A Theory of Negative Convenience Yields

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Abstract

This paper develops a structural model to explain the recent emergence of negative convenience yields (CYs) on sovereign debt, episodes in which government bonds trade at higher yields than synthetic, credit-risk-equivalent alternatives. Previously documented in the U.S., I show that similar yield inversions have also occurred in France, where public debt has recently traded at a discount to credit-equivalent synthetic instruments.

I present a dynamic, three-period model in which investors face institutional mandates that mechanically constrain portfolio rebalancing, generating inelastic demand. Liquidity is defined not by the feasibility of trade, but by an asset's state-contingent resale value. This reinterpretation is particularly relevant when public debt is held for insurance purposes: if prices collapse precisely in adverse states, the asset's insurance value erodes, even if it remains tradable.

When debt issuance increases, whether deterministically or stochastically, mandate-driven segmentation leads to lower resale prices and compresses the convenience yield. Under sufficient supply pressure, this erosion can invert the CY's sign. The model delivers a tractable decomposition of the preference premium (price equivalent of the CY) and shows that valuation anomalies can persist in equilibrium due to demand inelasticity and the absence of effective arbitrage.

Quantitative simulations replicate the CY inversions both observed in France and the U.S. The framework links debt supply dynamics, institutional rigidity, and liquidity premia, offering a unified explanation for how public debt can lose its pricing advantage despite sound fundamentals.

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1 Introduction

This paper provides a theoretical explanation for a recent and striking empirical fact: sovereign convenience yields can turn negative. Using the approach of Jiang, Lustig, et al. (2020), I document that since mid-2024, five-year French government bonds (OATs) have traded at yields exceeding those of synthetic, credit-risk-equivalent instruments constructed from overnight indexed swaps (OIS) and CDS spreads. This implies a negative convenience yield (CY), challenging not only the traditional view that public debt earns a yield discount due to its safety and liquidity, but also the arbitrage principle that such pricing gaps cannot persist in equilibrium. Figure 1 shows the sign reversal and Figure 2 decomposes the observed and synthetic yields.

Positive convenience yields have been extensively documented in U.S. and European sovereign bond markets (Krishnamurthy and Vissing-Jorgensen, 2012; Du, Im, and Schreger, 2017; Jiang, Lustig, et al., 2020). These premia are typically attributed to the role of public debt as collateral and as a safe, liquid store of value—services especially valuable in incomplete markets, as reviewed by Reis (2022). In theory, a positive CY reflects both credit safety and liquidity. The recent inversion in France, however, occurred without any observable rise in credit risk, pointing instead to an endogenous erosion of liquidity.

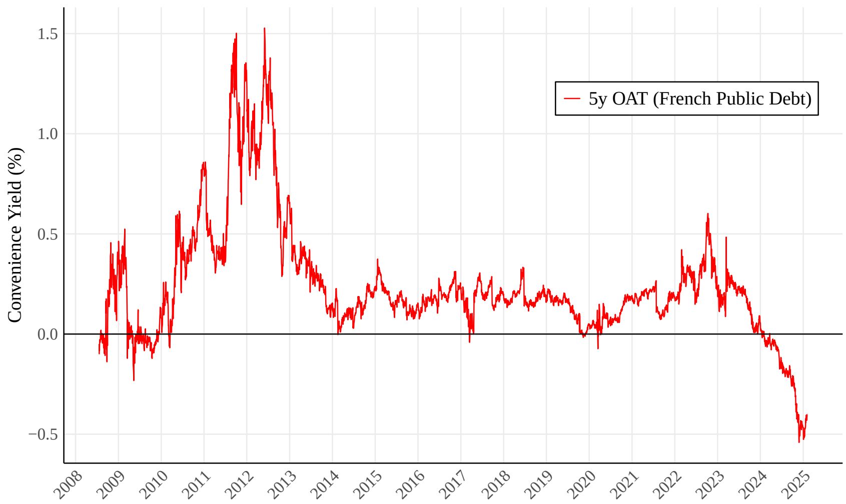


Figure 1: Convenience Yield on French 5-Year Bonds

This paper develops a dynamic, three-period model in which liquidity is defined not as the feasibility of trade, but as the asset's resale value in state-contingent equilibrium. The core friction is institutional: investors face portfolio mandates, such as regulatory liquidity rules or internal risk constraints, that fix the share of wealth allocated to public debt. When debt supply increases, these mandates constrain rebalancing, leading to inelastic demand, price pressure in secondary markets, and deterioration in resale value.

This interpretation of liquidity is particularly relevant in environments where public debt is held not for return, but for insurance. Consider an investor who allocates wealth to sovereign bonds with the expectation of selling them in adverse states to meet liquidity needs. If bond prices become highly sensitive to supply shocks and mandate-driven segmentation precisely in those states, the bond's insurance value breaks down. The asset remains tradeable, but its ability to transfer purchasing power across states is impaired. In this sense, liquidity is not about whether an asset can be sold, but about the price at which it can be sold when it is needed most.

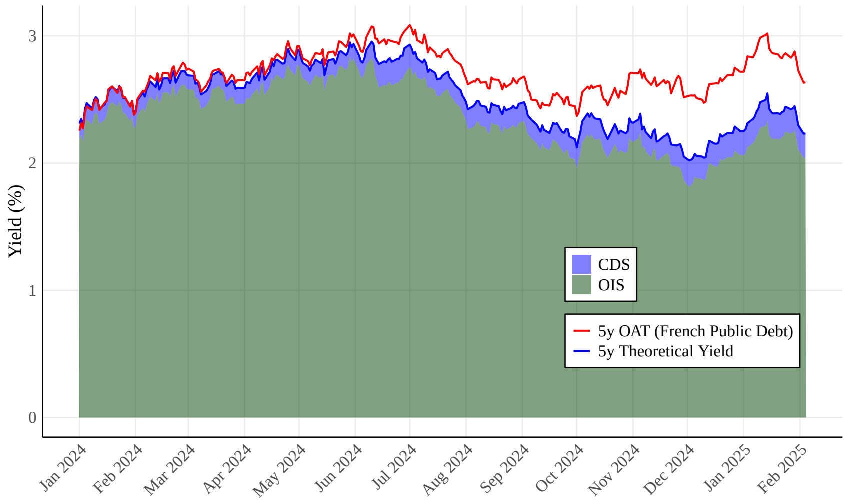


Figure 2: French Sovereign Yield and Credit-Equivalent Synthetic Decomposition

The model yields three core insights. First, sovereign bond valuations are highly sensitive to the level and expectation of future public debt issuance, even absent credit or trading risk. Second, liquidity is endogenous: it depends on whether an asset can be resold at a stable price in adverse states, which in turn reflects the rigidity of investor demand. Third, when issuance exceeds institutional absorption capacity, the convenience yield may compress and ultimately invert, reproducing the French episode and aligning with U.S. findings in Jiang, Richmond, and Zhang (2024).

This paper contributes to the existing literature in two ways. Empirically, it documents the first persistent instance of negative convenience yields in a major Eurozone bond market. Theoretically, it offers a tractable partial equilibrium framework in which mandate-driven segmentation and supply dynamics jointly generate endogenous liquidity premia. The model abstracts from sovereign default, execution frictions, and monetary factors, yet replicates yield inversions previously thought incompatible with arbitrage.

The mechanism generalizes the Inelastic Markets Hypothesis of Gabaix and Koijen (2023) to the sovereign bond space. In this environment, bond prices are not pinned down by marginal valuation or arbitrage, but by the mechanical absorption behavior of inelastic investors. As shown by Nenova (2025), institutional demand for sovereign debt is remarkably unresponsive to price signals, especially under regulatory mandates. When no marginal agent rebalances, price wedges persist, even when they violate textbook arbitrage logic.

More broadly, the paper shows that public debt may lose its liquidity premium not due to increased risk, but due to demand rigidity. When issuance expands in segmented markets, secondary prices adjust endogenously: liquidity deteriorates, valuation gaps emerge, and convenience yields can become negative. This mechanism has implications for debt management, liquidity regulation, and the interpretation of yield spreads in modern financial systems.

The remainder of the paper is organized as follows. Section 2 develops the baseline model under deterministic issuance. Section 3 extends the framework to incorporate issuance uncertainty and liquidity risk. Section 4 concludes.

Related Literature. This paper contributes to several strands of the literature on sovereign bond pricing, liquidity premia, and segmented markets.

The first connection is empirical. Krishnamurthy and Vissing-Jorgensen (2012) and Du, Im, and Schreger (2017) document persistent convenience yields on U.S. Treasuries, while Jiang, Lustig, et al. (2020) extend the analysis to European markets, including France. These studies show that convenience yields fluctuate with fiscal policy and macro-financial conditions. More recently, Jiang, Richmond, and Zhang (2024) report episodes of negative CY in U.S. data, attributing them to issuance surges and limited absorption capacity. The French inversion documented here confirms that these dynamics are not region-specific.

Second, the paper offers a structural explanation for CY inversions. While Brunnermeier, Merkel, and Sannikov (2022) and Arcidiacono, Bellon, and Gnewuch (2024) show that debt supply compresses convenience yields, but CYs remain non-negative. By modeling resale value as endogenous and demand as inelastic, this paper shows that even modest supply shifts can generate persistent price deviations.

Third, the model builds directly on the Inelastic Markets Hypothesis (Gabaix and Koijen, 2023), which emphasizes that prices are flow-sensitive when marginal demand is rigid. While originally developed for equity markets, recent applications to sovereign bonds include Nenova (2025), who provides fund-level evidence that sovereign bond demand is shaped by regulatory mandates and exhibits low elasticity—especially for highly rated debt. This motivates the modeling of mechanical portfolio constraints.

Finally, this paper departs from traditional microstructure views of liquidity. The OTC literature, including Duffie, Gârleanu, and Pedersen (2005), models illiquidity as stemming from search frictions or trading delays. Other frameworks link liquidity to market depth, suggesting that supply increases enhance liquidity by thickening markets (He, Krishnamurthy, and Milbradt, 2019). In contrast, this paper defines liquidity as expected resale value in segmented equilibrium. Inelastic mandates cause additional supply to depress prices in low-demand states, thereby reducing liquidity rather than improving it.

