useful for the analysis of inequality. In another paper Moscarini and Postel-Vinay (2016) develop a dynamic job-ladder model which uses firm size as a proxy for productivity, but which does not include worker heterogeneity. I choose a modified version of the Robin (2011) model as it has the key combination of aggregate shocks, ex-ante heterogeneity, tractability while including intuitive wage-setting, and a clear job-ladder mechanism as this allows me to examine in detail the implications of subsidising labour hoarding for unemployment, wages, and inequality. It is important to note however, that unlike in Moscarini and Postel-Vinay (2018a) and Moscarini and Postel-Vinay (2016), the job ladder I refer to in Robin (2011) is not formed by some firms being more productive than others, but by job-to-job transition increasing the share of the surplus a worker captures - therefore the job ladder mechanism will be not be seen in productivity, but will show up in wages.

Organisation. The remainder of the paper is organised as follows: in section two I present the Robin (2011) model, including my modifications to include a minimum wage, in section three I present my quantitative analysis and results, and in section four I conclude.

2. Model

My model is based off of Robin (2011), which I modify to allow for a legal minimum wage, taking inspiration from Lise, Robin, and Pascal (2018)'s modification of Lise and Robin (2017).

2.1. Setup

The Aggregate State. Time is discrete and indexed by $t \in M$. The aggregate state of the economy is an ergodic Markov chain $y_i \in \{y_1 < ... < y_N\}$ with transition matrix $\Pi = (\pi_{ij})$.

Following Robin (2011)'s notation, y_t denotes the stochastic process while y_i denotes a member of the support.

Workers and Firms. There is a unit mass of workers such that each worker of type $m \in \{1, ..., M\}$ has permanent productivity x_m, and such that there is measure $l(m)$ of a given type of worker. Firms are homogeneous, and so per-period match output is denoted $y_i(m)$, which in practice I set as $y_i(m) = x_m y_i$.

Turnover . Matches form and break at the beginning of each period following the realisation of the aggregate state. Let $u_t(m)$ denote the unemployment rate for workers of type m. Let $u_t = \sum_{m=1}^{M} u_t(m) l_m$ denote the aggregate unemployment rate. Matches separate exogenously at rate δ, and endogenously when the joint value of the match is weakly less than the maximum of the value of unemployment and $\underline{P}_i(m)$ - a minimum value of the match, enforced by a legal minimum wage. Concretely this means at the beginning of each period $\mathbf{1}\{P_t(m) \leq \max\{U_t(m), \underline{P}_t(m)\}\}[1 - u_t(m)]l_m$ workers of type m are endogenously laid off, while $\delta\mathbf{1}\{P_t(m) > \max\{U_t(m), \underline{P}_t(m)\}\}[1 - u_t(m)]l_m$ are exogenously laid off.

For simplicity and tractability vacancy creation is exogenous, unemployed workers meet potential employers at rate λ_0, while employed workers meet potential poachers at rate λ_1. Endogenising this through a matching function is certainly possible, but would likely require an approximation of the state space (Robin 2011; Pascal 2020; Lise, Robin, and Pascal 2018), and so for now I leave this for future work. Once an unemployed worker and firm meet, the worker will be hired if the value of the match is strictly greater than both the worker's value of unemployment and the minimum match value, thus at the beginning of each period $\lambda_0\mathbf{1}\{P_t(m) > \max\{U_t(m), \underline{P}_t(m)\}\}u_t(m)l(m)$ workers of type m are hired out of unemployment. Likewise a firm will attempt to poach an employed worker it meets if the value of the match is strictly greater than both the worker's value of unemployment and the minimum match value, thus at the beginning of each period firms attempt to poach $\lambda_1\mathbf{1}\{P_t(m) > \max\{U_t(m), \underline{P}_t(m)\}\}[1 - u_t(m)]l(m)$ workers of type m.

Wages. Wages are determined by Bertrand competition in the manner of Postel-Vinay and Robin (2002), except in the case where the minimum wage binds, in which case the firm must pay the worker a value of at least $\underline{P}_i(m)$. When hiring workers out of unemployment employers have full monopsony power, meaning workers are offered the maximum of their reservation wage and the legal minimum wage. Rent sharing further than that which is enforced by the minimum wage occurs due to on-the-job search, which triggers competition between employers. Because firms are identical and there are no mobility costs, Bertrand competition transfers the entire surplus to the worker, who is then paid the firm's reservation value.

Bertrand competition ensures that the continuation value of a match is independent of whether an employee is poached, as either the worker is not poached and the match continues, or they are poached and thus are paid the reservation value of the incumbent firm, which is exactly the same continuation value as if they hadn't been poached (Robin 2011; Lise and Robin 2017). Note that as firms are identical, the reservation values of the incumbent and poaching firms will be exactly the same, therefore whether the worker actually moves to the poacher is determined by a tie-breaking probability τ.

2.2. Unemployment Dynamics

Match Value. The joint value of a match in time t is given by:

$$P_i(m) = y_i(m) + \frac{1}{1+r}\sum_j \pi_{ij}[(1-(1-\delta)\mathbf{1}\{P_j(m) \geq \max\{U_j(m), \underline{P}_j(m)\})U_j(m) + \ldots \\ \ldots + (1-\delta)\mathbf{1}\{P_j(m) \geq \max\{U_j(m), \underline{P}_j(m)\})P_j(m)] \tag{1}$$

where r is the interest rate. Note that as set out above, the continuation value of the match $P_j(m)$ is independent of whether the worker is poached or not.

Value of Unemployment.. The value of unemployment, $U_i(m)$ is then given by:

$$U_i(m) = z_i(m) + \frac{1}{1+r}\sum_j \pi_{ij}[(1-\lambda_0)U_j + \ldots \\ \ldots + \lambda_0(\max\{U_j(m), \mathbf{1}\{P_j(m) \geq \max\{U_j(m), \underline{P}_j(m)\}\}\underline{P}_j(m)\})] \tag{2}$$

where $z_i(m)$ is home production. Note that due to the introduction of a minimum wage, there is now an extra term, as if the worker is hired from unemployment and the minimum match value $\underline{P}_j(m)$ is higher than their reservation wage, they will now get paid this value rather than the value of unemployment.

The Match Surplus. Using the notation $S_i(m) = P_i(m) - U_i(m)$, $\underline{S}_i(m) = \underline{P}_i(m) - U_i(m)$, and $X^+ = \max\{X, 0\}$, equation (1) can then be rewritten as follows:

$$P_i(m) = y_i(m) + \frac{1}{1+r}\sum_j \pi_{ij}[U_j(m) + (1-\delta)\mathbf{1}\{S_j(m) \geq \underline{S}_j^+(m)\}S_j(m)] \tag{3}$$

Likewise equation (2) can then be re-written:

$$U_i(m) = z_i(m) + \frac{1}{1+r}\sum_j \pi_{ij}[U_j(m) + \lambda_0\mathbf{1}\{S_j(m) \geq \underline{S}_j^+(m)\}\underline{S}_j^+(m)] \tag{4}$$

It is then possible to write the Bellman equation which defines the surplus function by subtracting equation (4) from (3):

$$
\begin{aligned}
S_i(m) = y_i(m) - z_i(m) & \\
&+ \frac{1-\delta}{1+r} \sum_j \pi_{ij} \mathbf{1}\{S_j(m) \geq \underline{S}_j^+(m)\} S_j(m) \\
&- \frac{1}{1+r} \sum_j \pi_{ij} \lambda_0 \mathbf{1}\{S_j(m) \geq \underline{S}_j^+(m)\} \underline{S}_j^+(m)
\end{aligned} \tag{5}
$$

All turnover decisions are determined by this equation along with the minimum viable surplus $\underline{S}_i(m)$, which I derive below, following the derivation of equilibrium wages. At this point we are in a position to set out the law of motion for the m type unemployment rate using the turnover rules set out above alongside our definition of the surplus function. Conditional on $y_t = y_i$ at the beginning of period t we can write:

$$
u_{t+1} = 1 - \mathbf{1}\{S_i(m) > \underline{S}_i^+(m)\}[\lambda_0 u_t(m) + (1-\delta)(1-u_t(m))] \tag{6}
$$

2.3. Wages

Worker Surplus. Let $W_i(w, m)$ denote the present value of a wage w in state i to a worker of type m. The per-period worker surplus is then $w - z_i(m)$. In the following period, the worker is laid off with probability $\mathbf{1}\{S_j(m) \leq \underline{S}_j^+(m)\} + \delta \mathbf{1}\{S_j(m) > \underline{S}_j^+(m)\}$, and gets their value of unemployment. Otherwise with probability λ_1, the worker receives an outside offer and takes the whole surplus, if the worker is not poached they will continue on the same wage unless either they or the employer has a credible threat to end the match, in which case the new wage will be the closest point in the new bargaining set to the old wage. This means that if $W_j(w, m) - U_j(m) < \underline{S}_j^+(m)$ either the worker has a credible threat to quit to unemployment, or the state steps in to enforce the minimum wage; meanwhile if $W_j(w, m) - U_j(m) > \max\{S_j(m), \underline{S}_j(m)\}$ the employer has a credible threat to fire the worker unless they accept renogotiation down to the point where they get the whole surplus - note the role of the max operator is just to denote that the employer cannot force renegotiation to below the minimum wage, however in practice if $\max\{S_j(m), \underline{S}_j(m)\} = \underline{S}_j(m)$ then the match will separate.

The worker surplus, $W_i(w,m) - U_i(m)$, therefore satisfies the Bellman equation

$$
\begin{aligned}
W_i(w,m) - U_i(m) = w - z_i(m) & \\
&+ \frac{1-\delta}{1+r} \sum_j \pi_{ij} \mathbf{1}\{S_j(m) > \underline{S}_j^+(m)\} \\
&\times [\lambda_1 S_j(m) + (1-\lambda_1)(W_j^*(w,m) - U_j(m))] \\
&- \frac{1}{1+r} \sum_j \pi_{ij} \lambda_0 \mathbf{1}\{S_j(m) > \underline{S}_j^+(m)\} \underline{S}_j^+(m)
\end{aligned} \tag{7}
$$

where

$$
W_j^*(w,m) - U_j(m) = \min\{\max\{W_j(w,m) - U_j(m), \underline{S}_j^+(m)\}, S_j(m)\} \tag{8}
$$

is the renegotiated worker surplus.

Equilibrium Wages. In each aggregate state y_i, a worker type x_m can only be offered one of two possible wages. Either the worker was offered a job while unemployed in which case they will be offered a wage $\underline{w}_i(m)$ such that $W_i(\underline{w}_i(m), m) = U_i(m) + \underline{S}_i^+(m)$, i.e. the max of their reservation wage and the legal minimum wage; or the worker was poached in which case they will see their wage rise to $\overline{w}_i(m)$ such that $W_i(\overline{w}_i(m), m) = U_i(m) + \max\{S_i(m), \underline{S}_i(m)\}$, such that they capture the entire surplus. I include the max operator in this latter equation for completeness to denote that even when capturing the whole surplus the worker cannot be paid below the legal minimum - in practice however such a match would never form.

