Global Migration and the Skill Premium

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Abstract

This paper develops a simple extension of the general equilibrium models common in the recent international trade and new economic geography literature. By considering a system where two types of workers contribute to the production function for each region, the papers suggests that previous estimations of the skill premium have over-estimated both the effect of the local skill availability and the productivity ratio. Furthermore it suggests that the different degree to which each region has access to the two types of workers affect the skill premium. Lastly, it suggests a econometric procedure to estimate the correct skill premium specification, based on previous estimation of the market access in the trade literature.

1 Introduction

The field of economics has always tended to simplify the modelling of the agents in favour of simplicity, tractability and elegance. This has been particularly true in general equilibrium models which have simplified the complexities of the labor market through a catch-all labor factor and its cost. These models predicted a world characterised by inter-regional differences and, at the same time, intra-regional uniformity. Specifically, each region was a unit with the same wage for all its inhabitants. On the contrary, each location in the world displays a disparity in income within the same labor market. This paper aims at providing an initial treatment of a general equilibrium model with two types of workers and its consequences. The first part will develop a general equilibrium model and will try to describe its mathematical properties, while the second part will build on previous results from the international trade literature to provide an estimation procedure of the skill premium. The main result of the paper will highlight how the wages of each type will depend, among other factors, on the number of agents of the other type and their retribution and the accessibility of a region to the global pool of workers of the same type. Additionally, the skill premium of each country depends also on the difference in accessibility between agents.

2 Literature review

This paper builds on and tries to contribute to three different strands of literature. The first one is related to the construction of general equilibrium models that try to explain the spatial distribution of the economy, and more widely, to the trade economics literature. The second one is the literature concerning migration and, in particular, the difference in migration between high-skill and low-skill agents. Lastly, this paper will try to expand our understanding of the determinants of the local skill premium, in particular in the context of a series of interdependent regions.

Following the work by Hanson (1997, 1998), the trade literature has developed theoretical models that allow them to consider economies composed by asymmetric regions which trade between themselves. In recent years, mathematical and economics advancements have led us to increasingly richer models of the spatial distribution of the economy, in particular following the contribution of Eaton and Kortum (2002) that managed to incorporate the gravity framework for trade between regions, thus simplifying future empirical analysis. Most recent models incorporate a set of agents that can relocate across a series of potentially asymmetrical regions which trade between them (Redding and Rossi-Hansberg, 2017). The agents display the same utility function and the same elasticities of substitution across different goods. They do differ, though, in the utility draw for each location. Models that introduce bilateral movement frictions, such as Anderson (2011) which extends the gravity framework explicitly to model migration, display different utilities level between agents based on the migration 'access' that each region of origin displays. The richest example of this is the model by Monte et al. (2018) which let agents choose both where they live and where they work according to a bilateral commuting utility friction. Collectively, though, all these models introduce location based utility costs and labor productivity, while keeping agents equal as long as they reside in the same region. Cucu (2020) tries to build a model which introduces different types of agents with different productivities but fails to provide any analytical result from that, even though his numerical simulations predict the existence and the uniqueness of the equilibrium. Other papers, such as Behrens et al. (2014, 2017) and Candau and Dienesch (2015) try to incorporate different types of agents in general equilibrium model, but do not follow the gravity approach. Allen and Arkolakis (2014) produce a rigorous mathematical approach to solve Armington models which is later expanded in Allen et al. (2016). They provide both the theorems to prove existence and uniqueness and examples of their application. I will try to build on Cucu (2020) to further the mathematical understanding of such models in a slightly simplified gravity framework. In particular this paper will try to formalize a mathematical proof of the existence of different equilibria and will further demonstrate that these models do not have a unique equilibrium.

The gravity framework greatly simplifies the empirical treatment of trade as indicated by Anderson (2011). Eaton and Kortum (2002) follows its theoretical model with a consistent gravity estimation. Redding and Venables (2004) manages to link the Eaton and Kortum model with a procedure to estimate the market potential. Before these papers the empirical prowess of the gravity framework was known and adopted, such as Tinbergen (1962) and Geraci and Prewo (1977), but the proposed theoretical foundations, such as Bergstrand (1985), were either incomplete or unsatisfactory. Anderson (2011) further discusses how the gravity framework could be extended to other areas which are characterized by factor flows. The paper explicitly proposes a model for migration, while also reviewing analogous attempts with respect to foreign direct investments and portfolio investments. With respect to the first one of the three, Beine et al. (2016) provides a review of empirical procedures for applying gravity estimation to migration flows. This paper will try to extend a set of empirical tools to deal with gravity and wage equations. In particular, those adopted by Redding and Venables (2004).