In sum, the paper reframes liquidity premia as equilibrium outcomes of mandate-driven segmentation and issuance dynamics. This helps rationalize recent anomalies in sovereign bond markets where convenience yields, once thought to reflect fundamental safety, can become negative absent any rise in default risk.

2 Model with Deterministic Debt Supply

This section develops a stylized three-period model to study how public debt supply shapes secondary market liquidity and the resulting convenience yield. The baseline environment assumes deterministic debt issuance, so liquidity conditions are known ex ante. The next section extends the model to introduce probabilistic issuance and endogenous liquidity risk.

The exposition proceeds in three parts. Section 2.1 outlines the environment and key assumptions. Section 2.2 and 2.3 presents the analytical results and provides numerical simulations to illustrate the main mechanisms, respectively for the second and the initial period.

2.1 Environment

We consider a three-period endowment economy with no storage and incomplete markets. Time is indexed by $t = 1, 2, 3$, and a unit mass of investors receives exogenous endowments in each period, which fully depreciate between periods. The model follows the timing structure of Arcidiacono, Bellon, and Gnewuch (2024), originally developed to study safe-asset spillovers, but fundamentally differs in its treatment of liquidity and pricing frictions. In particular, I introduce institutional mandates and define liquidity as endogenous resale value, rather than feasibility of trade.

Because agents cannot smooth consumption through either insurance mechanisms or storage technologies, they rely on financial assets to reallocate resources intertemporally and across states. The economy features two such assets, denoted a and b, representing a frictionless benchmark and government-issued public debt, respectively.

The core friction in the model arises from institutional investment mandates, which constrain investors' ability to adjust freely portfolios in response to income shocks. These mandates generate inelastic demand, in the spirit of the Inelastic Markets Hypothesis of Gabaix and Koijen (2023), and give rise to state-contingent resale value differences across otherwise identical assets.

Throughout, I abstract from sovereign default risk. Both assets a and b are assumed to share identical credit quality. Asset a represents a synthetic benchmark, a portfolio combining a liquid, risk-free instrument (proxied by the OIS in the data) and a CDS contract on public debt. Asset b reflects traded government bonds. All price differentials between a and b therefore stem from liquidity conditions, collateral constraints, and institutional segmentation, not from differences in creditworthiness.

Investors and Heterogeneity. Agents choose consumption and asset holdings to maximize lifetime expected utility. In period 1, each investor selects a consumption level c_1 and a portfolio (a_1, b_1), subject to a standard budget constraint and a collateral constraint tied to public debt holdings.

At the beginning of period 2, an idiosyncratic income shock occurs with probability θ, generating cross-sectional heterogeneity. The representative investor from period 1 splits into two types, indexed by $i \in \{L, H\}$:

- Type L receives an endowment $\omega_2 + \omega_{2,L}$, with $\omega_{2,L} < \bar{\omega}$;
- Type H receives an endowment $\omega_2 + \omega_{2,H}$, with $\omega_{2,H} \geq \bar{\omega}$.

In period 2, type-i agents choose consumption $c_{2,i}$ and adjust their portfolio positions in the secondary market. Let $\Delta^k_{2,i}$ denote the net demand for asset $k \in \{a, b\}$ by agent type i. These quantities are determined endogenously through market-clearing conditions and are subject to institutional mandates described below.

In period 3, all assets mature and agents consume their final endowments and asset payoffs. Figure 3 summarizes the sequence of events and frictions embedded in the model.

Assets. Both assets are purchased in period 1 and mature in period 3 with a face value of one. Asset a is non-pledgeable and functions purely as a store of value. It is in fixed supply and is not subject to additional issuance in period 2.

In contrast, asset b provides collateral services in period 1: a fraction $(1-\alpha)$ of its face value can be pledged, relaxing the collateral constraint. Only the remaining share αb_1 is marketable in period 2. In that period, the government may issue additional public debt Δ^b_{new}, which interacts with mandate-driven investor demand to determine the secondary market price of b.

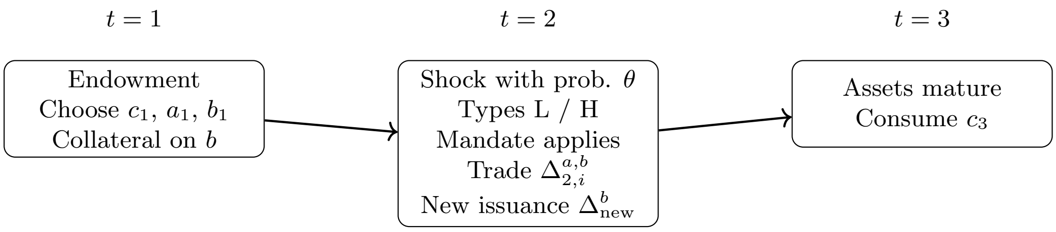


Figure 3: Timeline of Model Events

Financial Mandates. In period 2, investors are subject to a mechanical financial mandate requiring that a fixed share $\xi \in (0,1)$ of their idiosyncratic wealth $\omega_{2,i}$ be allocated to asset a, and the remaining share $1-\xi$ to asset b:

$$\frac{p^a_2 \Delta^a_{2,i}}{\omega_{2,i}} = \xi \quad \text{and} \quad \frac{p^b_2 \Delta^b_{2,i}}{\omega_{2,i}} = 1-\xi, \quad \forall\, i \in \{L, H\}.$$

If an investor's endowments falls below a threshold $\bar{\omega}$, the mandate cannot be satisfied. In this case, the investor is forced to liquidate her entire portfolio:

$$\Delta^a_{2,i} = \begin{cases} \xi \frac{\omega_{2,i}}{p^a_2} & \text{if } \omega_{2,i} \geq \bar{\omega}, \\ -a_1 & \text{otherwise} \end{cases} \qquad \Delta^b_{2,i} = \begin{cases} (1-\xi) \frac{\omega_{2,i}}{p^b_2} & \text{if } \omega_{2,i} \geq \bar{\omega}, \\ -\alpha b_1 & \text{otherwise} \end{cases}$$

That is, low-wealth agents (type L) liquidate all their holdings of both assets, while high-wealth agents (type H) absorb these sales in accordance with their mandates.

These frictions generate inelastic demand curves and disconnect second period asset prices from marginal utility, thereby endogenizing price effects from shifts in supply.

The assumption of fixed mandates reflects an empirically grounded constraint rather than a modeling simplification. As Nenova (2025) documents, mutual funds and other institutional investors exhibit persistently low substitution elasticities across sovereign bonds, particularly for highly rated debt. These patterns reflect regulatory liquidity coverage requirements, internal risk constraints, and prospectus-driven portfolio rules. Even in the presence of sizable yield differentials, rebalancing remains limited, and substitution occurs only slowly.

Liquidity. In this framework, liquidity is not defined by the feasibility of trade since both assets can be sold in the secondary market, but by the expected resale price across states. Liquidity therefore reflects intertemporal *price stability*, not transactional access. Since asset a is in fixed supply and not subject to new issuance, its price remains stable and serves as a benchmark. In contrast, the price of asset b depends on potential new issuance.

This difference generates a relative price effect in the secondary market: asset b may need to be sold at a discount when supply rises and constrained investors cannot absorb it. In such cases, illiquidity arises not from frictions in execution, but from insufficient marginal demand elasticity. Expected resale value becomes a state-contingent function of institutional structure and debt issuance policy.

Although assets a and b have identical payoffs, equilibrium prices may diverge. In principle, such price differences imply arbitrage. In practice, the absence of unconstrained investors in the model prevents this logic from closing. This is consistent with empirical evidence from Gabaix and Koijen (2023), who document that asset prices react strongly to flows when market segmentation and mandate rigidity limit arbitrage. Most institutional investors maintain fixed asset shares, while arbitrageurs, such as hedge funds and dealers, are either too small or face capital constraints. These frictions allow pricing anomalies to persist, even in the absence of default risk.

Investor Optimization. Given the environment described above, the representative investor solves a dynamic program over three periods. Preferences are time-separable with discount factor $\beta \in (0,1)$, and per-period utility is represented by a function $u(\cdot)$ satisfying standard assumptions. In period 1, the investor chooses consumption c_1 and asset holdings (a_1, b_1) to maximize expected lifetime utility, subject to a standard budget constraint and a collateral constraint tied to the pledgeable portion of public debt.

Formally, the problem is:

$$
\begin{aligned}
V_1 = \max_{c_1, a_1, b_1} \quad & u(c_1) + \beta\, \mathbb{E}[V_{2,i}(a_1, \alpha b_1)] \\
\text{s.t.} \quad & c_1 = \omega + \tilde{\omega}_1 - p_1^a a_1 - p_1^b b_1, \\
& \tilde{\omega}_1 = \phi b_1 (1-\alpha), \quad \text{(Collateral constraint)}
\end{aligned}
$$

where ϕ denotes the haircut on public debt used for collateral purposes, and $\alpha \in (0,1)$ is the share of debt that remains marketable in period 2.

The continuation value $V_{2,i}$ depends on the realization of the idiosyncratic income shock in period 2:

$$
\begin{aligned}
V_{2,L}(a_1, \alpha b_1) &= u(c_{2,L}) + \beta u(c_{3,L}), \\
V_{2,H}(a_1, \alpha b_1) &= u(c_{2,H}) + \beta u(c_{3,H}).
\end{aligned}
$$

The consumption profiles are given by:

$$
\begin{aligned}
c_{2,i} &= \omega_2 + \omega_{2,i} - p_2^a \Delta_{2,i}^a - p_2^b \Delta_{2,i}^b, \\
c_{3,i} &= \omega_3 + a_1 + \alpha b_1 + \Delta_{2,i}^a + \Delta_{2,i}^b.
\end{aligned}
$$

In period 2, agents do not reoptimize. Instead, portfolio reallocation and consumption are determined mechanically by the financial mandate and market-clearing conditions. I characterize the resulting equilibrium allocations and prices in the following subsection.

Because debt issuance is deterministic in this baseline setting, secondary market prices reflect anticipated liquidity conditions and the interaction of supply with inelastic investor demand, rather than conventional stochastic risk.