I now follow Robin (2011) in setting out how these wages can be solved for - now subject to the minimum wage. We can now denote the worker surplus when the economy is in state k and the worker has one of the equilibrium wages from state i as follows:

$$
\begin{aligned}
\underline{W}_{k,i} &= W_k(\underline{w}_i(m), m) - U_k(m) \\
\overline{W}_{k,i} &= W_k(\overline{w}_i(m), m) - U_k(m)
\end{aligned}
$$

and let also

$$
\begin{aligned}
\underline{W}_{k,i}^* &= \min\{\max\{\underline{W}_j(w,m) - U_j(m), \underline{S}_j^+(m)\}, S_j(m)\} \\
\overline{W}_{k,i}^* &= \min\{\max\{\overline{W}_j(w,m) - U_j(m), \underline{S}_j^+(m)\}, S_j(m)\}
\end{aligned}
$$

Making use of the definitions of wages, we can find that $\underline{W}_{i,i}(m) = \underline{S}_i^+(m)$ and $\overline{W}_{i,i}(m) = \max\{S_i(m), \underline{S}_i^+(m)\}$, these worker surpluses therefore satisfy the following modified Bellman equations:

$$
\begin{aligned}
\underline{W}_{k,i}(m) - \underline{S}_i^+(m) &= \underline{W}_{k,i}(m) - \underline{W}_{i,i}(m) \\
&= z_i(m) - z_k(m) + \frac{1-\delta}{1+r}\sum_j(\pi_{kj}-\pi_{ij})\mathbf{1}\{S_j(m) > \underline{S}_j^+(m)\} \\
&\times [\lambda_1 S_j(m) + (1-\lambda_1)\underline{W}_{j,i}^*(m)] \\
&- \frac{1}{1+r}\sum_j(\pi_{kj}-\pi_{ij})\lambda_0\mathbf{1}\{S_j(m) > \underline{S}_j^+(m)\}\underline{S}_j^+(m)
\end{aligned} \tag{9}
$$

and

$$
\begin{aligned}
\overline{W}_{k,i}(m) - \max\{S_i(m), \underline{S}_i^+(m)\} &= \overline{W}_{k,i}(m) - \overline{W}_{i,i}(m) \\
&= z_i(m) - z_k(m) + \frac{1-\delta}{1+r}\sum_j(\pi_{kj}-\pi_{ij})\mathbf{1}\{S_j(m) > \underline{S}_j^+(m)\} \\
&\times [\lambda_1 S_j(m) + (1-\lambda_1)\overline{W}_{j,i}^*(m)] \\
&- \frac{1}{1+r}\sum_j(\pi_{kj}-\pi_{ij})\lambda_0\mathbf{1}\{S_j(m) > \underline{S}_j^+(m)\}\underline{S}_j^+(m)
\end{aligned} \tag{10}
$$

These can then be simply solved using value-function iteration, starting from the solution to the linear system of equations formed by imposing that $\underline{W}_{j,i}^*(m) = \underline{W}_{j,i}(m)$ and $\overline{W}_{j,i}^*(m) = \overline{W}_{j,i}(m)$.

Having then calculated $\underline{W}_{j,i}(m)$ and$\overline{W}_{k,i}(m)$ for all k, i, and m, wages can then be calculated by combining equation (7) and the definitions of the equilibrium wages above as follows:

$$
\begin{aligned}
\underline{w}_i(m) &= \underline{S}_i^+(m) + z_i(m) - \frac{1-\delta}{1+r}\sum_j \pi_{ij}\mathbf{1}\{S_j(m) > \underline{S}_j^+(m)\} \\
&\times [\lambda_1 S_j(m) + (1-\lambda_1)\underline{W}_{j,i}^*(m)] \\
&+ \frac{1}{1+r}\sum_j \pi_{ij}\lambda_0\mathbf{1}\{S_j(m) > S_j^+(m)\}\underline{S}_j^+(m)
\end{aligned} \tag{11}
$$

and

$$
\begin{aligned}
\overline{w}_i(m) &= \max\{S_i(m), \underline{S}_i^+(m)\} + z_i(m) - \frac{1-\delta}{1+r}\sum_j \pi_{ij}\mathbf{1}\{S_j(m) > \underline{S}_j^+(m)\} \\
&\times [\lambda_1 S_j(m) + (1-\lambda_1)\overline{W}_{j,i}^*(m)] \\
&+ \frac{1}{1+r}\sum_j \pi_{ij}\lambda_0\mathbf{1}\{S_j(m) > S_j^+(m)\}\underline{S}_j^+(m)
\end{aligned} \tag{12}
$$

Minimum Wage. Finally, we have the necessary building blocks to derive $\underline{S}_i(m)$, the minimum surplus enforced by the minimum wage. First note that the minimum surplus is equal to the worker's surplus at the minimum wage. Then, denoting the legal minimum wage as w_{min}, we can write that $W_i(w_{min}, m) - U_i(m) = \underline{S}_i(m)$. Noting that the continuation value of the renegotiated minimum wage given by equation (8) is $W_j^*(w_{min}, m) - U_j(m) = \min\{\underline{S}_j^+(m), S_j(m)\}$, we can then use equation (7) to write that the Bellman equation which defines the minimum viable match surplus enforced by w_{min}:

$$\begin{aligned}\underline{S}_i(m) = w_{min} - z_i(m) + \frac{1-\delta}{1+r}\sum_j \pi_{ij}\mathbf{1}\{S_j(m) > \underline{S}_j^+(m)\}[\lambda_1 S_j(m) + (1-\delta)\underline{S}_j^+(m)] \\ - \frac{1}{1+r}\sum_j \pi_{ij}\lambda_0\mathbf{1}\{S_j(m) > \underline{S}_j^+(m)\}\underline{S}_j^+(m)\end{aligned} \tag{13}$$

All that remains is now to derive the model's laws of motion for wages.

2.4. Wage Dynamics

The support of the wage distribution is the union of all sets $\Omega_m = \{\underline{w}_i(m), \overline{w}_i(m), \forall i\}$ across all worker types. Let $g_t(w, m)$ denote the measure of workers of ability m employed at wage $w \in \Omega$ at the end of period $t-1$. Conditional on $y_t = y_i$, no worker can be employed if $S_i(m) \leq \underline{S}_i^+(m)$. The inflow to the stock of workers paid $\underline{w}_i(m)$ is otherwise made of all unemployed workers of type m which meet an employer, plus all employees paid a wage w such that $W_i(w, m) - U_i(m) < \underline{S}_i^+(m)$ who were not laid off but also not poached. We can therefore write:

$$\begin{aligned}g_{t+1}(\underline{w}_i(m), m) = \mathbf{1}\{S_i(m) > \underline{S}_i^+(m)\}\Bigg[\lambda_0 u_t l(m) + (1-\delta)(1-\lambda_1) \\ \times\left(g_t(\underline{w}_i(m), m) + \sum_{w\in\Omega_m}\mathbf{1}\{W_i(w, m) - U_i(m) < \underline{S}_i^+(m)\}g_t(w, m)\right)\Bigg]\end{aligned} \tag{14}$$

The inflow to the stock of workers paid $\overline{w}_i(m)$ contains all those workers of type m who are poached, as well as all those whose current wage is such that $W_i(w, m) - U_i(m) > S_i(m)$ who are forced to take a pay-cut to avoid being fired, the only reason to outflow is layoff. We can therefore write:

$$\begin{aligned}g_{t+1}(\overline{w}_i(m), m) = \mathbf{1}\{S_i(m) > \underline{S}_i^+(m)\}(1-\delta)\Bigg[\lambda_1(1-u_t(m))l(m) + (1-\lambda_1) \\ \times\left(g_t(\overline{w}_i(m), m) + \sum_{w\in\Omega_m}\mathbf{1}\{W_i(w, m) - U_i(m) > S_i(m)\}\right)\Bigg]\end{aligned} \tag{15}$$

For all wages in the support for type m but not given out in state i, i.e. $\forall w \in \Omega_w \setminus \{\underline{w}_i(m), \overline{w}_i(m)\}$, only workers whose wage lies between $\underline{w}_i(m)$ and $\overline{w}_i(m)$, and who are

not laid off or poached, keep their wage:

$$
\begin{aligned}
g_{t+1}(w,m) = {} & \mathbf{1}\{S_i(m) > \underline{S}_i^+(m)\}(1-\delta)(1-\lambda_1) \\
& \times \mathbf{1}\{\underline{S}_i^+(m) \leq W_i(w,m) - U_i(m) \leq S_i(m)\} g_t(w,m)
\end{aligned} \tag{16}
$$

As a confirmation we can see that summing over all wages and dividing by $l(m)$ yields the law of motion for the unemployment rates set out in equation (6).

2.5. Parametrization

I parametrize the model following the parametric structure and estimates in Robin (2011), which I briefly summarize here. The only exception is that for computational performance I use a coarser grid for worker types, setting $M = 50$ rather than $M = 500$. I set the unit of time to a quarter, and the annual interest rate to 5%.

The Aggregate Productivity Process. Let F denote the equilibrium distribution of y_t and let C denote a parametric copula. Let $a_1 < \ldots < a_N$ define a grid on $[\varepsilon, 1-\varepsilon]$. Then $y_i = F^{-1}(a_i)$ and $\pi_{ij} \propto c(a_i, a_j)$, with the normalization $\sum_j \pi_{ij} = 1$. Following Robin (2011) I use $N = 100$ and $\varepsilon = 0.002$; F is a log-normal distribution with parameters 0 and σ, and c is a Gaussian copula density with parameter ρ. Following Robin (2011)'s estimates I set $\sigma = 0.023$ and $\rho = 0.94$.

Worker Heterogeneity. As mentioned above I set $y_i(m) = y_i x_m$, where x_m is a linear grid of M points on $[\underline{x}, 1-\underline{x}]$. Following Robin (2011) I assume a beta distribution such that

$$
l(m) = \text{betapdf}(x_m - \underline{x}, \nu, \mu)
$$

with the normalization $\sum_m l(m) = 1$. Here I depart from Robin (2011) by using a coarser grid of $M = 50$ points for computational feasibility. Following the estimates in Robin (2011) I set $\underline{x} = 0.73$, $\nu = 2.00$, and $\mu = 5.56$.

Home Production. Home production, or the opportunity cost of employment, is given the parametric structure $z_i(m) = z_0 + \alpha[y_i(m) - z_0]$. The indexation of home production on productivity prevents the reservation wage of high-skill workers from being lower in booms than in busts, and the reservation wage of high-skilled workers from being lower than that of low-skilled workers. Once again following Robin (2011)'s estimates, I set $\alpha = 0.64$ and $z_0 = 0.77$.

Turnover Parameters. Following Robin (2011)'s calculations I set $\lambda_1 = 0.12 \times \lambda_0$, and set $\tau = 0.5$. Then following Robin (2011)'s estimates I set $\delta = 0.042$ and $\lambda_0 = 0.99$.

3. Quantitative Analysis and Results

I now turn to my analysis of the model's implications for the effects of subsidising labour hoarding in recessions.

3.1. Modelling the Subsidy

Subsidising Labour Hoarding . I model the subsidy to labour hoarding as a simple lump-sub subsidy to the match surplus, denoted ψ, given out when the aggregate state drops below a certain threshold. In order to define the threshold, and to structure the following analysis I categorise the aggregate state into low, medium and high categories such that the threshold between the low and medium state, denoted y_{low}, is the first tertile of the distribution of the aggregate state and the threshold between the medium and high state, denoted y_{high}, as the second tertile. This is a relatively restrictive structure but reflects the limited information governments have when setting such policies, as well as the recent crisis-driven use of such policies (i.e. their use when a deep recession occurs). While in future work it might be desirable to examine the impact of choosing different thresholds or a smoother introduction of the policy, for present purposes modelling the policy this way provides a simple structure with which to analyse the effects at play. The subsidy is funded by a proportional tax on production, denoted γ, which is the same in all states such that the government's budget balances in the long run.