With respect to migration, economists have long worked with concepts introduced by Ravenstein (1885, 1889), such as *push and pull factors*. In these frameworks agents are subjected to economic forces of attraction and repulsion from each possible migration option (including the origin region) and can then maximise their expected utility when deciding the destination region. While the basic idea of maximising utility of the potential migrant, the pull-push framework has been substituted by a human capital investment one (Gaston and Nelson, 2013). An important step forward has been the adaptation of Borjas (1987) of the Roy (1951) model. In this model each agent compares its productivity with the level of inequalities of a potential host country. The conclusion is that productive agents self select into more unequal countries and vice-versa. Abramitzky et al. (2012) have confirmed Borjas' intuition through an analysis of Norwegian migrants in the US in the Nineteenth Century. On the other hand, migration economists have underlined the complexity of the decision of an agent to migrate, even when isolating the measurable economic factors from everything else. As an example, Stark and Taylor (1989, 1991) highlight how the choice of migrating is taken when considering one's own income relative to the peers in the origin region. Other non-economic factors that determine self-selection in migration has been a focus of Chen and Rosenthal (2008) and Beine et al. (2008), which differentiate between skill level of the migrants. Furthermore, Kerr et al. (2016) discusses the implications of the increasing high-skill migration flows. Another strand of literature argues for a household, rather than an individual, approach of migration. In this framework the migrants are seen as an household human capital investment with the aim of risk-mitigation (Stark and Bloom, 1985; Rosenzweig and Stark, 1989) through future remittances (Durand et al., 1996; Yang, 2008, 2011). Radu and Straubhaar (2012) highlights the deficiencies and progress of economists to tackle non-economic determinants of migration. While acknowledging the micro-foundation of the migration choice, this paper will follow the general trade view of migration as a trade factor flow. The introduction in the theoretical framework of a *bilateral migration friction* as in Anderson (2011) and in empirical estimation of the origin and destination fixed effects will contribute to the alleviation of these hurdles.

Kerr et al. (2017) laments the absence of good quality data with respect to global migration flows. Most of the analysis regarding skill-specific migration, in particular with respect to the 'brain-drain', follow Carrington and Detragiache (1998, 1999) in trying to capture a partial snapshot of migration stocks by collecting the immigration data from censuses and surveys of a selected range of destination countries. The most used collections of migration data follow this approach (Docquier and Marfouk, 2006; Docquier et al., 2009; Brücker et al., 2013) in providing a set of bilateral stocks of immigrants data with respect of a selected number of destination countries. This paper will contribute to the literature by highlighting the consequences of a skill-based discrepancy in migration determinants on the destination countries while adapting econometric analysis of trade gravity to migration gravity.

Previous labor literature (Katz and Murphy, 1992) reports and measure an existing wage ratio between agents with different level of education. Previous models do not introduce the possibility of different type of workers in each location and how the local productivity ratio with respect to a single location affects the distribution of the remaining regions.

The richness of the model would be useful to further our interpretation of the local wage ratio. Previous research on the ties between the labor markets and globalization focuses mostly on the effect of migration on a specific skill level. Borjas et al. (1996) runs a simple area comparison analysis, but point out that the outcomes depend critically on the econometric approach adopted. More recently, the review of Kerr et al. (2017) shows that there is not a consensus of the effect of high-skill migration on the corresponding wage, while McLaren (2017) focuses more on the effect of globalization on the different sectors of the economy. Black et al. (2014) consider the changes in wage inequalities in the U.S. after a local productivity shock through the lenses of the Roback model by focusing on the different housing prices and nominal wages, but fail to capture fully the dynamics of different capabilities of migration across skill-levels. Lewis (2013), instead, ties the effect of migration on the wage ratio through the labor complementarity to capital, but does not include a general equilibrium approach.

Previous studies on the sources of the wage inequalities, such as Autor et al. (2008) fail to consider the role of migration in a general equilibrium framework. If different skill groups have different abilities to migrate responding to a change in relative productivity or relative wages, then, it is easy to infer that this difference will have an effect on the final wage ratio. This paper, therefore, aims to fill the gap of the literature by considering in the same framework migrants of different skill level and their global effect on the skill premium. It will also provide a series of definitions taken from the international trade literature which can improve our understanding of the issue.