Summary Metrics. To capture the value investors attach to holding public debt, I define two complementary metrics.

The *convenience yield* (CY) quantifies the yield advantage of holding asset b over the benchmark a:

$$CY \equiv r^a - r^b = \frac{1}{p_1^a} - \frac{1}{p_1^b}.$$

The *preference premium* (PP) captures the price wedge between the benchmark asset a and public debt b in the primary market:

$$\pi^b \equiv p_1^a - p_1^b.$$

While CY highlights the yield-based perspective, the preference premium offers a more tractable measure of how collateral services, liquidity frictions, and redistributional forces shape asset valuations.

2.2 Results Period 2

This subsection characterizes the equilibrium in period 2 by deriving closed-form expressions for asset prices and consumption allocations under the institutional constraints outlined above. These closed-form expressions inform the investor's optimization in period 1, which is solved by backward induction in 2.3. To validate the analytical results and assess comparative statics, I complement the derivations with numerical simulations. Formal proofs of Proposition 2 and B.2 can be found in Appendix B and all simulation details are reported in Appendix C.

Period 2 Equilibrium. In the absence of new issuance for asset a, all units liquidated by type L agents must be absorbed by type H agents:

$$\theta\Delta^a_{2,L} + (1-\theta)\Delta^a_{2,H} = 0 \quad \Longleftrightarrow \quad \Delta^a_{2,H} = \frac{\theta}{1-\theta}a_1.$$

In contrast, asset b is subject to new issuance in period 2. Type H agents must absorb both the liquidated positions of type L and the additional supply:

$$\theta\Delta^b_{2,L} + (1-\theta)\Delta^b_{2,H} = \Delta^b_{\text{new}} \quad \Longleftrightarrow \quad \Delta^b_{2,H} = \frac{\theta\alpha b_1 + \Delta^b_{\text{new}}}{1-\theta}.$$

Prices are pinned down by the fixed portfolio shares of type H investors. Substituting the market-clearing quantities into their mandated allocations yields:

$$p_2^a = \xi\,\frac{1-\theta}{\theta}\,\frac{\omega_{2,H}}{a_1},$$
$$p_2^b = (1-\xi)(1-\theta)\,\frac{\omega_{2,H}}{\theta\alpha b_1 + \Delta^b_{\text{new}}}.$$

Given these allocations, period-2 consumption follows directly. Type L agents, unable to meet their required portfolio shares, liquidate all holdings and consume the proceeds. Type H agents absorb the full supply and rebalance in line with their fixed portfolio shares. The resulting consumption paths are:

Type L (fire-sale agent):

$$\begin{aligned}
c_{2,L} &= \omega_2 + \omega_{2,L} + p_2^a a_1 + p_2^b \alpha b_1 \\
&= \omega_2 + \omega_{2,L} + \xi \frac{1-\theta}{\theta}\omega_{2,H} + (1-\xi)(1-\theta)\frac{\omega_{2,H}\alpha b_1}{\theta\alpha b_1 + \Delta^b_{\text{new}}}, \\
c_{3,L} &= \omega_3.
\end{aligned}$$

Type H (absorbing agent):

$$\begin{aligned}
c_{2,H} &= \omega_2 + \omega_{2,H} - p_2^a \Delta^a_{2,H} - p_2^b \Delta^b_{2,H} \\
&= \omega_2, \\
c_{3,H} &= \omega_3 + a_1 + \alpha b_1 + \Delta^a_{2,H} + \Delta^b_{2,H} \\
&= \omega_3 + \frac{1}{1-\theta}(a_1 + \alpha b_1 + \Delta^b_{\text{new}}).
\end{aligned}$$

These equilibrium prices allow us to characterize the endogenous liquidity differential between the two assets. In particular, I derive a closed-form expression for the secondary market price wedge, δ^b, which captures how supply conditions and mandate-driven inelastic demand jointly determine relative valuation.

Liquidity. To quantify the relative liquidity of public debt b compared to the benchmark asset a, I define the secondary market price spread:

$$\delta^b \equiv p_2^b - p_2^a = (1-\theta)\omega_{2,H}\left(\frac{1-\xi}{\theta\alpha b_1 + \Delta^b_{\text{new}}} - \frac{\xi}{\theta a_1}\right).$$

This wedge reflects the difference in resale prices at time $t = 2$ and arises endogenously from the model's supply dynamics and institutional constraints. A positive value, $\delta^b > 0$, indicates that asset b enjoys a liquidity premium over the benchmark. In contrast, $\delta^b < 0$ signals a liquidity discount, consistent with a diminished store-of-value function in secondary markets. The formal characterization follows below.

Proposition 1. *(Characterization of the Secondary Market Price Wedge)*

Although assets a and b are substitutes in terms of payoffs, their equilibrium prices diverge due to differences in supply and the structure of portfolio mandates. The resulting endogenous price wedge δ^b satisfies:

- Liquidity premium:

$$\delta^b > 0 \quad \Longleftrightarrow \quad \frac{\xi}{1-\xi} > \frac{\theta a_1}{\theta\alpha b_1 + \Delta^b_{\text{new}}}.$$

- Liquidity discount:

$$\delta^b < 0 \quad \Longleftrightarrow \quad \frac{\xi}{1-\xi} < \frac{\theta a_1}{\theta\alpha b_1 + \Delta^b_{\text{new}}}.$$

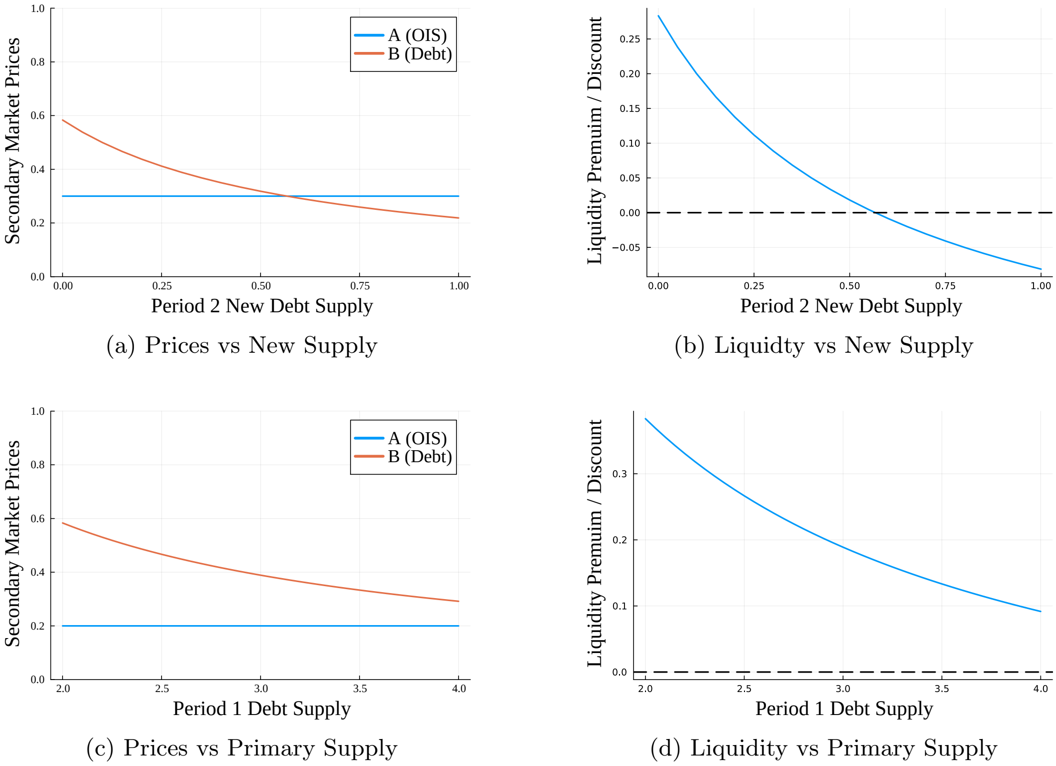
- Sensitivity to supply conditions:

$$\frac{\partial \delta^b}{\partial \Delta^b_{\text{new}}} < 0, \qquad \frac{\partial \delta^b}{\partial b_1} < 0.$$

These results provide a formal characterization of how the secondary market liquidity wedge δ^b responds to changes in public debt supply. A higher value of δ^b indicates that asset b commands a resale premium and serves more effectively as a store of value within segmented markets. Conversely, a negative wedge reflects impaired resale conditions and reduced insurance value. In both cases, increases in debt supply, whether through initial issuance b_1 or secondary issuance Δ^b_{new}, tighten resale conditions and depress the relative liquidity of public debt.

Simulation results in Figure 4 confirm the theoretical predictions of Proposition 1. Panel (4a) illustrates that, while the price of asset a remains constant, p^b_2 declines systematically with Δ^b_{new} and Panel (4b) traces the corresponding fall in the liquidity wedge δ^b, which turns negative as secondary issuance increases. Panels (4c) and (4d) report the same pattern for primary supply b_1: prices of asset b fall, and the liquidity wedge declines, confirming the signs of $\partial\delta^b/\partial b_1$ and $\partial\delta^b/\partial\Delta^b_{\text{new}}$.

Although a positive or negative δ^b may suggest the presence of arbitrage, such discrepancies persist in equilibrium due to institutional segmentation and the absence of unconstrained arbitrageurs. This persistence reflects a core implication of the Inelastic Markets Hypothesis Gabaix and Koijen (2023): even in the absence of risk, price wedges can emerge endogenously from institutional constraints on portfolio choice.