The Bellman equation defining the Surplus function therefore becomes:

$$\begin{aligned} S_i(m) = \psi \mathbf{1}\{y_i \leq y_{low}\} + (1-\gamma) y_i(m) - z_i(m) \\ + \frac{1-\delta}{1+r} \sum_j \pi_{ij} \mathbf{1}\{S_j(m) \geq \underline{S}_j^+(m)\} S_j(m) \\ - \frac{1}{1+r} \sum_j \pi_{ij} \lambda_0 \mathbf{1}\{S_j(m) \geq \underline{S}_j^+(m)\} \underline{S}_j^+(m) \end{aligned} \tag{17}$$

While in the long run the government must satisfy the budget constraint:

$$\sum_t \psi \mathbf{1}\{y_t \leq y_{low}\} \Big[\sum_m (1-u_t(m)) l(m) \Big] = \sum_t \sum_m \gamma y_i(m) (1-u_t(m)) l(m) \tag{18}$$

To examine the effects of the policy, I simulate a very long economy of 7000 periods, taking draws from the Markov chain to determine the aggregate state, before dropping the first 1000 periods to remove dependence on starting values. Taking this approach I first solve for the taxes required to balance the government's budget constraint in the long run at different values of the policy, before plotting the policy's effect on unemployment, wages, income, and finally social welfare.

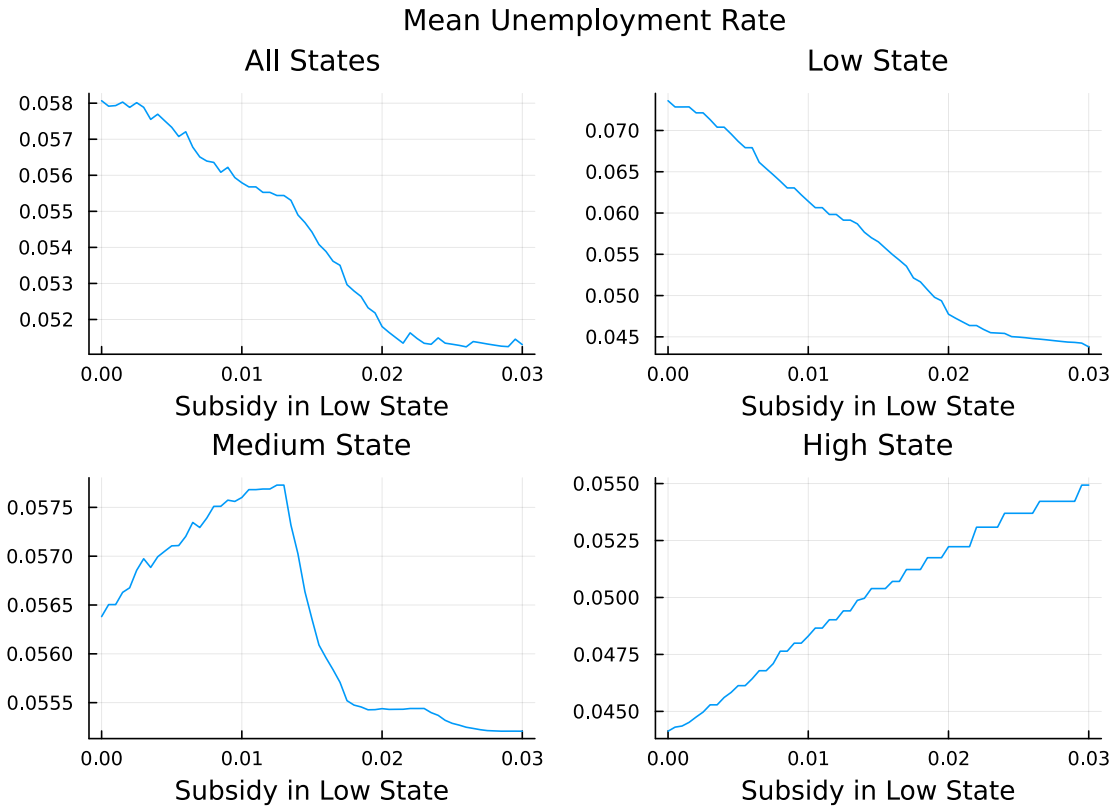

FIGURE 1. Unemployment Rate by Subsidy in Low, Medium and High States

3.2. Effects without a Minimum Wage

3.2.1. Unemployment

A first basic question to ask is whether subsidising labour hoarding is effective in reducing unemployment, and if so: when and for whom? There are three main ways the policy may reduce unemployment over the business cycle. First, during the low state the subsidy will directly decrease endogeneous separations and increase hiring. Second, the policy may decrease unemployment outside of the low state, as those who avoided being fired in the low state due to the policy will no longer have to go through the search and matching process to become employed. Finally, the higher taxes needed throughout the business cycle to fund the policy may lead to fewer hirings and more separations.

Aggregate Unemployment. Figure 1 shows the policy's effect on the aggregate unemployment rate, immediately noticable is that the policy reduces unemployment overall, with the average unemployment rate dropping from 5.8% to close to 5.1%. The difference is particularly stark in the low state, when the subsidy is actually paid out, where the average unemployment rate drops from over 7% down to 4.5% at higher levels of the policy. In the

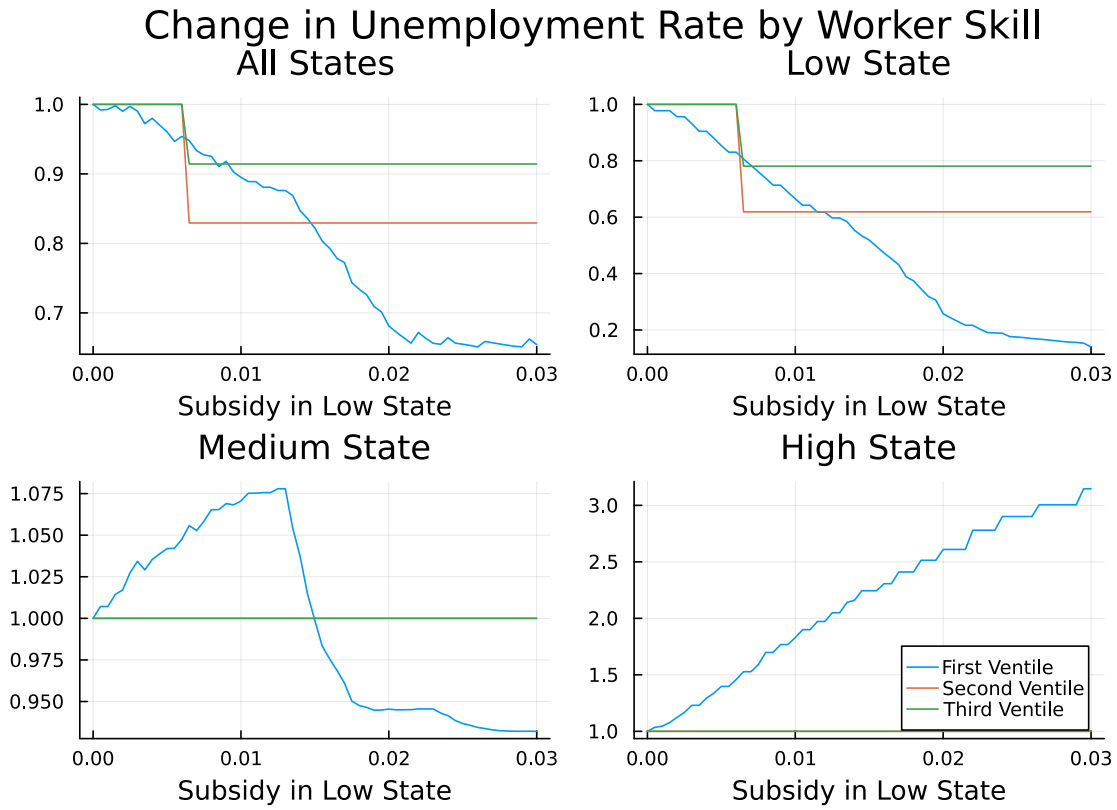

FIGURE 2. Unemployment Rate by Subsidy in Low, Medium and High States

Note: Unemployment is normalised to one in when the subsidy is set to zero.

medium state we see both potential effects playing out - at lower levels of the policy the higher taxes increase unemployment, however as the policy increases the effect flips as lower unemployment in the low state feeds through - notably the combination of these two effects means that the changes caused by the policy in the medium state are of the smallest magnitude. Unsurprisingly in the high state the impact of higher taxes means that the policy unambiguously increases the unemployment rate.

Unemployment by Skill Level. A natural follow up-question is to ask, given that the policy seems to effectively reduce unemployment, is who it reduces unemployment for? This is not necessarily a trivial question - while only the bottom levels of the worker skill distribution are vulnerable to endogenous unemployment at any given moment (Robin 2011), the range of vulnerable is wider in the low state - when the policy reduces unemployment the most - and narrower in the high state - when the policy increases unemployment the most. Figure 2, which breaks down the changes in unemployment across the first three ventiles of the skill distribution, confirms this basic picture: while the policy reduces unemployment across all of the first three ventiles in the low state, only the first ventile is

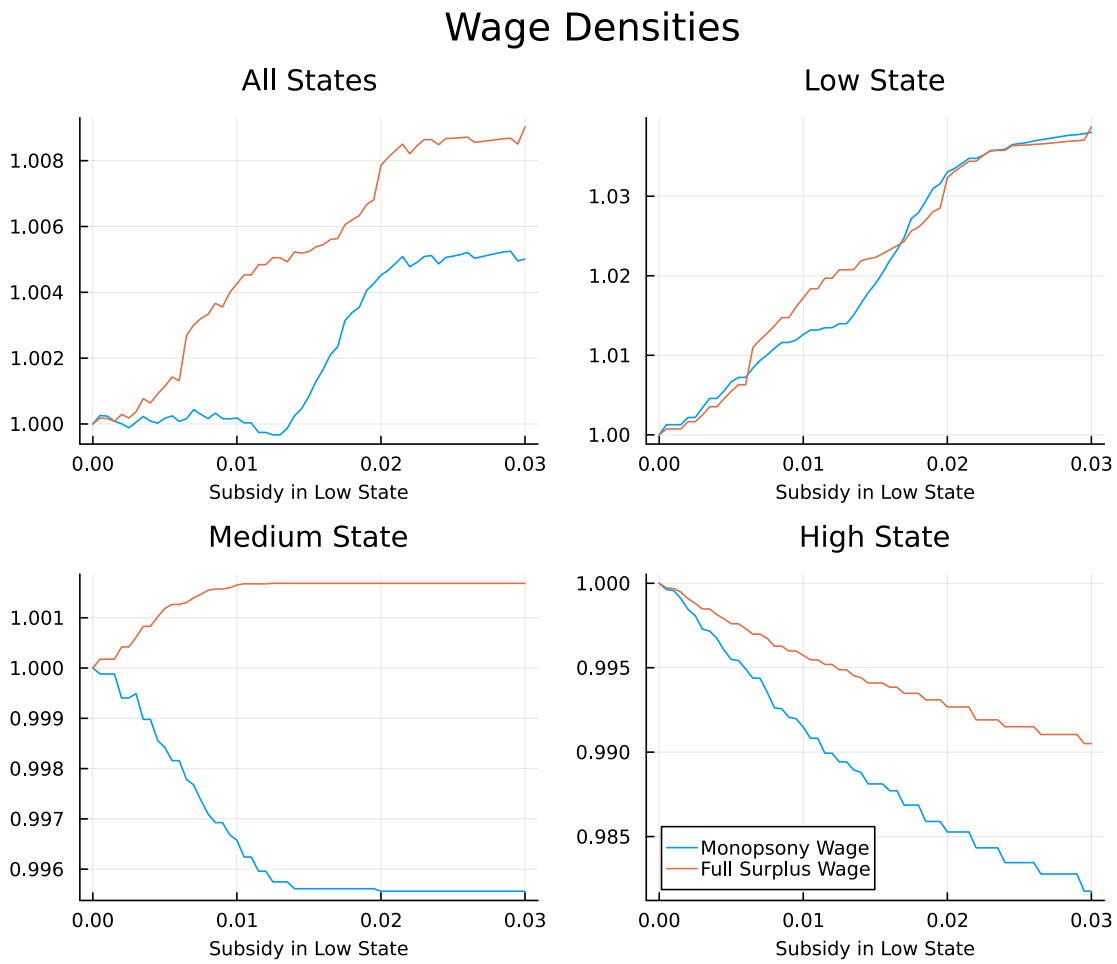

FIGURE 3. Wage Densities for Monopsony and Full Surplus Wages

subject to the changes in the medium state, or the increase in unemployment in the high state. In particular it is striking that in the high state the unemployment rate for the first ventile triples. Nonetheless, in the case without a minimum wage at least, the biggest proportional reduction overall is for the first ventile - which sees its average unemployment rate drop by up to 30% (note percent not percentage points). We will see below that this picture does not necessarily hold in the presence of a minimum wage.