The paper is divided in two main sections. In the first one (section 3) I will lay out the theoretical model and discuss its properties. In particular, subsections 3.1 to 3.5 describe the individual components of the model and the equations that characterize the general equilibrium, subsection 3.6 provides a demonstration of the existence of an equilibrium and the lack of uniqueness. In the second section (section 4) I build and test and empirical framework. In particular, subsections 4.1 and 4.2 compute the wage equations of the model, define the *skill-specific migration access* and provide two special cases, subsection 4.3 describes the estimation procedure, subsections 4.4 to 4.7 run and comment the empirical analysis and section 4.8 provides an alternative model and its empirical consequences.

3 The theoretical framework

In the world there are two types of agents defined by their skill level S. They can be either skilled, $S = s$, or unskilled, $S = u$. They are generated in a region exogenously and can decide where to move. The world is composed by a set N of regions. We will index them by i when referring to regions of destination, by n when referring to regions of origin, by k as the origin of a good, and t in any other case. Regions can be asymmetrical with regards to frictions, amenities, productivities and other exogenous factors. The main difference with Cucu (2020) is the absence of a housing market. According to Redding and Rossi-Hansberg (2018), the housing market acts as a disagglomeration force, but the existence of a Fréchet distributed local amenities and the movement frictions guarantee that there does not exist a single equilibrium where all the agents move to a single location.

The model this paper proposes enjoys existence of an equilibrium, but it demonstrate also that it lacks uniqueness. This becomes a clear issue when confronted with the possibility of a simulation. Cucu (2020), with a more complex model, shows numerically the existence of a unique equilibrium. This paper does not delve in the question of which specific feature of Cucu's model leads to the uniqueness of the equilibrium, but it provides a starting point for this inquiry.

3.1 The agent preferences

Each agent ω draws its preferences from the following utility function

$$U_{S,n,i}(\omega) = \frac{b_i(\omega)}{\kappa_{S,n,i}(\omega)} C_i(\omega) \tag{1}$$

Where $b_i(\omega)$ indicates the agent draw for amenities in location i. $\kappa_{n,i}(\omega)$ indicates, instead, the frictions with respect of moving from region n to region i both in economic and in emotional terms. We let this value be different for skilled and unskilled agents. $C_{S,i}(\omega)$ instead indicate the consumption of a continuum of goods $j \in [0, 1]$ and has the following CES formulation

$$C_i(\omega) = \left[\int_0^1 c_i(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}} \tag{2}$$

The corresponding price index is

$$P_i = \left[\int_0^1 p_i(j)^{1-\rho} dj \right]^{\frac{1}{1-\rho}} \tag{3}$$

The shock is drawn from the following Fréchet distribution

$$G_i(b) = e^{-B_n b^{-\epsilon}} \tag{4}$$

3.2 Production

Production happens with perfect competition according to a linear production function. The input of the production function l_k, is a CES combination of high and low skilled labor, defined as following

$$l_k(j) = \left[\eta_k l_{k,s}(j)^{\frac{\rho-1}{\rho}} + (1-\eta_k) l_{k,u}(j)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \tag{5}$$

Where $\eta_i \in (0,1)$ takes into account the relative productivity of the two types of workers and it is location specific. From now on I will define l_k as labor units. Given this function, each producer optimizes the input such that they can minimize the cost.

$$\begin{aligned} &\text{Min}_{l_{i,s}(j), l_{i,u}(j)} \quad w_{i,s} l_{i,s}(j) + w_{i,u} l_{i,u}(j) \\ &\text{s.t.} \quad l_k(j) = \left[\eta_k l_{k,s}(j)^{\frac{\rho-1}{\rho}} + (1-\eta_k) l_{k,u}(j)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \end{aligned} \tag{6}$$

The solution is the following

$$\frac{l_{k,u}(j)}{l_{k,s}(j)} = \Big(\frac{1-\eta_k}{\eta_k} \frac{w_{k,s}}{w_{k,u}} \Big)^{\rho} \tag{7}$$

The corresponding optimal price index of each labor unit is

$$w_k = \left[\eta_k^{\rho} w_{k,s}^{1-\rho} + (1-\eta_k)^{\rho} w_{k,u}^{1-\rho} \right]^{\frac{1}{1-\rho}} \tag{8}$$

Productivity z for each location i and product j is drawn from the following Fréchet distribution

$$F_k(z) = e^{-A_k z^{-\theta}} \tag{9}$$

Goods are homogeneous between them. The price of buying a good j produced in location k from location i is, therefore

$$p_{k,i}(j) = \frac{d_{k,i} w_k}{z_k(j)} \tag{10}$$

Where $d_{k,i}$ indicates the trade friction of buying a good produced in location k from location i