(a) Prices vs New Supply

(b) Liquidty vs New Supply

(c) Prices vs Primary Supply

(d) Liquidity vs Primary Supply

Figure 4: Impact of Debt Supply on Prices and Liquidity

2.3 Results Period 1

Period 1 Maximization Program. Combining the constraints from period 1 and the equilibrium values derived from period 2, the representative investor's problem can be written as:

$$
\begin{aligned}
V_1 = \max_{a_1, b_1} \, & u\left(\omega - p_1^a a_1 - p_1^b b_1 + \phi b_1 (1-\alpha)\right) \\
& + \theta\beta \, u\left(\omega_2 + \omega_{2,L} + p_2^a a_1 + p_2^b \alpha b_1\right) + \theta\beta^2 \, u(\omega_3) \\
& + (1-\theta)\beta \, u\left(\omega_2 + \omega_{2,H} - p_2^a \Delta_{2,H}^a - p_2^b \Delta_{2,H}^b\right) \\
& + (1-\theta)\beta^2 \, u\left(\omega_3 + a_1 + \alpha b_1 + \Delta_{2,H}^a + \Delta_{2,H}^b\right).
\end{aligned}
$$

The expression captures the investor's tradeoff between immediate consumption and future utility, accounting for collateral services in period 1 and liquidity conditions in period 2. The continuation value reflects the heterogeneity induced by the income shock and its interaction with market structure, mandates, and supply conditions.

Throughout, investors are modeled as atomistic: they take both current and future prices as given and do not internalize their impact on aggregate market outcomes. While the representative investor understands that she may sell her portfolio in the secondary market, especially in adverse income realizations, she does not anticipate that she will absorb fire-sale positions from others or play a price-setting role. Formally, when solving the period-1 problem, she treats second-period demand by high-wealth agents, $\Delta_{2,H}^k$ for $k \in \{a, b\}$, as independent of her own asset choices, implying $\partial \Delta_{2,H}^k / \partial b_1 = 0$.

The current model abstracts from strategic investor behavior by assuming atomistic agents who take prices as given and do not internalize their marginal impact on market outcomes. This simplifies the equilibrium selection problem and rules out multiple equilibria by design. However, in richer environments, especially those with a finite set of large institutions or anticipatory arbitrageurs, multiple self-fulfilling equilibria could arise, driven by expectations about future resale value or issuance responses. Incorporating such dynamics would require modeling belief feedback loops and potentially introduce coordination failures in secondary markets. While this paper focuses on tractability and intuition, exploring strategic extensions remains a promising direction for future research.

First-Order Conditions. The optimal asset allocations satisfy the following conditions. For public debt b:

$$
p_1^b = \alpha\left(\theta \cdot m_{1,2}^L \cdot p_2^b + (1-\theta) \cdot m_{1,3}^H\right) + \phi(1-\alpha).
$$

For the benchmark liquid asset a:

$$
p_1^a = \theta \cdot m_{1,2}^L \cdot p_2^a + (1-\theta) \cdot m_{1,3}^H.
$$

where:

- $m_{1,2}^L = \beta \, \frac{u'(c_{2,L})}{u'(c_1)}$ is the stochastic discount factor from period 1 to 2 in the fire-sale scenario;
- $m_{1,3}^H = \beta^2 \, \frac{u'(c_{3,H})}{u'(c_1)}$ is the discount factor from period 1 to 3 in the hold-to-maturity scenario.

This decomposition highlights the distinct sources of valuation for the two assets. Asset a functions as the benchmark liquid instrument, valued exclusively for its resale and maturity payoffs, and unaffected by either collateral constraints or new issuance. In contrast, asset b provides collateral services in period 1, enhancing its budgetary value ex ante, but also faces adverse resale conditions due to secondary market issuance in period 2.

This difference in valuation channels leads to distinct responses of p_1^a and p_1^b to anticipated changes in future debt supply, as characterized formally in the following proposition.

Proposition 2. *(Impact Secondary Market Supply on Primary Prices)*

The effect of future debt issuance Δ^b_{new} on period-1 asset prices arises through a combination/balance of channels. For asset b, three forces are at play: (i) the impact of resale price erosion on liquidity, (ii) the marginal utility of type H investors in period 3, and (iii) the marginal utility of type L investors in period 2. For asset a, only the latter two matter.

- The period-1 price of public debt, p_1^b, decreases in the level of future debt issuance Δ^b_{new} if and only if:

$$\frac{\partial p_1^b}{\partial \Delta^b_{\text{new}}} < 0 \quad \Leftrightarrow \quad \theta \left(\frac{\partial m^L_{1,2}}{\partial \Delta^b_{\text{new}}} \cdot p_2^b + m^L_{1,2} \cdot \frac{\partial p_2^b}{\partial \Delta^b_{\text{new}}} \right) + (1-\theta) \cdot \frac{\partial m^H_{1,3}}{\partial \Delta^b_{\text{new}}} < 0.$$

  This condition reflects the balance of three forces:

$$\underbrace{\left| \theta \cdot m^L_{1,2} \cdot \frac{\partial p_2^b}{\partial \Delta^b_{\text{new}}} \right|}_{\text{(i) Liquidity deterioration}} + \underbrace{\left| (1-\theta) \cdot \frac{\partial m^H_{1,3}}{\partial \Delta^b_{\text{new}}} \right|}_{\text{(ii) Decreasing Maturity kernel}} > \underbrace{\left| \theta \cdot \frac{\partial m^L_{1,2}}{\partial \Delta^b_{\text{new}}} \cdot p_2^b \right|}_{\text{(iii) Increasing Fire-sale kernel}}.$$

  The inequality is shown to hold analytically under standard assumptions (see Appendix B.1) and confirmed in simulations (Figure 5a - 5b).

- The period-1 price of the benchmark liquid asset, p_1^a, increases in Δ^b_{new} if and only if:

$$\frac{\partial p_1^a}{\partial \Delta^b_{\text{new}}} > 0 \quad \Leftrightarrow \quad \theta \cdot \frac{\partial m^L_{1,2}}{\partial \Delta^b_{\text{new}}} \cdot p_2^a + (1-\theta) \cdot \frac{\partial m^H_{1,3}}{\partial \Delta^b_{\text{new}}} > 0.$$

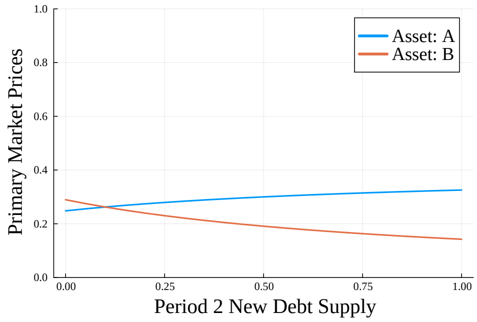
  The logic here reflects a tradeoff between:

$$\underbrace{\left| \theta \cdot \frac{\partial m^L_{1,2}}{\partial \Delta^b_{\text{new}}} \cdot p_2^a \right|}_{\text{(iii) Increasing Fire-sale kernel}} > \underbrace{\left| (1-\theta) \cdot \frac{\partial m^H_{1,3}}{\partial \Delta^b_{\text{new}}} \right|}_{\text{(ii) Decreasing Maturity kernel}}.$$

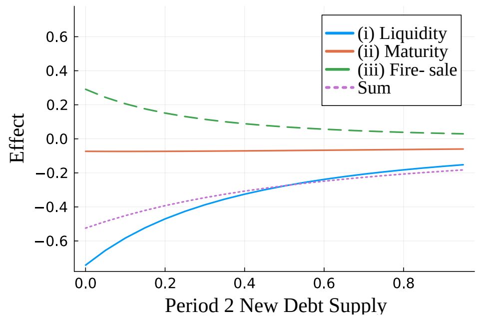
  Numerical simulations confirm that this inequality also holds under standard calibrations (Figure 5a - 5c).

When future issuance Δ^b_{new} rises, the secondary market price p_2^b declines due to mandate-driven inelastic demand. This erosion in resale value weakens asset b's effectiveness as an insurance device in the fire-sale state (type L). At the same time, increased issuance raises final-period consumption for absorbing agents (type H), reducing marginal utility $u'(c_{3,H})$ and lowering the maturity kernel $m^H_{1,3}$.

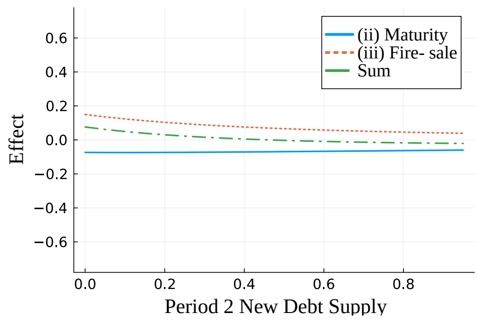
A third channel acts in the opposite direction: tighter liquidity increases marginal utility for type L agents, thereby raising the Fire-sale kernel $m^L_{1,2}$ and partially offsetting the decline



(a) Prices vs New Supply



(b) $\frac{\partial p_1^b}{\partial \Delta_{new}^b}$ Decomposition



(c) $\frac{\partial p_1^a}{\partial \Delta_{new}^b}$ Decomposition

Figure 5: Impact Secondary Market Debt Supply on Primary Prices

*Note: All parameter values used in simulations are reported in Appendix C. Panels (b) and (c) decompose the pricing response by channel. The variable **Sum** denotes the total derivative* $\frac{\partial p_1^k}{\partial \Delta_{new}^b}$ *for* $k \in \{a, b\}$.

in p_1^b. Nevertheless, under standard assumptions (Appendix B.1) and across numerical simulations (Figure 5b), this countervailing force remains too weak to dominate the liquidity and Maturity kernel effects.

Turning to the benchmark liquid asset a, we observe a symmetric logic operating in reverse. The condition $\partial p_1^a / \partial \Delta_{\text{new}}^b > 0$ highlights asset a's role as a liquidity benchmark. As secondary issuance of asset b increases and its liquidity deteriorates, the marginal utility of type L agents rises, increasing $m_{1,2}^L$ and raising the value of asset a.

This upward adjustment in p_1^a may be marginally offset by a decline in $m_{1,3}^H$. However, our baseline simulations show that the term $(1-\theta) \cdot \partial m_{1,3}^H / \partial \Delta_{\text{new}}^b \approx 0$, reflecting high final-period consumption $c_{3,H}$ and the resulting flatness of the marginal utility curve and of the Maturity kernel (Figure 5c). The impact of future issuance on p_1^a thus operates almost entirely through changes in period-2 marginal utility.

We now turn to the role of the initial quantity of public debt b_1 acquired in period 1, and its effect on asset prices. Because b_1 influences both resale prices and consumption in every period, it affects current valuations through a combination of redistributional and liquidity-based forces. Proposition 3 formalizes this relationship and presents supporting simulation evidence. Derivation details can be found in Appendix B.2.