3.2.2. Wages

Wage Mechanisms. The policy has several potential effects on wages that it is important to consider. First, by making it viable to employ lower productivity workers - when their reservation wages are at their lowest - the policy may reduce average wages through a composition effect that may persist into the medium and high states. On the other hand however, the policy may have effects through the job ladder - by avoiding firings and

increasing the employment rate, the policy may lead to a higher poaching or quit rate, leading to more workers leveraging job offers into capturing the full surplus. Meanwhile by increasing firms' reservation values in the low state the subsidy will help workers capture higher wages when they are poached in the low-state, wages which they will then likely keep into the medium and high states until they are poached again, while also reducing the pay-cuts already poached workers are forced to take to keep their jobs when the aggregate state is low. Thus the subsidy may effectively prop-up the job ladder in the low state, while also preventing workers from falling down the within-firm "wage ladder" through renegotiation. Finally however, the increased taxes necessary to fund the policy will decrease firms' reservation values, meaning that when workers capture the full surplus in the medium and high states they are paid less - while increased unemployment in the high state may also mean less workers are poached when the rewards to being poached are highest.

That the job ladder effects laid out above do indeed operate can be seen in figure 3, which shows that while the policy increases the number of people earning both types of wages, the increase is notably more pronounced for those earning the full surplus wage. This pattern is present for some values of the subsidy in the low state, but it is particularly notable in the medium state, where the subsidy in the low state actually increases the number of workers capturing the full surplus in the medium state, despite the increased taxes, while fewer people earn the monopsony wage.

Average Wages and Wage Inequality. Figure 4 shows that overall the forces pushing up average wages dominate, except for in the low state when average wages decline. This suggests that in the low state the composition effect dominates, as lower productivity, lower wage workers are brought into employment. That wages go up in the medium and high states when the subsidy is not being paid out suggests that either increased poaching or fewer wage cuts are dominating the reduced full surplus wages caused by tax increases.

Figure 5 breaks down the changes in average wages by worker-skill quintile, in particular we can see that while average wages fall for all in the low state and increase for all in the high state as a result of the policy, the decreases are largest for the lowest skill workers and the increases are largest for the highest skilled workers. As high wage workers do not benefit from increased poaching as they are not at risk of unemployment, this suggests that, although the poaching effect is clearly present (see figure 3), the main effect pushing up wages is the avoidance or shrinkage of pay cuts during the low state. Notably the negative effect on wages for low skilled workers persists into the medium state, as you have more of the lowest productivity workers already in employment, having been hired in the low state when their reservation wage was the lowest.

These results therefore suggest the subsidy leads to an increase in wage inequality,

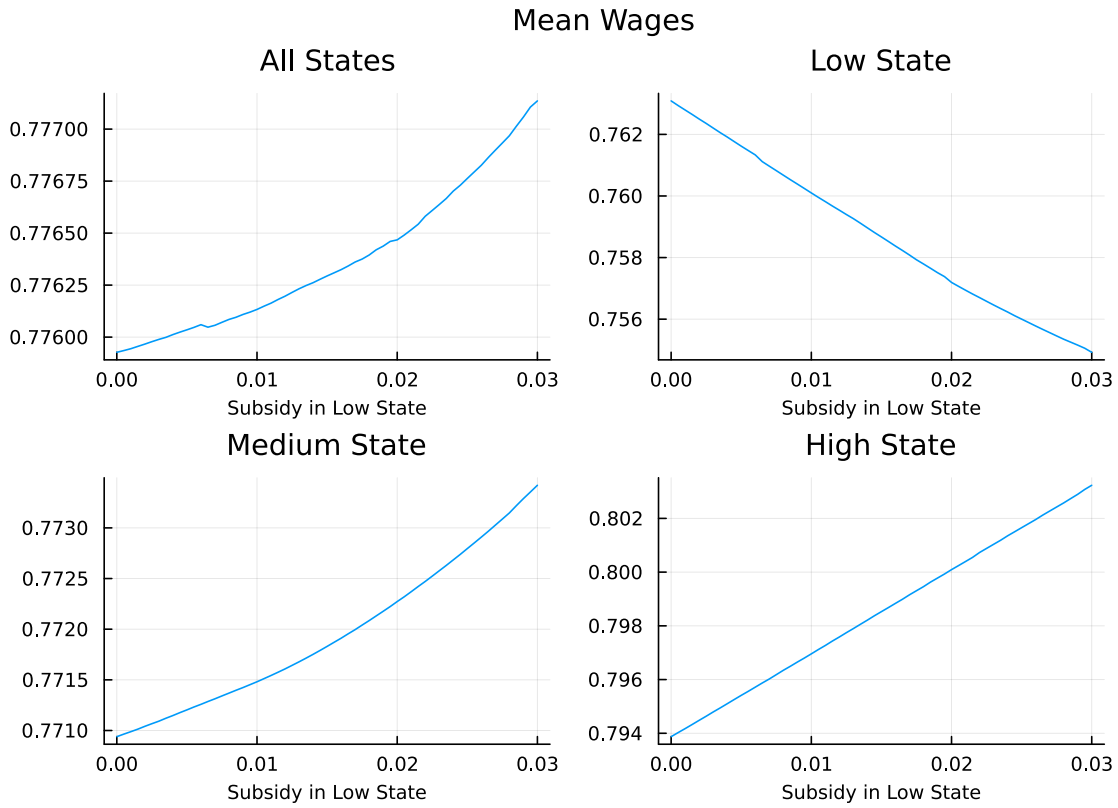

FIGURE 4. Average Wages in Low, Medium and High States

with higher skilled workers' wages increasing and lower skilled workers' wages decreasing. In particular, it is those for whom the policy most decreases unemployment who also see their wages decline. This is contrary to the post-COVID experience in the US, which has seen a remarkable degree of wage compression (Autor, Dube, and McGrew 2023), however it is perhaps more in line with the great recession experience of Italy, where workers put on short time work saw their wages stagnate, though those workers were not necessarily at the bottom of the income distribution (Giupponi and Landais 2023; Giupponi, Landais, and Lapeyre 2022). It is also somewhat contrary to the intuition provided by Hall and Kudlyak (2022), that those who are laid off only temporarily in recessions before returning to their old jobs see their incomes rebound far more strongly; though this could be explained by framing this policy as bringing, in their terms, the long-term unemployed "without jobs" into low-wage employment. However, it should be noted that these are perhaps the results most sensitive to the inclusion of a minimum wage; indeed, as I will show below, a binding minimum wage can entirely flip the model's predictions for the subsidy's effect on wage inequality.

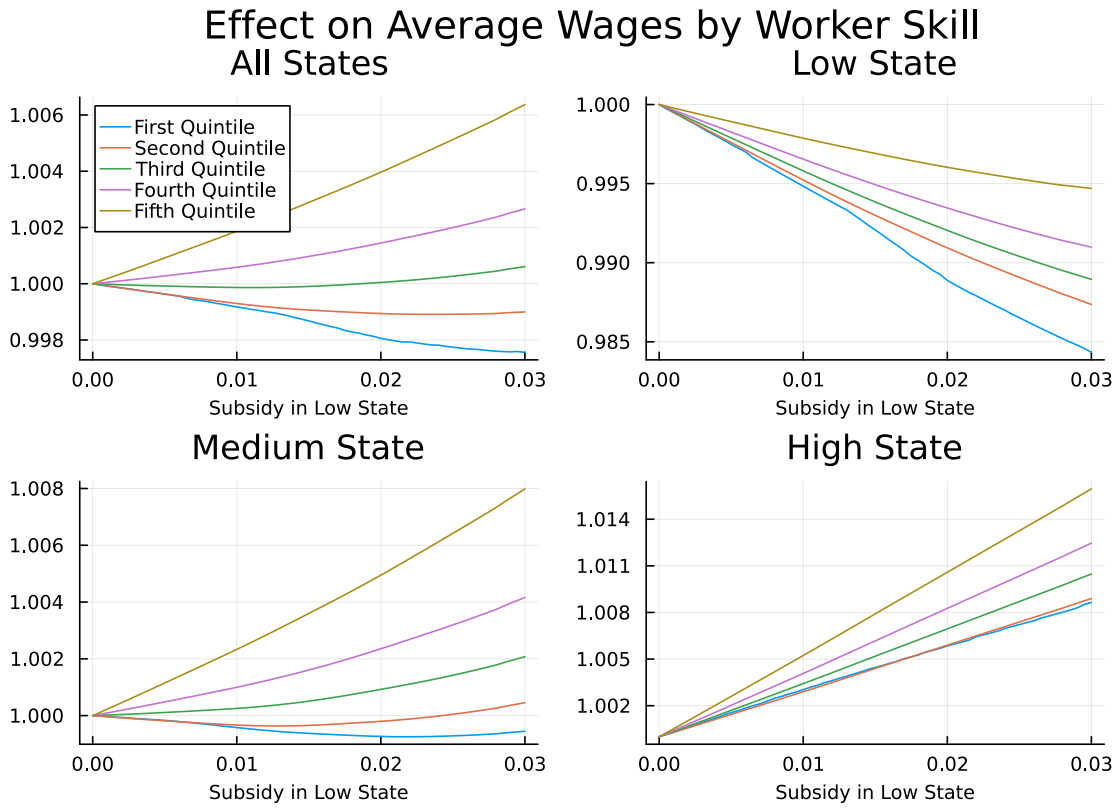

FIGURE 5. Average Wages by Worker Skill in Low, Medium and High States

Note: Wages are normalised to one in when the subsidy is set to zero.

3.2.3. Income and Production

So far we have seen that subsidising labour hoarding in recessions seems to increase both employment and wages, but with highly heterogeneous effects across worker skill levels and across aggregate states of the economy. Before turning to overall social welfare it is useful to clarify the aggregate effects of these changes by calculating the subsidy's effect on aggregate wage income, market production, and the resulting labour share - with which we can further see which of the effects discussed above dominate. Note that it is important to emphasize that the changes discussed in this section should not be confused for changes in social welfare - as they do not consider changes in worker or social surplus, this will instead be tackled in the next section where wage income and market production will be considered in conjunction with home production in a measure of social welfare.