3.3 Expenditure shares and price indices

As in the Eaton and Kortum (2002) model the representative agent of location i, sources each good from the lowest-cost supplier location. Given the properties of the Fréchet distribution, one can write the share of expenditure of location i on goods produced in location k as

$$\pi_{k,i} = \frac{A_k(d_{i,k}w_k)^{-\theta}}{\sum_{t\in N} A_t(d_{i,t}w_t)^{-\theta}} \tag{11}$$

As a consequence one can rewrite the price index of region i as

$$P_i = \gamma\Big[\sum_{k\in N} A_k(d_{k,i}w_k)^{-\theta}\Big]^{-\frac{1}{\theta}} \tag{12}$$

Where

$$\gamma = \left[\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)\right]^{\frac{1}{1-\sigma}}$$

And $\Gamma(\cdot)$ is the Gamma function. We ensure $\theta > \sigma - 1$ in order to have a finite value of the price index. Using the share of expenditure and the fact that $d_{i,i} = 1$, we can rewrite the price index as

$$P_i^{-\theta} = \frac{\gamma^{-\theta}A_i w_i^{-\theta}}{\pi_{i,i}} \tag{13}$$

3.4 Residential choices and income

I follow the mechanism of Anderson (2011) to build a gravity migration framework. The indirect utility function of an agent can be written as

$$U_{S,n,i}(\omega) = \frac{b_n(\omega)}{\kappa_{n,i}}\frac{w_{S,i}}{P_i} \tag{14}$$

As it is a monotonic transformation of the amenity shock draw, this function inherits the Fréchet distribution. The parameters are

$$G_{S,i,n} = e^{-\psi_{S,i,n}U^{-\epsilon}}, \qquad \psi_{S,i,n} = B_i\left(\frac{w_{S,i}}{\kappa_{S,i,n}P_i}\right)^{\epsilon} \tag{15}$$

We can then write the probability of an agent of type S with its origin in n to move to region i

$$\lambda_{S,i,n} = \frac{l_{S,i,n}}{l_{S,n,0}} = \frac{B_i\left(\frac{w_{S,i}}{\kappa_{S,i,n}P_i}\right)^{\epsilon}}{\sum_{t\in N} B_t\left(\frac{w_{S,t}}{\kappa_{S,t,n}P_t}\right)^{\epsilon}} \tag{16}$$

Expected utility for an agent of type S with an origin in n is then

$$\bar{U}_{S,n} = \delta\Bigg[\sum_{i\in N} B_i\left(\frac{w_{S,i}}{\kappa_{S,i,n}P_i}\right)^{\epsilon}\Bigg]^{\frac{1}{\epsilon}} \tag{17}$$

Where

$$\delta = \Gamma\left[\frac{\epsilon-1}{\epsilon}\right]$$

Analogously to the expenditure share, we require $\epsilon > 1$ to ensure a finite value for the Gamma function.

I finally provide the last equations that describe the model. Income generated in each location is equal to the income spent on goods across all the different locations

$$w_i l_i = \sum_{k\in N} \pi_{k,i} w_k l_k \tag{18}$$

And the population of each location is equal to the sum of the incoming migration

$$l_{S,i} = \sum_{n\in N} \lambda_{S,i,n} l_{S,n,0} = \sum_{n\in N} l_{S,n,i} \tag{19}$$

3.5 General equilibrium

Combining the different equations above one can write the system that characterises the general equilibrium of the model. In particular the total expenditure for region i (18), the share of expenditures (11), the probability of moving from region n to region i (16), the optimal ratio between types of workers (7) and the price index (13).

The final equilibrium can be characterised by

- The migration flows per type, $l_{S,i,n}$
- The population of each region per type $l_{S,i}$
- The production levels l_i
- The income per type, $w_{S,i}$
- The unit production cost per region w_i
- The trade shares, $\pi_{i,k}$
- The price index per region P_i

Such that the following equations hold for each region

The labor income

$$w_i l_i = \sum_{k \in N} \pi_{k,i} w_k l_k \tag{20}$$

The expenditure shares

$$\pi_{k,i} = \frac{A_k (d_{i,k} w_k)^{-\theta}}{\sum_{t \in N} A_t (d_{i,t} w_t)^{-\theta}} \tag{21}$$