Proposition 3. *(Impact of Initial Debt Supply on Primary Market Prices)*

The effect of initial public debt holdings b_1 on period-1 prices operates through three channels: the secondary market price p_2^b, and the pricing kernels $m_{1,2}^L$ and $m_{1,3}^H$.

- For public debt b, the sign of $\frac{\partial p_1^b}{\partial b_1}$ is theoretically ambiguous and depends on the relative strength of the following components:

$$\textbf{(i)} \quad \underbrace{\theta \cdot m_{1,2}^L \cdot \frac{\partial p_2^b}{\partial b_1}}_{\text{Liquidity deterioration}} \qquad \textbf{(ii)} \quad \underbrace{\theta \cdot \frac{\partial m_{1,2}^L}{\partial b_1} \cdot p_2^b}_{\text{Fire-sale kernel adjustment}} \qquad \textbf{(iii)} \quad \underbrace{(1-\theta) \cdot \frac{\partial m_{1,3}^H}{\partial b_1}}_{\text{Maturity kernel adjustment}}$$

  Analytically, the liquidity effect (i) is strictly negative:

$$\theta \cdot m_{1,2}^L \cdot \frac{\partial p_2^b}{\partial b_1} < 0.$$

  As shown in simulations (Figure 6b):

$$\theta \cdot \frac{\partial m_{1,2}^L}{\partial b_1} \cdot p_2^b \approx 0 \quad \text{and} \quad (1-\theta) \cdot \frac{\partial m_{1,3}^H}{\partial b_1} \approx 0$$

  The pricing kernel effects (ii) and (iii) are quantitatively small, whereas the liquidity term (i) dominates. As a result:

$$\frac{\partial p_1^b}{\partial b_1} < 0$$

- For the benchmark asset a, which is unaffected by collateral constraints or secondary market issuance, only the pricing kernel channels are relevant:

$$\textbf{(i)} \quad \underbrace{\theta \cdot \frac{\partial m_{1,2}^L}{\partial b_1} \cdot p_2^a}_{\text{Fire-sale kernel adjustment}} \qquad \textbf{(ii)} \quad \underbrace{(1-\theta) \cdot \frac{\partial m_{1,3}^H}{\partial b_1}}_{\text{Maturity kernel adjustment}}$$

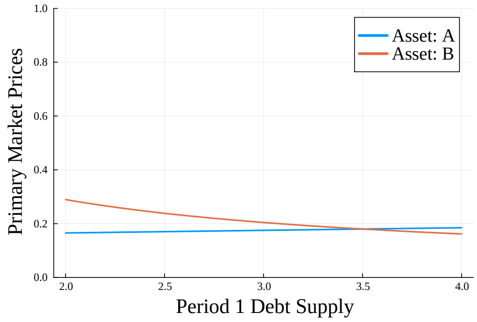
  Simulations indicate (Figure 6c):

$$\theta \cdot \frac{\partial m_{1,2}^L}{\partial b_1} \cdot p_2^a \approx 0 \quad \Rightarrow \quad \frac{\partial p_1^a}{\partial b_1} \approx 0$$

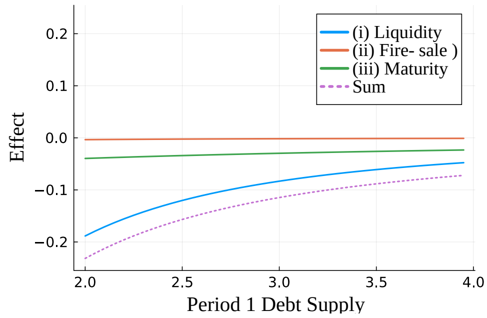
The decline in p_1^b is driven almost entirely by the liquidity channel: as b_1 increases, the resale value p_2^b declines, depressing the asset's primary market price. The marginal utility effects are nearly flat across states, as consumption changes proportionally in the numerator and denominator of the pricing kernels (Figure 6b). The effect is qualitatively and quantitatively similar to that of Δ_{new}^b (Figure 5a - 6a).

In contrast, asset a is insulated from liquidity deterioration and only responds via changes in pricing kernels, which are quantitatively minor, resulting in an effectively flat price response (Figure 6a - 6c).

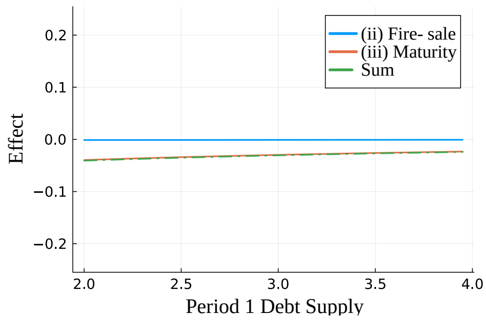
Having established how both secondary issuance and initial debt supply shape primary market valuations, I now turn to two summary metrics that capture the relative pricing of public debt in equilibrium. These indicators distill the effects of liquidity, collateral services, and segmentation into interpretable price-based measures.



(a) Prices vs Initial Supply



(b) $\frac{\partial p_1^b}{\partial b_1}$ Decomposition



(c) $\frac{\partial p_1^a}{\partial b_1}$ Decomposition

Figure 6: Impact of Initial Debt Supply on Primary Prices

Note: All parameter values used in simulations are reported in Appendix C. Panels (b) and (c) decompose the pricing response by channel. The variable ***Sum*** *denotes the total derivative* $\frac{\partial p_1^k}{\partial b_1}$ *for* $k \in \{a, b\}$.

Preference Premium. The preference premium (PP) is defined as the primary market price wedge between the benchmark asset a and public debt b: $\pi^b \equiv p_1^a - p_1^b$. Formaly:

$$\pi^b = \underbrace{\theta \cdot m_{1,2}^L \cdot \delta^{\alpha b}}_{\text{(i) Liquidity}} + \underbrace{\phi(1-\alpha)}_{\text{(ii) Collateral}} - \underbrace{(1-\theta)(1-\alpha) \cdot m_{1,3}^H}_{\text{(iii) Lost maturity share}},$$

where $\delta^{\alpha b} = \alpha \cdot p_2^b - p_2^a$ is a scaled version of the secondary market liquidity spread. The factor α reflects the share of asset b that remains marketable in period 2, excluding the portion pledged as collateral in period 1.

The preference premium reflects the interaction of three forces: (i) the collateral benefit of public debt in period 1, which supports its price; (ii) its effective resale value in period 2, scaled by the marketable share α; and (iii) the lost maturity payoff associated with the pledged share.

Although both the preference premium and the convenience yield (CY) capture deviations from benchmark pricing, the PP is more tractable and provides a clear decomposition of underlying mechanisms. This makes it particularly useful for understanding shifts in relative asset valuation.

Proposition 4. *(Sign and Mechanisms of the Preference Premium)*

The sign of π^b depends on the relative magnitude of these forces:

- If $\delta^{\alpha b} > 0$, then:

$$\pi^b > 0 \quad \Leftrightarrow \quad \theta \cdot m^L_{1,2} \cdot \delta^{\alpha b} + \phi(1-\alpha) > (1-\theta)(1-\alpha) \cdot m^H_{1,3}.$$

- If $\delta^{\alpha b} < 0$, then:

$$\pi^b > 0 \quad \Leftrightarrow \quad \phi(1-\alpha) > \left|\theta \cdot m^L_{1,2} \cdot \delta^{\alpha b}\right| + (1-\theta)(1-\alpha) \cdot m^H_{1,3}.$$

A positive preference premium signals strong valuation support from collateral utility and stable resale prospects. A negative premium reflects that liquidity deterioration dominates, which mirrors the empirical regime shift observed in French bond markets.

From previous simulations, Figure 2 and 3, we know that

$$\frac{\partial(1-\theta)(1-\alpha) \cdot m^H_{1,3}}{\partial \Delta^b_{new}} \quad \text{and} \quad \frac{\partial(1-\theta)(1-\alpha) \cdot m^H_{1,3}}{\partial b_1} \approx 0$$

Therefore, the sign of the preference premuim depens mainly on the relative strengh of:

$$\underbrace{\theta \cdot m^L_{1,2} \cdot \delta^{\alpha b}}_{\text{(i) Liquidity}} \quad \text{and} \quad \underbrace{\phi(1-\alpha)}_{\text{(ii) Collateral}}$$

The Convenience Yield. In the introduction, I documented a recent inversion in the French Convenience Yield, i.e. public debt now trades at a yield premium relative to liquid benchmarks, i.e.:

$$CY \equiv r^a - r^b = \frac{1}{p^a_1} - \frac{1}{p^b_1} < 0.$$

The model provides a mechanism for this shift. In particular, Proposition 5 shows that increases in debt supply reduce the convenience yield, through the channels that shape the preference premium [1].

Proposition 5. *(Impact of Debt Supply on the Convenience Yield)*

As long as the conditions from Propositions 2 and 3 hold, the convenience yield is decreasing in both anticipated secondary issuance Δ^b_{new} and initial public debt holdings b_1:

$$\frac{\partial CY}{\partial \Delta^b_{\text{new}}} = -\frac{1}{(p^a_1)^2} \cdot \frac{\partial p^a_1}{\partial \Delta^b_{\text{new}}} + \frac{1}{(p^b_1)^2} \cdot \frac{\partial p^b_1}{\partial \Delta^b_{\text{new}}} < 0,$$

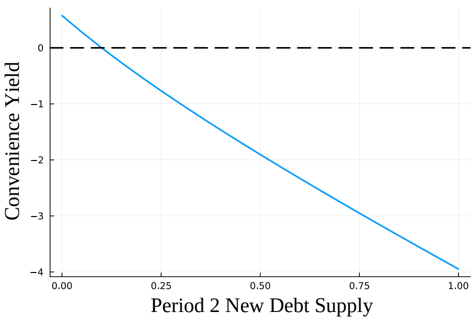
$$\frac{\partial CY}{\partial b_1} = -\frac{1}{(p^a_1)^2} \cdot \frac{\partial p^a_1}{\partial b_1} + \frac{1}{(p^b_1)^2} \cdot \frac{\partial p^b_1}{\partial b_1} < 0.$$

These results are confirmed in numerical simulations (see Figure 7).