Production and the Labour Share. Figure 6 shows that, as lower unemployment and higher wages imply in the context of the model, both net market production and wage income increase as a result of the policy. However it also shows that this is entirely driven by the low state, in the medium and high states both wage income and net market production

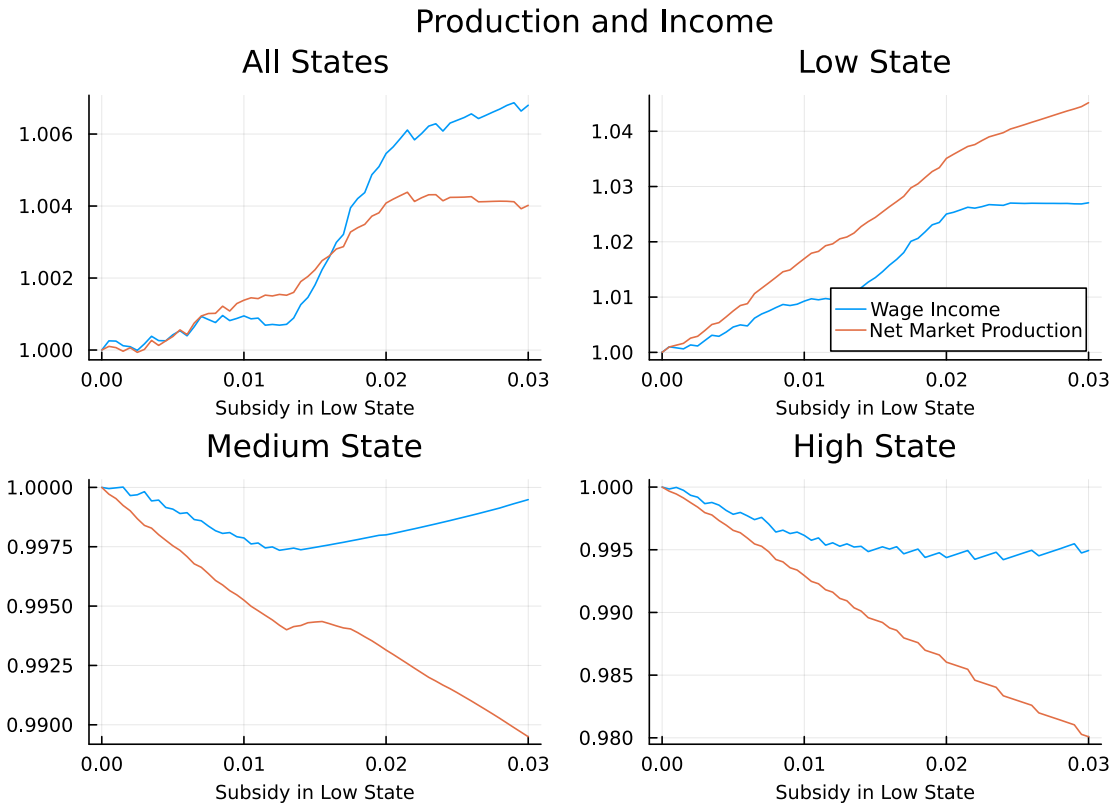

FIGURE 6. Production and Wage Income in Low, Medium and High States

Note: Production and wage income are both normalised to one in when the subsidy is set to zero.

fall. For net market production this is a clear result of the higher taxes necessary to fund the policy, which both directly eat into production and, potentially decrease it through making more matches infeasible. While this is offset somewhat by lower unemployment in the medium state (there is a clear discontinuity in the medium state at the point where the subsidy starts to decrease unemployment), this is not enough to prevent the overall negative effect.

We can also see that while for most levels of the policy it has limited effects on the labour share overall, at higher levels it does lead to an increase in wage income relative to net market production. This is driven primarily by the medium and higher states, where higher average wages do clearly eat into the profit share, whereas in the low state wage income increases but does not keep pace with overall production.

Wage Income by Skill Level. To further understand what is driving changes in wage income, it is once again helpful to break down wage income by skill level. Figure 7 shows that the group which has by far the biggest changes in its wage income due to the policy in every state is the lowest skilled quintile of workers, who see by far the biggest increase in wage

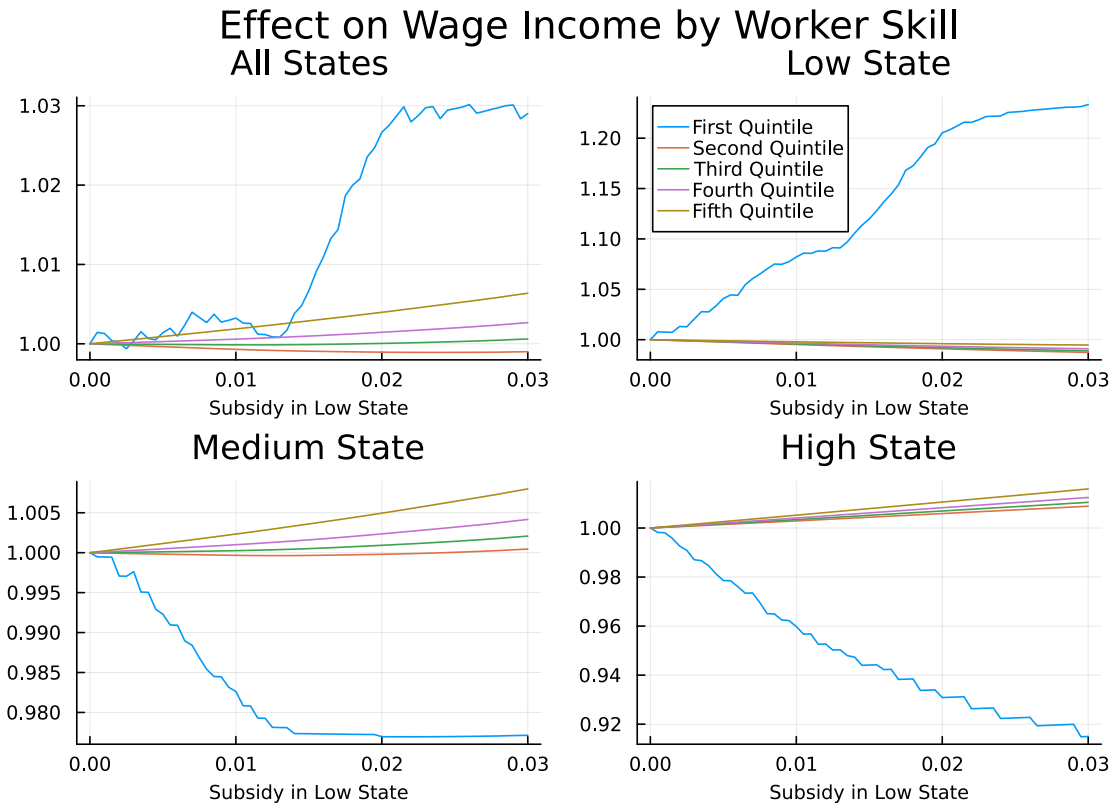

FIGURE 7. Wage Income by Worker Skill in Low, Medium and High States

Note: Wage income is normalised to one in when the subsidy is set to zero.

income overall, driven by a large increase the low state (in contrast to the slight decreases for everyone else). The lowest skilled quintile also see by far the biggest change in wage income in the medium and high states, sharply dropping off in contrast to the slight increases for everyone else. The reversed trajectories of the lowest quintile emphasise that while the modest effects for everyone else are driven by the changes in average wages discussed above, for the lowest quintile it is the changes in unemployment that matter most - as many more of them hired on low wages.

3.2.4. Social Welfare

Having considered the subsidy's effects on unemployment, wages, production, and income, a natural final step is to bring these together in a measure of social welfare. How to calculate social welfare in this model is not trivial however - assuming a standard utilitarian social planner approach of simply summing together net market and home production means treating surplus that is entirely captured by the firm due to monopsony power the same as that paid out in wages, which obscures many of the dynamics discussed above, meanwhile only caring about workers welfare may obscure negative effects on production

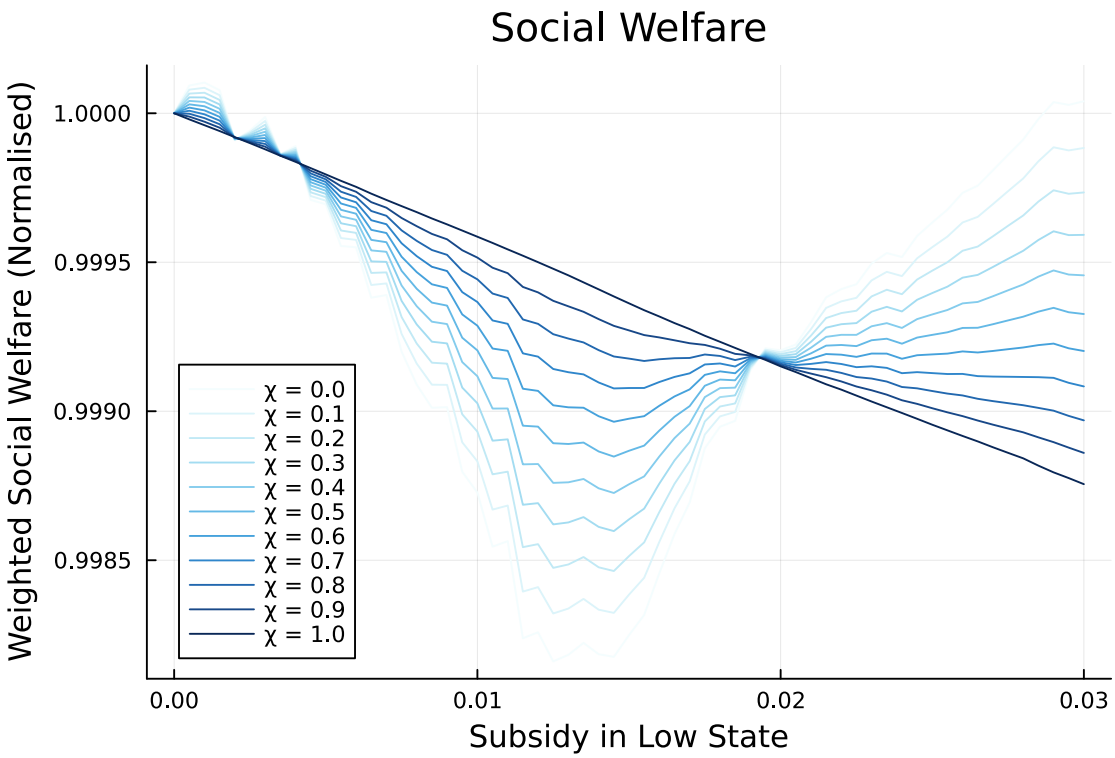

FIGURE 8. Effect of Subsidy on Social Welfare

Note: Social Welfare is normalised to one in when the subsidy is set to zero.

and it is reasonable to expect that many governments might care about both to varying extents. More broadly given that, as illustrated in figure 6, the policy clearly has an impact on the labour share, meaning it may be useful to have a welfare measure which allows us to examine who is "winning" and who is "losing" from such a policy. For these reasons I calculate the following weighted measure of social welfare which imposes a weight $\chi \in [0,1]$ on market income captured by firms:

(19)

$$SWF_t = \sum_m z_t(m)u_t(m) + \sum_m \sum_{w \in \Omega_m} w g_t(w,m) + \chi \left(\sum_m y_t(m)(1-u_t(m)) - \sum_m \sum_{w \in \Omega_m} w g_t(w,m) \right)$$

for a range of values of χ. Intuitively when $\chi = 1$ the social planner only cares about overall production, whereas when $\chi = 0$, the social planner only cares about workers.

Figure 8 makes clear that in the overwhelming majority of cases the policy has negative effects on welfare, whether one cares about overall utilitarian welfare or the welfare of workers. Other than at very low levels of the policy, where it is marginally welfare improving if one has a sufficient preference towards wage income, and at very high levels of the policy, where it is marginally welfare improving if one only cares about workers, the effects are unambiguously negative for the range of policies considered.

The unambiguously negative effects using the utilitarian welfare function ($\chi = 1$) are not surprising, given that the policy is effectively to tax efficient matches to subsidise inefficient ones. While workers' welfare starts to increase at very high levels of the subsidy (likely driven by higher wages at the top of the skill distribution), figure 1 shows that at this level unemployment in the high state is higher than in the low state - illustrating that this is an implausibly high level of the policy. Meanwhile, the more preference is given to workers the worse the policy is for much of the range considered - a fact which is also not entirely surprising given that the largest effects laid above consist of subsidising people to enter very low wage work. This element of the picture however is significantly altered once a minimum wage is introduced, which is the case I now turn to.