The migration flows

$$\frac{l_{S,i,n}}{l_{S,n,0}} = \frac{B_i \left(\frac{w_{S,i}}{w_i \kappa_{S,i,n}} \right)^{\epsilon} \left(\frac{A_i}{\pi_{i,i}} \right)^{\frac{\epsilon}{\theta}}}{\sum_{t \in N} B_t \left(\frac{w_{S,t}}{w_t \kappa_{S,t,n}} \right)^{\epsilon} \left(\frac{A_t}{\pi_{t,t}} \right)^{\frac{\epsilon}{\theta}}} \tag{22}$$

And the optimal ratio between production inputs

$$\frac{l_{i,u}}{l_{i,s}} = \Big(\frac{1 - \eta_i}{\eta_i} \frac{w_{i,s}}{w_{i,u}} \Big)^{\rho} \tag{23}$$

3.6 Existence and uniqueness

In this section we will provide proof of existence and uniqueness of the partial equilibrium of $\{l_{S,i}, w_{S,i}, w_i, \pi_{i,k}, P_i, l_{s,i,n}\}$ for all S, i, n, k and $l_{u,i,n}$ for all i and for all $n \in N - \{n^*\}$. The only variable that is fixed exogenously is $l_{u,i,n}$ for a specific reference region n^*. With respect to l_i we guarantee existence, but it is not defined exogenously. Secondly, we will show the existence of multiple general equilibria.

Proposition 1 (Existence and uniqueness of the partial equilibrium). *Given the spatial model defined by the system of equations* (20) − (21) − (22) − (23) *and the set of endogenous variables that describe it as listed in subsection 3.5, there exist a unique partial equilibrium if the set of variables l_{S,i,n^*} is defined exogenously for one element of S and for a single origin region $n^* \in N$.*

Proof. For the first part we will rely on the theorem drawn from Allen et al. (2016). The theorem is fully stated in the appendix. We can combine equations (12) − (13) − (16) − (17) to get

$$\bar{W}_{S,n}^{-\theta} = \frac{B_i^{-\frac{\theta}{\epsilon}} \left(\frac{w_{S,i}}{\kappa_{S,n,i}} \right)^{-\theta} (l_{S,i,n})^{\frac{\theta}{\epsilon}}}{\gamma^{-\theta} \sum_{k \in N} A_k (d_{k,i} w_k)^{-\theta}} \tag{24}$$

Where

$$\bar{W}_{S,n} = \left[\left(\frac{\bar{U}_{S,n}}{\delta} \right) l_{S,n,0}^{-\frac{1}{\epsilon}} \right] \tag{25}$$

If we combine (24) for $S = u$ and for a specific region $n^* \in N$ and (18), then we have a structure similar to the one of Allen et al. (2016). In order to apply their theorem we need to select two endogenous variables and fix all the other ones. Given the system I select $\{w_i, w_{u,i}\}$ to be our endogenous variables, while I fix $\{\pi_{k,i}, l_{u,n^*,i}\}$ for all $k,\ i \in N$.

The system can be rewritten, then, as

$$w_i = \sum_{k \in N} \xi^{\pi}_{k,i} w_k \tag{26}$$

$$w_{u,i}^{-\theta} = \lambda_{u,n^*} \sum_{k \in N} \xi^{W}_{k,i} w_k^{-\theta} \tag{27}$$

Where

$$\xi^{\pi}_{k,i} = \frac{\pi_{k,i} l_k}{l_i}$$

$$\xi^{W}_{u,k,i} = \frac{A_k d_{k,i}^{-\theta} \kappa_{u,n^*,i}^{-\theta}}{l_{u,n^*,i}^{\frac{\theta}{\epsilon}} B_i^{\frac{\theta}{\epsilon}}}$$

$$\lambda_{u,n^*} = \bar{W}_{u,n^*}^{-\theta} \gamma^{-\theta}$$

Given that all the parameters and variables are restricted to positive values, then we can already invoke the first part of the theorem by Allen et al. (2016), to prove the existence of at least one equilibrium. In order to prove uniqueness we need to define the following matrices. We define $\mathbf{\Gamma}$ as the matrix of the exponents of the variables on the left hand sides of the equations. Analogously, we define $\mathbf{B}$ to be the matrix of the exponents on the right hand side of the system.

$$\mathbf{\Gamma} = \begin{bmatrix} 1 & 0 \\ 0 & -\theta \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 1 & 0 \\ -\theta & 0 \end{bmatrix}$$