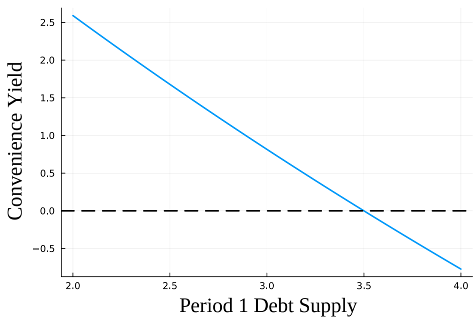
In equilibrium, increases in debt supply reduce the price of public debt p^b_1 due to anticipated resale pressure, while the price of the benchmark asset a either remains stable or increases. This asymmetry reflects the structure of institutional mandates, which impose fixed portfolio shares in period 2. Although both assets are subject to the same mandate rules, the supply elasticity of asset b introduces a secondary market wedge. In the absence of unconstrained arbitrageurs, this wedge cannot be eliminated through trade. As a result, public debt

[1] As $CY = \pi^b/(p^a_1 \cdot p^b_1)$ with p^a_1 and $p^b_1 > 0$.

valuations deteriorate as supply increases, leading to a compressed or negative convenience yield.



(a) Secondary Market Supply



(b) Primary Market Supply

Figure 7: Impact of Debt Supply on the Convenience Yield

Note: All parameter values used in simulations are reported in Appendix C.

These results are consistent with the empirical evidence presented in the introduction for France, where the convenience yield turned negative in mid-2024, and with the findings of Jiang, Richmond, and Zhang (2024), who attribute similar inversions in the United States to large-scale public debt issuance. In the model, increases in debt supply lead to weaker secondary market prices, a decline in the liquidity value of public debt, and a compression or an inversion of the yield spread relative to benchmark assets. This mechanism arises in the absence of credit risk and highlights how inelastic institutional demand, can endogenously generate pricing anomalies in sovereign bond markets.

3 Model with Liquidity Risk

This section extends the baseline model by introducing uncertainty over future public debt issuance. While Section 2 assumes that the secondary market supply Δ^b_{new} is known at $t = 1$, we now treat Δ^b_{new} as a random variable drawn from a known distribution. This modification introduces liquidity risk into the model, reflecting the fact that investors must value public debt under uncertainty about future market conditions.

The exposition proceeds in two parts. Section 3.1 outlines the minimal deviations from the deterministic setup. Section 3.2 presents the analytical implications and simulation results, with particular attention to the role of the variance and the mean of debt supply.

3.1 Environment

The structure of the economy remains unchanged relative to Section 2, except that period-2 debt issuance Δ^b_{new} is now a random variable. Let N denote its distribution on $\mathbb{R}$ with mean μ and variance σ^2.

Figure 8 summarizes the revised timeline of events.

Investor Optimization. Let $j \in \text{Supp}(N)$ be a realization of Δ^b_{new}. The representative investor's problem becomes:

$$V_1 = \max_{c_1, a_1, b_1} u(c_1) + \mathbb{E}_{j \sim N} \left[\theta \cdot V^{(j)}_{2,L}(a_1, \alpha b_1) + (1 - \theta) \cdot V^{(j)}_{2,H}(a_1, \alpha b_1) \right],$$

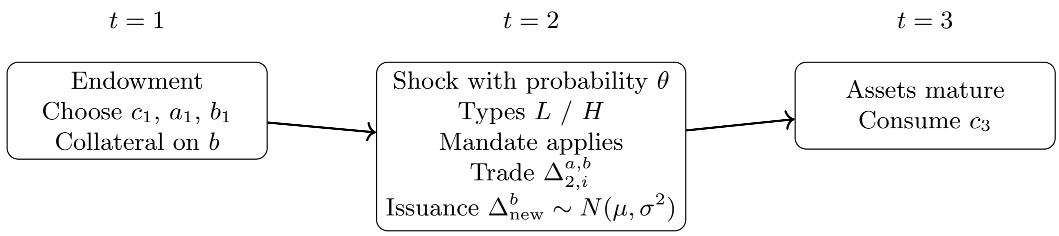


Figure 8: Timeline of Model Events with Probabilistic Debt Supply

subject to the standard budget and collateral constraints:

$$c_1 = \omega + \phi(1-\alpha)b_1 - p_1^a a_1 - p_1^b b_1.$$

3.2 Analytical Results and Simulations

The key departure from the deterministic model is that asset valuations must now be formed over state-contingent returns. Formally, all expressions from Section 2 generalize to expectations over j.

Primary Market Pricing. The price of asset b in period 1 becomes:

$$\begin{aligned} p_1^b = {} & \underbrace{\alpha\theta \cdot \mathrm{Cov}(m^L_{1,2,j}, p^b_{2,j})}_{\text{(i) Liquidity risk}} + \underbrace{\alpha\theta \cdot \mathbb{E}[m^L_{1,2,j}] \cdot \mathbb{E}[p^b_{2,j}]}_{\text{(ii) Expected fire-sale value}} \\ & + \underbrace{\alpha(1-\theta) \cdot \mathbb{E}[m^H_{1,3,j}]}_{\text{(iii) Expected maturity value}} + \underbrace{\phi(1-\alpha)}_{\text{(iv) Collateral benefit}} . \end{aligned}$$

The price of asset a, unaffected by issuance uncertainty, simplifies to:

$$p_1^a = \underbrace{\theta \cdot \mathbb{E}[m^L_{1,2,j}] \cdot p_2^a}_{\text{Expected fire-sale value}} + \underbrace{(1-\theta) \cdot \mathbb{E}[m^H_{1,3,j}]}_{\text{Expected maturity value}} .$$

Preference Premium. The preference premium becomes:

$$\begin{aligned} \pi^b_N &= p_1^b - p_1^a \\ &= \underbrace{\alpha\theta \cdot \mathrm{Cov}(m^L_{1,2,j}, p^b_{2,j})}_{\text{(i) Liquidity risk}} + \underbrace{\alpha\theta \cdot \mathbb{E}[m^L_{1,2,j}] \cdot \mathbb{E}[\delta^b_j]}_{\text{(ii) Expected liquidity spread}} \\ &\quad + \underbrace{\phi(1-\alpha)}_{\text{(iii) Collateral benefit}} - \underbrace{(1-\theta)(1-\alpha) \cdot \mathbb{E}[m^H_{1,3,j}]}_{\text{(iv) Maturity loss}}, \end{aligned}$$

where $\delta^b_j = \alpha \cdot p^b_{2,j} - p^a_2$ measures the scaled state-contingent liquidity wedge between the assets.

The new term in both expressions is the covariance between the pricing kernel $m^L_{1,2,j}$ and resale price $p^b_{2,j}$. This term captures the risk that public debt becomes less liquid precisely when it is most valuable to constrained agents. Since $m^L_{1,2,j}$ is increasing and $p^b_{2,j}$ is decreasing in the issuance shock j, the covariance is theoretically negative. This "liquidity risk" lowers the value of asset b ex ante, reinforcing the depreciation mechanisms documented in proposition 4 for the deterministic case.

Relation to the Deterministic Case. The deterministic model of Section 2 can be interpreted as a special case of this stochastic environment in which the variance of secondary issuance vanishes. Formally, letting $\sigma^2 \to 0$ in the distribution N collapses all expectations to point evaluations, and the covariance term disappears:

$$\mathbb{E}[f(\Delta_j^b)] \to f(\Delta_{\text{new}}^b), \quad \text{Cov}(m_{1,2,j}^L, p_{2,j}^b) \to 0.$$

In this limit, all pricing expressions and equilibrium conditions revert to those derived under certainty. Thus, the stochastic model presented here nests the earlier results as a knife-edge case where future liquidity conditions are perfectly known.

We use simulation-based comparative statics to examine how the first and second moments of future debt issuance affect sovereign bond valuations. Specifically, we analyze how variation in the mean (μ) and standard deviation (σ) of the issuance distribution $N \sim \mathcal{N}(\mu, \sigma^2)$ influences the preference premium π_N^b and the convenience yield. This allows us to isolate the role of expected issuance versus issuance uncertainty in shaping asset prices under liquidity risk.

We simulate the model over a grid of (μ, σ) values, holding all other parameters fixed. Results are reported below.

Proposition 6. *(Numerical Finding: Increasing Mean Issuance)*

As the mean issuance μ increases, the preference premium π_N^b declines monotonically. This reflects a deterioration in the expected liquidity spread, while other components remain stable:

(i) **Liquidity risk:** negligible and invariant to μ.

(ii) **Expected liquidity spread:** declines as μ rises.

(iii) **Collateral benefit:** fixed by construction.

(iv) **Maturity loss:** stable across μ.

Panel 9a confirms that the decline in π_N^b is driven entirely by term (ii), with all other components approximately constant. This mirrors the comparative statics in the deterministic model (Proposition 4) and reinforces the dominant role of the expected supply level in equilibrium valuations.

Proposition 7. *(Numerical Finding: Increasing Issuance Uncertainty)*

As the standard deviation σ increases via a mean-preserving spread, the preference premium π_N^b remains approximately constant. All components of the decomposition are stable:

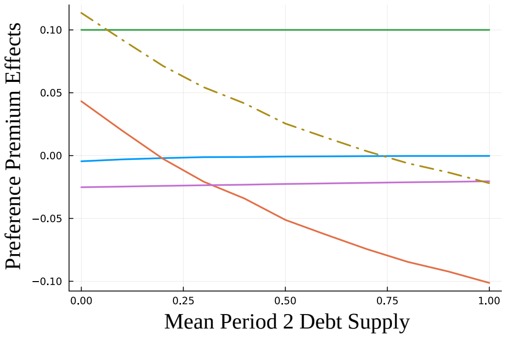
(i) **Liquidity risk:** quantitatively negligible and flat in σ.

(ii) **Expected liquidity spread:** unchanged.