3.3. Effects with a Minimum Wage

I now turn to the effects of subsidising labour hoarding during recessions in an economy with a binding minimum wage. The minimum wage changes the dynamics at hand in several ways which I lay out as I go - most important however, are that it implies exiting unemployment is directly welfare improving for at least the portion of the population for whom the minimum wage binds, and that it may imply, particularly in downturns, that there may be matches for which the joint surplus is positive but which do not form as it is below the minimum level enforced by the minimum wage. For reasons of computational feasibility I restrict my analysis below to subsidies up to 0.01, i.e. the lower third of the policies considered above.

3.3.1. Implementing the Minimum Wage

My aim in implementing a minimum wage is primarily to illustrate the difference it makes to the mechanisms at hand and thus the potential importance of it and other forms of wage rigidity when considering the effects of policies which subsidise labour hoarding, my priority is therefore to choose a minimum wage which is binding in the context of the model and can illustrate a wide range of potential mechanisms. My aim is therefore not to choose a minimum wage to match a particular minimum wage law in the world or to try and match the model to data, while this would certainly be desirable for the time being I leave this to future work. Instead I choose a minimum wage such that it is clearly binding, and creates excess unemployment in downturns. To do this I set $w_{min} = 0.76$, which is slightly below both the mean and median wages in the model - while this is certainly high compared to most existing minimum wage laws, this allows us to clearly see how a minimum wage changes the implications of the subsidy - while it still does not cause hugely higher unemployment except in the low state.

That $w_{min} = 0.76$ is binding is clearly illustrated by the bottom left panel of figure 9, which shows its impact on the wage distribution for those earning the monopsony wage

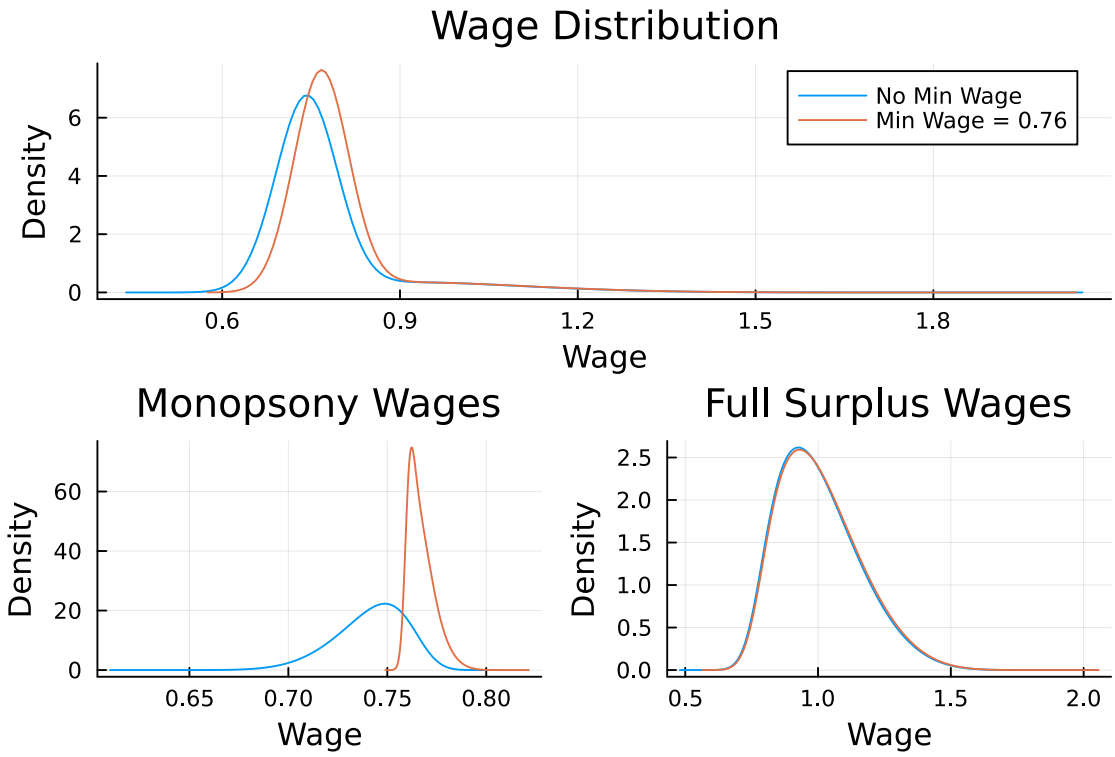

FIGURE 9. Effect of Minimum Wage on Wage Distribution

(that given out to those being hired out of unemployment) - the spike in the density plot at the minimum wage also demonstrates, unsurprisingly given that employers have total monopsony power, that there is at least a segment of workers that the minimum wage makes better off without making unemployed. The minimum wage has little-to-no effect on the distribution of the wages of those workers capturing the full surplus.

As set out in table 1, the minimum wage increases the average unemployment rate and the average unemployment rate in the low state, as well as raising significantly the maximum unemployment rate achieved in the simulation and very slightly increasing it in the medium state; however, it has no effect on either the mean unemployment rate in the high state or the minimum unemployment rate achieved. This therefore illustrates that while the minimum wage does prevent matches which would have a positive surplus from forming, it does so primarily in the low state.

3.3.2. Unemployment

The minimum wage may alter the subsidy's effect on unemployment in two main ways. First by expanding the range of workers at risk of being made endogenously unemployed in the low state, it may mean the subsidy reduces unemployment in the low state for higher skilled workers. Secondly however, by raising the productivity threshold necessary

TABLE 1. Effect of Minimum Wage on Unemployment

	No Minimum Wage	**Minimum Wage = 0.76**
Mean Unemployment Rate	5.81%	5.99%
Mean Unemp. Rate in Low State	7.36%	7.87%
Mean Unemp. Rate in Medium State	5.64%	5.69%
Mean Unemp. Rate in High State	4.41%	4.41%
Minimum Unemployment Rate	4.09%	4.09%
Maximum Unemployment Rate	12.09%	16.4%

Note: Authors' calculations from simulating 7000 periods of the model taking draws from the Markov chain before dropping the first 1000 to remove dependence on starting values.

to become employable, it may reduce the subsidy's impact for those at the very bottom of the skills distribution.

Aggregate Unemployment. As figure 10 makes clear the effect of the subsidy on unemployment is far more ambiguous in the case of the minimum wage, while it still reduces unemployment in the low state, this effect is reduced from more than a one percentage point reduction with the subsidy set to 0.01 without the minimum wage, to a 0.3 percentage point reduction with the minimum wage. Meanwhile the increase in unemployment in the medium and high states is relatively unchanged, leaving the the overall effect approximately zero for most sizes of the subsidy considered, with unemployment increasing as the subsidy approaches 0.01.

Unemployment by Skill Level. Breaking down the changes in the unemployment rate by skill level in figure 11 it is clear what is driving the differences: while the increases in unemployment in the medium and high states are relatively similar to the equivalent sections of figure 2, the decrease in unemployment for the lowest ventile in the low state is far less pronounced. Meanwhile, although uneven, the second and third ventiles still see significant reductions, but no larger than they did without the minimum wage. Indeed the reduction in unemployment during the low state for the first ventile is so diminished that overall the policy slightly increases unemployment for the lowest skilled over the entire business cycle, while reducing it overall for those in the second and third ventiles. This is in line with empirical findings in the literature, that rather than protecting those at the very bottom of the income distribution - policies which seek to insure jobs rather than workers, such as short-time-work, generally seem to benefit insiders slightly further up the distribution who are nonetheless at risk of layoff in recessions (Cahuc and Carcillo 2011; Giupponi, Landais, and Lapeyre 2022).

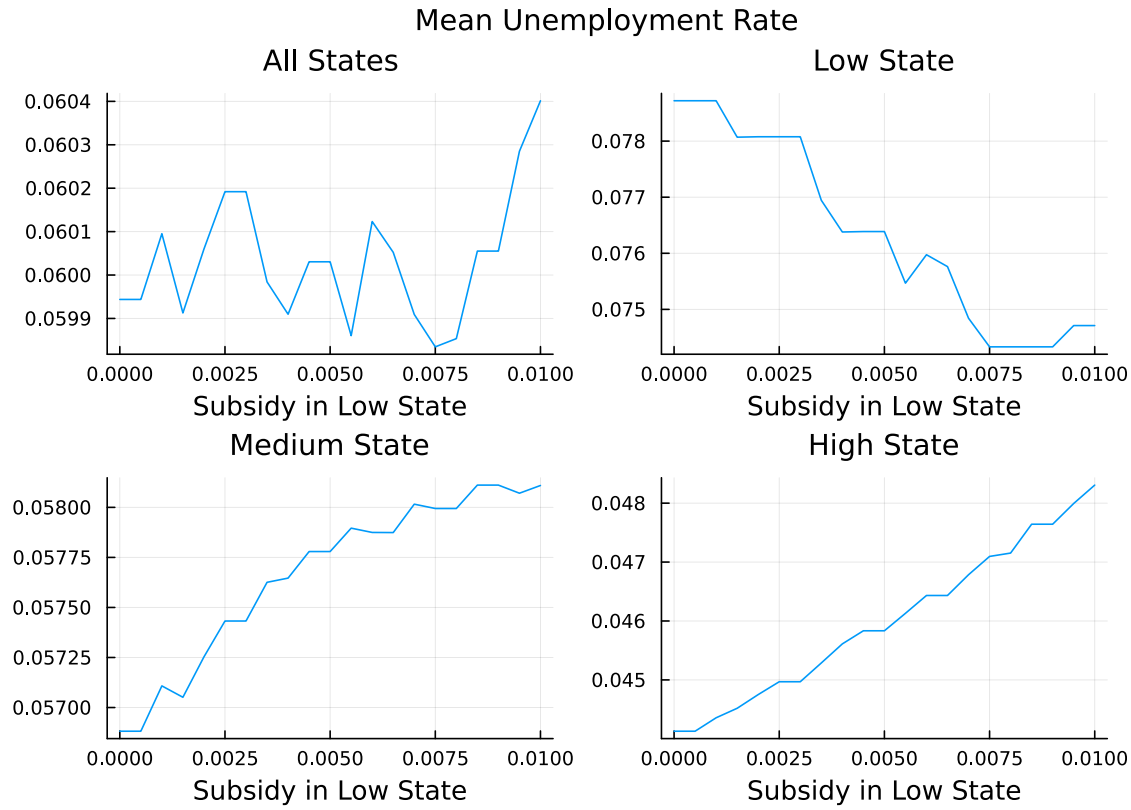

FIGURE 10. Effect of Subsidy on Unemployment with Minimum Wage

3.3.3. Wages

There are two primary ways the minimum wage will change the effect of the subsidy on wages is by putting a floor on the wages of the lowest skilled, it may greatly reduce the composition effect which drove the reduction in wages above. Secondly, by raising the monopsony wage in the low state for all types whose reservation wage in the low state is below 0.76, it may decrease the relative impact of the subsidy increasing firms' reservation value in the low state, i.e. if medium-to-higher skilled workers on the monopsony wage are now getting paid more in the low state due to the minimum wage, it may matter less whether those of them on the full surplus wage have to take a pay-cut or not. This latter effect may also lower the relative job-ladder impact of preventing lower-skill workers from becoming unemployed. Thus overall we can expect that the subsidy will likely have a less negative effect on the average wages of lower-skilled workers, as it no longer brings them into low-wage work, while it may have a less positive impact on the wages of higher wage workers.

Average Wages. Even before breaking average wages down by skill level, we can see in figure 12 that the subsidy's effects are dramatically different with the minimum wage.