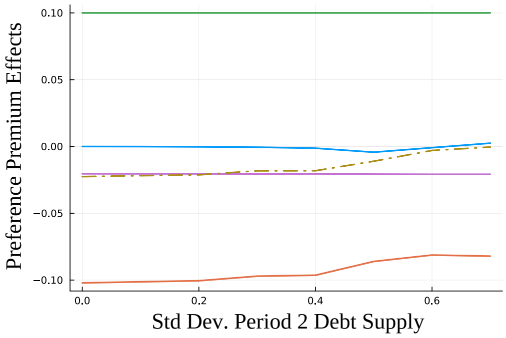
(iii) **Collateral benefit:** constant by definition.

(iv) **Maturity loss:** invariant to σ.

Panel 9b shows no meaningful sensitivity to issuance volatility. A slight uptick in the liquidity spread appears at high values of σ, but this is likely numerical noise rather than a robust equilibrium effect. In this setting, second-moment uncertainty does not materially affect prices.



(a) Effect of μ (mean issuance)



(b) Effect of σ (issuance volatility)

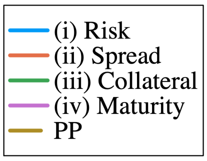
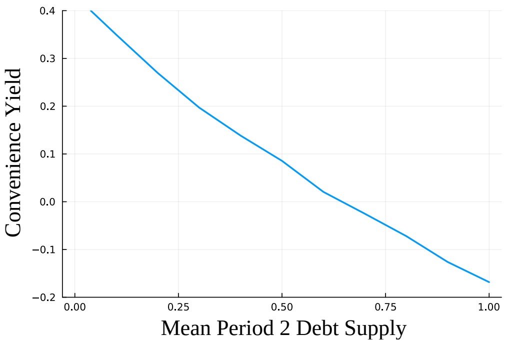


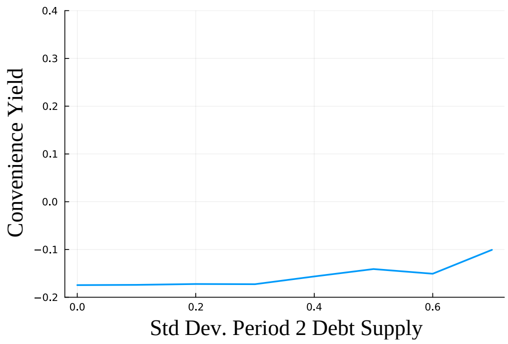
Figure 9: Decomposition of the Preference Premium π_N^b

Note: Simulations hold all other parameters constant. Panel (a) varies the mean μ of secondary issuance; Panel (b) varies the standard deviation σ while keeping the mean fixed.

Convenience Yield. The convenience yield exhibits identical behavior. Figure 10 plots the yield differential under varying μ and σ. Consistent with the preference premium decomposition, convenience yields respond strongly to the mean of issuance but are largely unresponsive to changes in volatility. Liquidity premia in this environment are primarily expectations-driven.



(a) Effect of μ



(b) Effect of σ

Figure 10: Evolution of the Convenience Yield π_N^b

Note: Panel (a) increases the mean of issuance; Panel (b) increases variance with fixed mean.

4 Conclusion

This paper develops a tractable dynamic model to explain how sovereign convenience yields can become negative—even in the absence of credit risk, search frictions, or monetary distortions. Using French data, I document an inversion in mid-2024 where government bonds traded at higher yields than synthetic credit-equivalent benchmarks. This empirical anomaly poses a direct challenge to the standard view that public debt earns a premium due to its safety and liquidity.

The model shows that when institutional investors face binding portfolio mandates, their demand for sovereign bonds becomes inelastic. In such an environment, increases in debt supply depress secondary market prices by overwhelming constrained absorption capacity. As a result, the expected resale value of sovereign bonds declines, even though the assets themselves remain fundamentally safe. This erosion in state-contingent liquidity mechanically compresses, and may even reverse, the convenience yield.

The framework also clarifies why such pricing distortions can persist. Under standard asset pricing logic, a negative convenience yield implies an arbitrage opportunity: long the government bond, short the synthetic hedge. But when markets are segmented and mandates bind, no marginal investor is able or willing to exploit the wedge. In this sense, the model complements the Inelastic Market Hypothesis of Gabaix and Koijen (2023), which emphasizes that flows can move prices sharply in the presence of rigid demand and a scarcity of arbitrage capital. Sovereign bond markets, as documented by Nenova (2025), embody precisely these features: demand is driven by regulation and risk budgets, not marginal pricing.

More broadly, the findings suggest that the liquidity premium on sovereign bonds is not a structural constant, but a fragile equilibrium outcome. When supply surges and absorption is constrained, public debt can lose its pricing advantage, not because it becomes risky, but because it becomes hard to resell at favorable prices. This insight has implications for fiscal policy, regulatory design, and the interpretation of yield spreads in segmented markets.

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Appendix

A Data and Computation of the Convenience Yield

This appendix details the data sources, construction methodology, and computation of the convenience yield used in Section 1. The goal is to isolate the non-credit pricing component of French government bonds by comparing observed yields with synthetic, credit-risk-equivalent counterparts.

Data Sources The analysis uses proprietary Bloomberg data provided by the Banque de France, covering:

- The 5-year yield on French government bonds (OATs),
- The 5-year Overnight Indexed Swap (OIS) rate on the euro,
- The 5-year Credit Default Swap (CDS) spread on French sovereign debt.

All data are reported at a daily frequency and span from January 2008 to May 2025. Yields and spreads are quoted in annualized percentage points.

Methodology and Yield Decomposition Following Jiang, Lustig, et al. (2020), the yield on a sovereign bond is decomposed as:

$$\mathrm{CY} = \mathrm{Yield}_{\mathrm{Theoretical}} - \mathrm{Yield}_{\mathrm{OAT}} = \mathrm{OIS} + \mathrm{CDS} - \mathrm{Yield}_{\mathrm{OAT}}$$

where:

- CY is the *convenience yield*, interpreted as the yield discount investors are willing to accept for holding public debt due to its safety and liquidity attributes.
- OIS is the risk-free rate,
- CDS is the credit risk compensation.

A positive CY indicates that the government bond trades at a premium to the synthetic instrument, consistent with its money-like properties. A negative CY implies a yield inversion, i.e., public debt trades at a discount relative to the risk-adjusted synthetic, suggesting a loss of its liquidity premium.

Empirical Pattern and Additional Evidence Figure 11 presents the full decomposition of the observed OAT yield and its synthetic benchmark since 2008. It extends Figure 2 in the main text and highlights the long-run evolution of yield components.
As shown, the convenience yield has historically remained positive, reflecting France's status as a core Eurozone issuer. However, since mid-2024, the CY has turned negative despite stable CDS spreads, suggesting an erosion in the bond's liquidity premium.

These findings underscore the necessity of theoretical models that allow for endogenous illiquidity in frictionless execution environments, as developed in Sections 2–3.

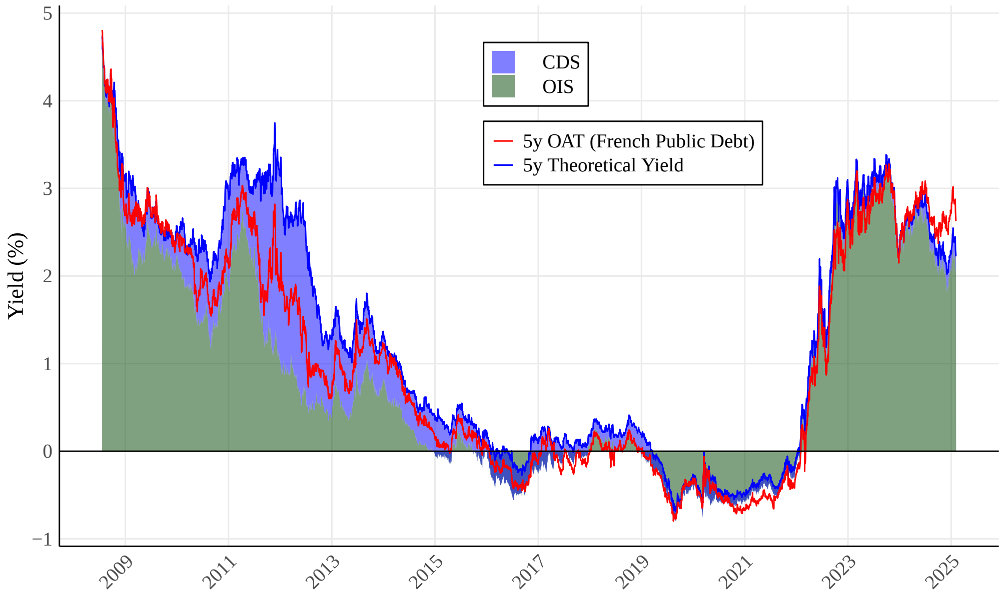

Figure 11: Decomposition of French 5-Year Government Bond Yield: 2008–2025
Note: Theoretical yields are constructed from OIS and CDS data.

B Analytical Proofs

For brevity, I provide a formal proof only for Proposition 2 and 3, as it involves less direct comparative statics. The other propositions are either direct derivations from the model's equilibrium conditions or follow from algebraic manipulation and are verified through numerical simulations. Full analytical proofs are available upon request.