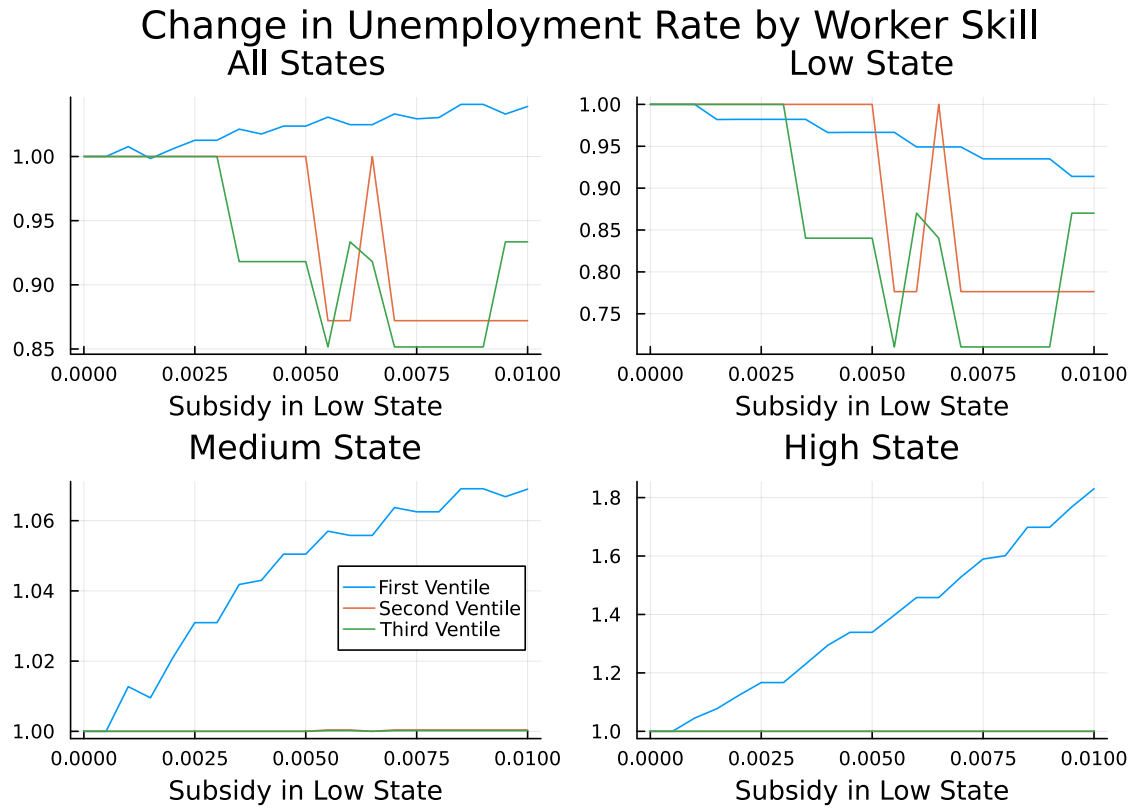

FIGURE 11. Effect of Subsidy on Unemployment by Worker Skill Level with Minimum Wage

Note: Unemployment is normalised to one when the policy is zero.

While wages still increase with the subsidy, this is now primarily driven by the low state, where before wages were decreasing, while wages are now slightly decreasing in the medium state where they were previously mildly increasing, while wages remain somewhat increasing in the high state. This already suggests the composition effect which drove the reduction in wages in the low state is no longer a strong force, and that the effects driving higher low state wages - i.e. a higher quit rate and higher firm reservation values, are dominating.

Wage Inequality. Breaking down changes in wages by skill level in figure 13, we already see a dramatically different picture to before. The effects of the policy now point solidly towards wage compression. In particular the first and second quintiles see their average wages increase in all three aggregate state categories. For the very lowest-skilled in the first quintile the increase in the medium and high states may be partially a composition effect as the very lowest skilled are made unemployed, but for much of these two categories this will not be the case as figure 11 makes clear. Thus this is clearly the case of our two "ladder" type mechanisms, the increased poaching rate and increased low-state firm reservation

FIGURE 12. Effect of Subsidy on Average Wages with Minimum Wage

values, driving up wages at the bottom end of the distribution. While these effects will also play a role further up the distribution, the reduction of firm reservation values in the medium and high states due to increased taxes, seems to have an effect, for the top three quintiles in the medium state, and for the very top quintile in the high state; however, while overall the biggest gains accrue to those lower down the distribution, only the very top quintile sees their wages drop overall.

These results are more in line with the post-COVID wage compression seen in the US (Autor, Dube, and McGrew 2023), and the stark difference between them and the results without the minimum wage emphasise the importance of considering potential sources of wage rigidity when analysing policies which seek to preserve jobs.

3.3.4. Income and Production

Production and the Labour Share. The results on total wage income and net market production, presented in figure 14 likewise presents a somewhat different picture than in the case without a minimum wage. Whereas in figure 6, both overall wage income and net market production were increasing, now - unsurprisingly since the effect on overall unemployment is no longer clearly negative, the subsidy has a clearly negative effect on

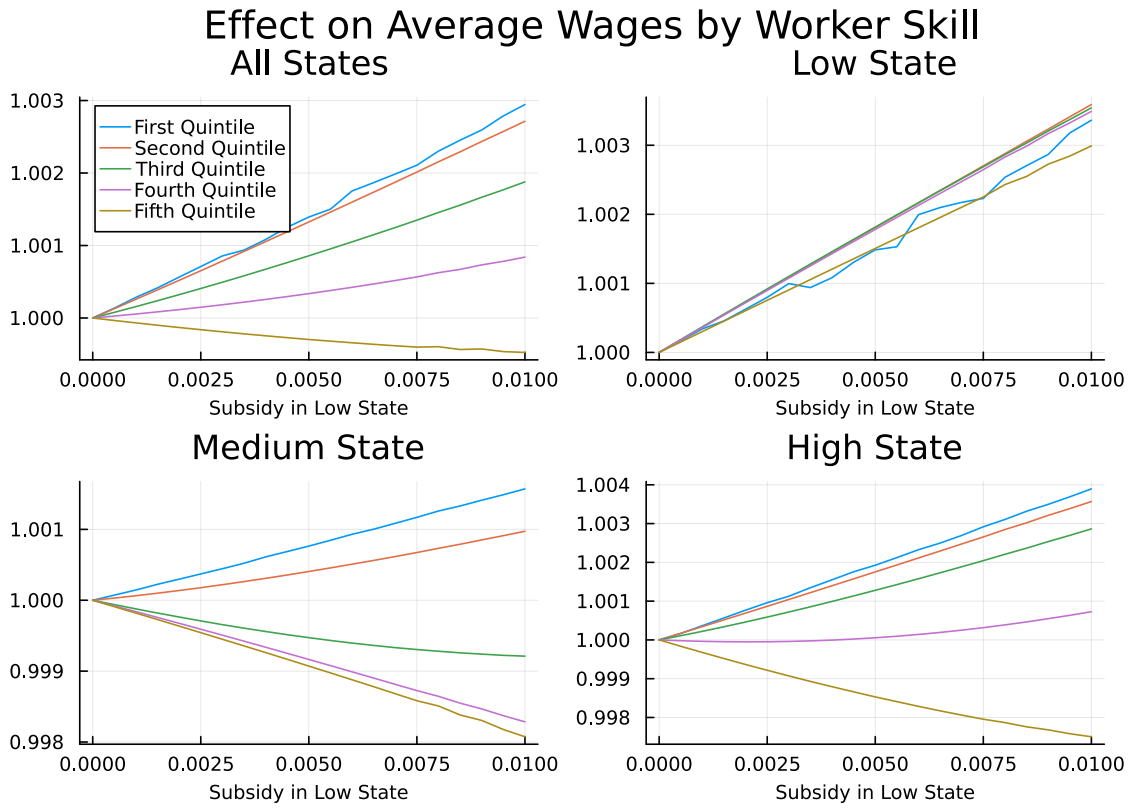

FIGURE 13. Effect of Subsidy on Average Wages by Worker Skill Level with Minimum Wage

Note: Average Wages are normalised to one when the policy is zero.

net production. Wage income however is still increasing, driven primarily by it's increase in the low state and that while both net market production and wage income fall in the medium state, wage income falls by far less than overall production. Notably this means that with the minimum wage the subsidy causes an increase in the labour share - whereas for the equivalent subsidy size without the minimum wage in figure 6, the labour share was largely unchanged or decreasing.

Wage Income by Skill Level. Once again breaking the changes in wage income down by skill level in figure 15 the picture is very different than in figure 7. Whereas before the overwhelming reduction in unemployment hugely boosted wage income for the lowest quintile despite their falling wages, now the opposite is true - the drop in overall employment for the lowest quintile is enough to decrease their overall wage income, despite rising wages and despite large gains in the low state. Unlike figure 7 however, this effect is no longer overwhelming, the second lowest quintile indeed sees a proportional increase in wage income of a similar magnitude. Overall in fact it is only the lowest and highest quintiles which see their total wage income decrease, while the three middle

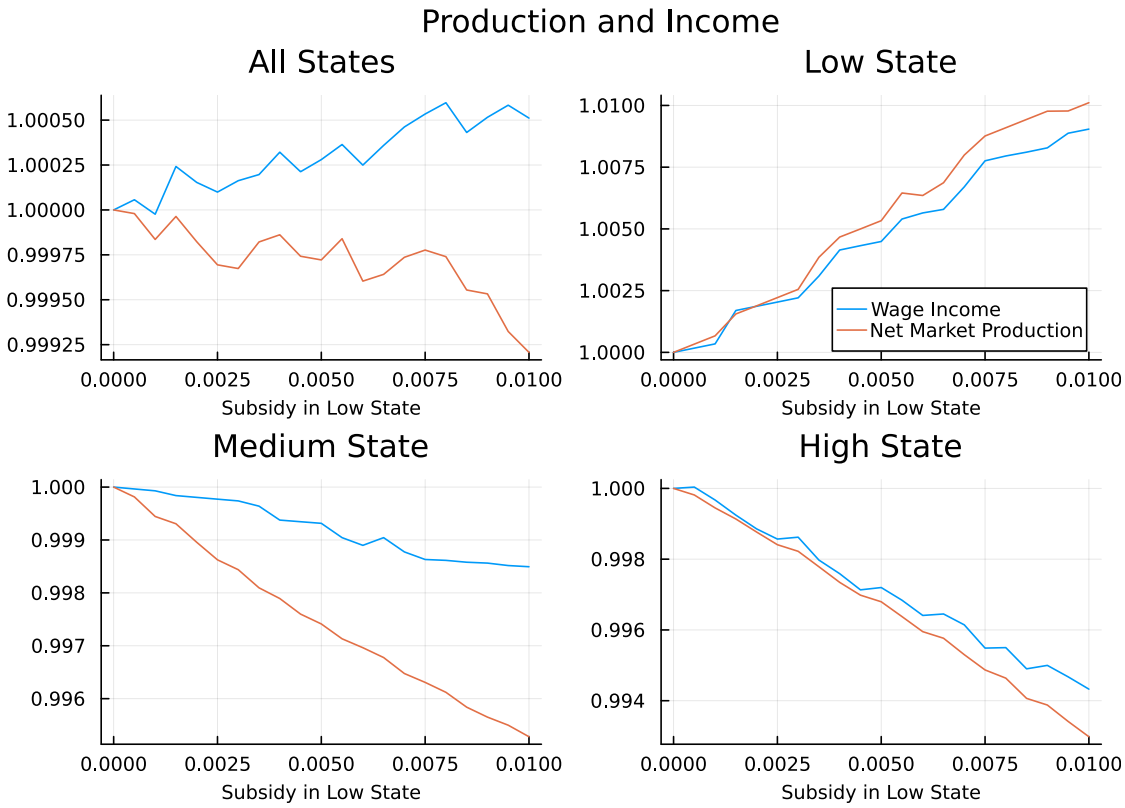

FIGURE 14. Effect of Subsidy on Production and Wage Income with Minimum Wage

Note: Wage Income and Net Production are normalised to one when the policy is zero.

quintiles all see theirs increase, with the second quintile doing best, followed by the third. These results therefore re-emphasise the model's agreement, already suggested by the unemployment results in figure 11,with empirical findings that the primary group which benefits from job-insurance policies are insiders slightly above the bottom of the income distribution (Cahuc and Carcillo 2011; Giupponi, Landais, and Lapeyre 2022).

3.3.5. Social Welfare

I use the same weighted measure of social measure presented in equation (19), where $\chi = 1$ represents a utilitarian social planner, while $\chi = 0$ represents a social planner only interested in the welfare of workers. The minimum wage may change the impact of the subsidy on social welfare in several important ways. Firstly it may mean that those matches which form due to the subsidy may be those which would have had a positive surplus without the subsidy, but were not able to form due to the minimum wage boosting the welfare impact of the subsidy. On the other hand the subsidy's more limited impact on employment over the business cycle may meaning that it decreases production overall may counteract this effect. With $\chi < 1$ however, the minimum wage may imply a more

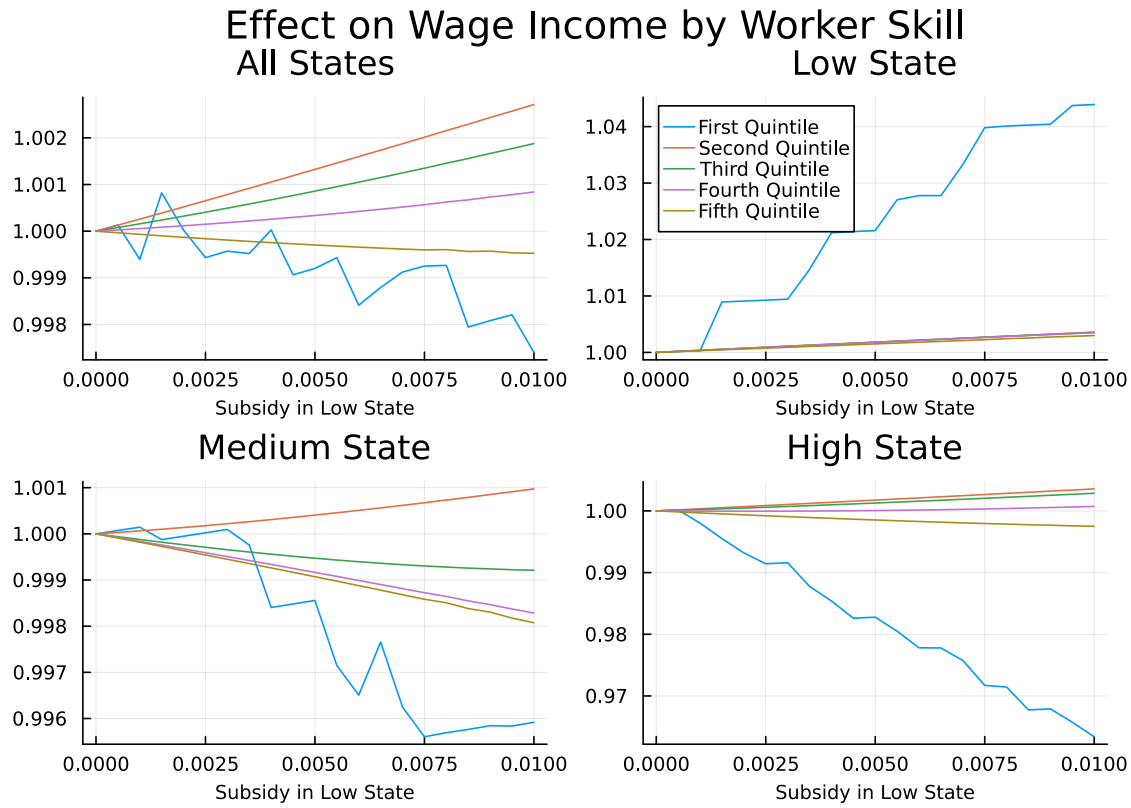

FIGURE 15. Effect of Subsidy on Wage Income by Worker Skill Level with Minimum Wage

Note: Wage Income is normalised to one when the policy is zero.

positive impact of the subsidy on welfare, as it causes a clearer shift to the labour share, while it's reduction of unemployment in the low state may bring large welfare gains to workers who's opportunity cost of work may be far below the minimum wage when the aggregate state is low.

The social welfare results, presented in figure 16, are indeed significantly different from the case without a minimum wage. While with $\chi = 1$ the results are unambiguously negative and similar to those in figure 8, for $\chi \leq 0.5$ the policy is welfare improving across the range of subsidies considered. It should be noted that the magnitudes involved are somewhat smaller than the changes in figure 8, however nonetheless this is a sharply different result which highlights the importance of considering wage rigidities, such as those caused by minimum wages, when considering the welfare and distributional effects of policies which aim to subsidise labour hoarding.

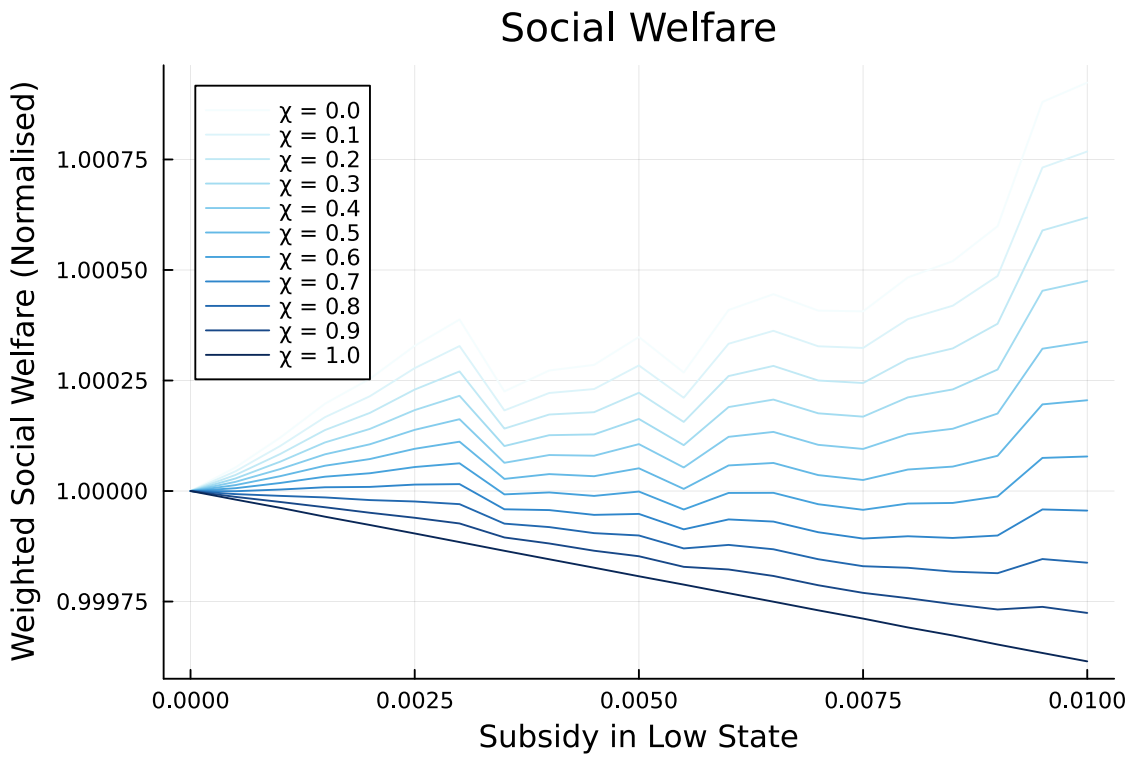

FIGURE 16. Effect of Subsidy on Social Welfare with Minimum Wage

Note: Social Welfare is normalised to one in when the subsidy is set to zero.

4. Conclusion

To conclude, during the pandemic induced recession of 2020-2021, policies aimed at preserving worker-firm matches grew in popularity around the globe. While the form these policies took varied, from short-time-work schemes, to recall unemployment insurance, to low interest loans - their key commonality was that they sought to subsidise labour hoarding during the recession. While there is a growing literature examining the effects of such policies from both an empirical and theoretical perspective, there has so far been relatively little examination of their potential dynamic effects, in particular to my knowledge there has been no structural examination of their potential effects on the job ladder - i.e. the process by which workers increase their wage via. job-to-job transitions. I contribute to this literature by examining the effects of subsidising labour hoarding in recessions within a job-search model with heterogeneous workers, aggregate shocks, and on-the-job search - namely the Robin (2011) model, which I modify to include a legal minimum wage. The model includes a clear job-ladder mechanism via a sequential auctions framework, as well as wage renegotiation according to the evolution of the aggregate state. The model thus yields rich results on unemployment, wages and inequality which can be used to understand the dynamic mechanisms through which subsidising

labour hoarding in recessions affect the labour market throughout the business cycle.

I find that without a minimum wage, the model predicts a significant reduction in unemployment - achieved primarily by shifting low skill workers into low wage employment. Meanwhile the increase in firms' low-state reservation values brought on by the subsidy, benefits the wages of higher skilled workers who are most likely to be poached (as they are most likely to be in employment), leading to an increase in wage inequality. In the case without a minimum wage the effects on social welfare, whether examined by a utilitarian social planner or one focussed on worker's welfare specifically, are unambiguously negative except at very low or very high levels of the policy - which may be slightly beneficial to workers. This is unsurprising seeing as the main effects of the policy in this case are to subsidise low productivity matches by taxing high productivity ones, while shifting people into work with wages scarcely better than their reservation wage.

My results in the case where there is a binding minimum wage are rather different. In this case the reduction in unemployment is far less certain: the reduction in unemployment in the low state is now more modest overall and it is largest for those just above the bottom of the skill distribution, indeed over the whole business cycle the policy has a negative effect overall on the employment of the bottom ventile of workers. These results are in line with empirical results finding that labour hoarding policies seem to mostly protect insiders rather than the most marginal employees (Cahuc and Carcillo 2011; Giupponi, Landais, and Lapeyre 2022). The wage results are also rather different, finding wage compression rather than an increase in wage inequality, suggesting that the job ladder effects of the policy may benefit those lower down the skill distribution - a finding which is in line with the US experience following the pandemic (Autor, Dube, and McGrew 2023). Overall I find that the policy causes an increase in the labour share and an increase in wage income for the middle three quintiles of the worker skill distributions, with only the top and bottom quintiles seeing their wage income fall. I also find that the policy, while still welfare negative from the utilitarian social planner's point of view - is now welfare improving for workers.

There are several potential avenues for future work following this paper. In particular, the major omission of the model considered here is that vacancy creation is exogenous. Endogenising vacancy creation through a standard matching function is certainly possible however, though it would require an approximation of the state space, and this would provide an important insights as to how the mechanisms highlighted here would change were firms able to adjust their vacancies in response. Another natural next step is to compare the model to the data and to calculate counterfactual policy experiments relative to the data. Re-estimating the model on wage data as well as turnover data, including choosing a minimum wage so as to best fit the data, would help prepare the model for such counter-factual analysis. Including a richer sense of sorting alongside wages by

including heterogeneous firms, such as in Pascal (2020)'s work building off of Lise and Robin (2017), would also provide a better insight into the labour productivity effects of labour hoarding policies and their relationship with the wage effects. To gain a better view of the specific trade-offs involved in short-time-work, it would also be useful to construct a model with aggregate shocks and on-the-job search, that also allowed for adjustment of labour demand along the intensive margin, with diminishing marginal returns to hours worked by a given worker. Finally, an important area of research for policymaking is the degree of substitutability or complementarity between policies which aim to insure workers in recessions, and those such as labour hoarding subsidies, which seek to insure firms (Giupponi, Landais, and Lapeyre 2022). Therefore, a potentially important step is to integrate unemployment insurance into the same model, and examine both its effects alone in relation to the effects found here, as well as the joint effects of including both policies.

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