B.1 Proof of Proposition 2

We examine how the secondary market price of public debt, p_2^b, responds to changes in new issuance Δ_{new}^b, holding all else constant. The relevant equilibrium condition is:

$$p_2^b = \alpha \left(\theta m_{1,2}^L p_2^b + \theta m_{1,2}^L \delta p_2^b + (1-\theta) m_{1,3}^H\right) + \phi(1-\alpha),$$

where ϕ captures the collateral value, and $m_{1,2}^L, m_{1,3}^H$ are the stochastic discount factors of the low-type and high-type agents, respectively. Define the equilibrium function:

$$F(p_2^b, \Delta_{\text{new}}^b) = p_2^b - \alpha \left(\theta m_{1,2}^L p_2^b + \theta m_{1,2}^L \delta p_2^b + (1-\theta) m_{1,3}^H\right) - \phi(1-\alpha).$$

Applying the Implicit Function Theorem:

$$\frac{\partial p_2^b}{\partial \Delta_{\text{new}}^b} = -\frac{\partial F / \partial \Delta_{\text{new}}^b}{\partial F / \partial p_2^b}.$$

The denominator is:

$$\frac{\partial F}{\partial p_2^b} = 1,$$

and the numerator is:

$$-\frac{\partial F}{\partial \Delta^b_{\text{new}}} = \alpha\left[\theta \frac{\partial m^L_{1,2}}{\partial \Delta^b_{\text{new}}} p^b_2 + \theta m^L_{1,2} \frac{\partial p^b_2}{\partial \Delta^b_{\text{new}}} + (1-\theta)\frac{\partial m^H_{1,3}}{\partial \Delta^b_{\text{new}}}\right].$$

Hence, we obtain:

$$\frac{\partial p^b_2}{\partial \Delta^b_{\text{new}}} = \alpha\left[\theta \frac{\partial m^L_{1,2}}{\partial \Delta^b_{\text{new}}} p^b_2 + \theta m^L_{1,2} \frac{\partial p^b_2}{\partial \Delta^b_{\text{new}}} + (1-\theta)\frac{\partial m^H_{1,3}}{\partial \Delta^b_{\text{new}}}\right]$$

We decompose the effect of Δ^b_{new} on the stochastic discount factors. For the low type:

$$\frac{\partial m^L_{1,2}}{\partial \Delta^b_{\text{new}}} = \frac{\partial m^L_{1,2}}{\partial c_{2,L}} \cdot \frac{\partial c_{2,L}}{\partial p^b_2} \cdot \frac{\partial p^b_2}{\partial \Delta^b_{\text{new}}},$$

where:

$$\frac{\partial m^L_{1,2}}{\partial c_{2,L}} < 0, \quad \frac{\partial c_{2,L}}{\partial p^b_2} < 0.$$

Thus,

$$\frac{\partial m^L_{1,2}}{\partial \Delta^b_{\text{new}}} > 0.$$

Similarly, for the high type:

$$\frac{\partial m^H_{1,3}}{\partial \Delta^b_{\text{new}}} = \frac{\partial m^H_{1,3}}{\partial c_{3,H}} \cdot \frac{\partial c_{3,H}}{\partial \Delta^b_{\text{new}}},$$

with:

$$\frac{\partial m^H_{1,3}}{\partial c_{3,H}} < 0, \quad \frac{\partial c_{3,H}}{\partial \Delta^b_{\text{new}}} > 0.$$

Hence,

$$\frac{\partial m^H_{1,3}}{\partial \Delta^b_{\text{new}}} < 0.$$

We conclude that:

$$\frac{\partial p^b_2}{\partial \Delta^b_{\text{new}}} < 0 \quad \text{if and only if} \quad \left|\theta \frac{\partial m^L_{1,2}}{\partial \Delta^b_{\text{new}}} p^b_2\right| + \left|(1-\theta)\frac{\partial m^H_{1,3}}{\partial \Delta^b_{\text{new}}}\right| > \left|\theta m^L_{1,2} \frac{\partial p^b_2}{\partial \Delta^b_{\text{new}}}\right|.$$

Therefore, if the following condition holds, the previously stated condition also holds:

$$\left|\theta \frac{\partial m^L_{1,2}}{\partial \Delta^b_{\text{new}}} p^b_2\right| > \left|\theta m^L_{1,2} \frac{\partial p^b_2}{\partial \Delta^b_{\text{new}}}\right|.$$

Assuming log utility, we have:

$$\frac{1}{c_{2,L}} > \frac{\beta \alpha b_1 p^b_2}{c^2_{2,L}},$$

which simplifies to:

$$\omega_2 + \omega_{2,L} + p^a_2 a_1 + \alpha b_1 p^b_2 (1-\beta) > \theta.$$

This inequality always holds under standard parameterizations, confirming that:

$$\frac{\partial p^b_2}{\partial \Delta^b_{\text{new}}} < 0. \quad \text{Q.E.D.}$$

B.2 Proof of Proposition 3

We examine how the initial public debt position b_1 affects the primary market price p_1^b. Define the equilibrium function:

$$F(p_1^b, b_1) = p_1^b - \alpha \left[\theta m_{1,2}^L(b_1, p_2^b) p_2^b(b_1) + (1-\theta) m_{1,3}^H(b_1)\right] - \phi(1-\alpha),$$

where p_2^b, $m_{1,2}^L$, and $m_{1,3}^H$ are all endogenous functions of b_1. The primary price satisfies $F(p_1^b, b_1) = 0$ in equilibrium. By the IFT, we have:

$$\frac{\partial p_1^b}{\partial b_1} = -\frac{\partial F/\partial b_1}{\partial F/\partial p_1^b}.$$

We compute:

$$\frac{\partial F}{\partial p_1^b} = 1 - \alpha \left[\theta \frac{\partial m_{1,2}^L}{\partial p_1^b} p_2^b + (1-\theta)\frac{\partial m_{1,3}^H}{\partial p_1^b}\right].$$

Given our parametrization, where marginal utilities are small and most parameters lie below one—it follows that:

$$\frac{\partial F}{\partial p_1^b} > 0.$$

Therefore, the sign of $\frac{\partial p_1^b}{\partial b_1}$ is determined solely by the numerator.

We compute:

$$-\frac{\partial F}{\partial b_1} = \alpha \left[\theta \left(\frac{\partial m_{1,2}^L}{\partial b_1} p_2^b + m_{1,2}^L \frac{\partial p_2^b}{\partial b_1}\right) + (1-\theta)\frac{\partial m_{1,3}^H}{\partial b_1}\right].$$

This decomposition aligns with the proposition:

(i) $\theta m_{1,2}^L \cdot \frac{\partial p_2^b}{\partial b_1}$ — liquidity deterioration channel,

(ii) $\theta \cdot \frac{\partial m_{1,2}^L}{\partial b_1} \cdot p_2^b$ — fire-sale pricing kernel channel,

(iii) $(1-\theta) \cdot \frac{\partial m_{1,3}^H}{\partial b_1}$ — maturity pricing kernel channel.

The first term (i) is unambiguously negative, as shown in Proposition 2. The second and third terms involve higher-order derivatives of utility:

$$\frac{\partial m_{1,2}^L}{\partial b_1} = \frac{\partial m_{1,2}^L}{\partial c_{2,L}} \cdot \frac{\partial c_{2,L}}{\partial p_2^b} \cdot \frac{\partial p_2^b}{\partial b_1},$$
$$\frac{\partial m_{1,3}^H}{\partial b_1} = \frac{\partial m_{1,3}^H}{\partial c_{3,H}} \cdot \frac{\partial c_{3,H}}{\partial b_1}.$$

Under CRRA or log utility, signs of these terms depend on how changes in b_1 redistribute consumption over time and across types. In simulations, these effects are often dominated by the first term, but in general the total expression is ambiguous.

The sign of $\frac{\partial p_1^b}{\partial b_1}$ is ambiguous. It is governed entirely by the numerator of the IFT expression. The denominator is strictly positive under standard assumptions, so comparative statics hinge on the interaction of supply, marginal utilities, and consumption responses.

C Calibration and Simulation Details

The simulations presented in this paper are not intended to match empirical moments or perform quantitative policy analysis. Their role is purely illustrative: to isolate the underlying mechanisms of the model, assess the relative magnitude of the valuation components, and characterize the conditions under which the convenience yield (CY) may turn negative. The parameterization is therefore deliberately minimalistic, designed for tractability and internal consistency.

C.1 Deterministic Simulations

Preferences and Market Environment. Agents exhibit time-separable preferences with logarithmic utility, yielding marginal utility $u'(c) = 1/c$. The discount factor is set to $\beta = 0.96$, and the probability of an income shock is $\theta = 0.5$. The economy spans three periods with deterministic endowments: $\omega_1 = 6$, $\omega_2 = 2$, and $\omega_3 = 3$, with an idiosyncratic adjustment of $\omega_{2L} = 0$ and $\omega_{2H} = 1$.

Institutional Parameters. Portfolio mandates require that a fraction $\xi = 0.3$ of investor wealth be allocated to the benchmark liquid asset. A haircut of $\phi = 1$ is applied to collateralized public debt, and the share of debt retained for trading is set at $\alpha = 0.6$, ensuring both market-based and collateral channels operate in equilibrium.

Solution Strategy. The model is solved numerically using `NLsolve.jl` in `Julia`. For fixed asset holdings, the equilibrium prices (p_1^a, p_1^b) and their secondary market counterparts (p_2^a, p_2^b) are determined by a two-equation system derived from first-order conditions and market clearing. Comparative statics are conducted by varying either the initial debt issuance b_1 or new issuance Δ^{new} in period 2.

For each simulation, I compute the convenience yield, along with the liquidity spread and the preference premium. Simulations are retained only if convergence is achieved and prices remain strictly positive.

Interpretation. This procedure identifies regions of the parameter space where supply shocks, combined with institutional rigidity, reduce the expected resale value of sovereign bonds. In these settings, liquidity deteriorates endogenously. The model admits configurations where the convenience yield is negative, even in the absence of default risk or search frictions.

C.2 Stochastic Supply Simulations

I next extend the model to incorporate uncertainty over future debt issuance. These simulations examine how variation in the first and second moments of public debt supply affects equilibrium valuations, preference premia, and the sign of the convenience yield.

Stochastic Structure. Let $\Delta_j^b \sim \mathcal{N}(\mu, \sigma^2)$ represent the realization of new issuance in period 2. For each pair (μ, σ), I draw $N = 1000$ samples from this distribution. The case $\sigma = 0$ nests the deterministic benchmark.

Equilibrium Computation. For each draw Δ_j, I compute equilibrium prices and marginal utilities, including the stochastic discount factors $m^L_{1,2,j}$ and $m^H_{1,3,j}$, bond prices $p^b_{1,j}$, secondary market prices $p^b_{2,j}, p^a_{2,j}$ and the realized convenience yield / preference premium.

Implementation. Simulations are implemented in `Julia`, using `NLsolve.jl` for fixed-point solution, `Distributions.jl` for sampling, and `Statistics.jl` for moments. Plots are rendered via `Plots.jl` with the `GR` backend.