Given that $\mathbf{\Gamma}$ is invertible, then we can define the following matrix $\mathbf{A}$

$$\mathbf{A} = \mathbf{B}\mathbf{\Gamma^{-1}} = \begin{bmatrix} 1 & 0 \\ -\theta & 0 \end{bmatrix}$$

Then we compute the eigenvalues of the matrix constructed by the absolute values of $\mathbf{A}$ and we pick the largest defined as $\rho(\mathbf{A^P})$. It is easy to show that

$$\rho(\mathbf{A^P}) = 1$$

Then, applying the theorem by Allen et al. (2016), this ensures that the general equilibrium with respects of the two variables chosen is unique, but we are not able to compute it by an iterative procedure

We can proceed analogously by considering (20) and (21) and variables $\{w_i, \pi_{k,i}\}$. the system is

$$w_i = \sum_{k \in N} \xi^{\pi'}_{k,i} \pi_{k,i} w_k \tag{28}$$

$$w_k^{-\theta} \pi_{k,i}^{-1} = \sum_{t \in N} \xi^{E}_{i,k,t} w_t^{-\theta} \tag{29}$$

Where

$$\xi^{\pi'}_{k,i} = \frac{l_k}{l_i}$$

$$\xi^{E}_{i,k,t} = \frac{A_t d_{i,t}^{-\theta}}{A_k d_{i,k}^{-\theta}}$$

Then we can define the respective matrices $\mathbf{\Gamma'}$ and $\mathbf{B'}$

$$\mathbf{\Gamma'} = \begin{bmatrix} 1 & 0 \\ -\theta & -1 \end{bmatrix}$$

$$\mathbf{B'} = \begin{bmatrix} 1 & 1 \\ -\theta & 0 \end{bmatrix}$$

$$\mathbf{A'} = \mathbf{B'}\mathbf{\Gamma'^{-1}} = \begin{bmatrix} 1-\theta & -1 \\ -\theta & 0 \end{bmatrix}$$

The conditions imposed by the first two parts of theorem 3 of Allen et al. (2016) are respected for $\theta \leq 1$, which means that the equilibrium exists and is unique. It is also easy to show that considering the three equations together and the first part of theorem 3, we ensure existence of an equilibrium for the three variables considered. Let us now proceed by contradiction to show that these conditions ensure the uniqueness of the equilibrium. Let us assume that there exist two vectors $\{\pi_{k,i}, w_i, w_{u,i}\}$ such that there is a general equilibrium for equations $(26)-(27)-(29)$ by fixing the same vector l_{u,i,n^*}. This would mean that there exist two vectors $\{\pi_{i,k}, w_i\}$ that satisfy equilibrium $(28)-(29)$ and two vectors $\{w_i, w_{u,i}\}$ that satisfy $(26)-(27)$ both of which are untrue as we have demonstrated above. Therefore the partial equilibrium for $(26)-(27)-(29)$ is unique.

We can now use this result together with the other conditions of the model to demonstrate existence and uniqueness of the rest of the variables. First, $\{w_{u,i}, w_i, \pi_{k,i}\}$ let us compute $l_{u,i,n}$ for all $n \in N - \{n^*\}$ together with equation (22) for $S = u$. Then, equations (8) and (12) ensure the existence and uniqueness of $\{w_{s,i}, P_i\}$ as they are sums of known or fixed variables. As the price index of each location exists and it is unique, then we can infer through (16) for $S = s$, that $l_{s,i,n}$ also exists and is unique. Given that this process is true for all locations $n \in N$ then $l_{s,i}$ exists and is unique, which together with (7) this is true also for $l_{u,i}$.

It is important to prove that the top-down solution for $\{w_{s,i}, l_{u,i}\}$ recovered by equations (7)-(8) is the same as the bottom-up one obtained by obtaining $w_{u,i}$ through (8) and then summing over (16) for $S = s$ over all regions of origin. To do so we state the following lemma

Lemma 1. *Given the spatial model defined by* (20) - (21) - (22) - (23) *and the partial equilibrium* $\{w_i, P_i, w_{u,i}, l_{u,i}l_{u,i,n}, \pi_{i,k}\}$ *for all* i, n, k*, then there exist a vector* $\{l_{s,i}, w_{s,i}, l_{s,i,n}\}$ *that satisfies both* (7) *and* (16) *at the same time for all* i, n*.*

To demonstrate so we consider the following system

$$\begin{cases} l_{s,i} = l_{u,i}\left(\frac{1-\eta_i}{\eta_i}\frac{w_{s,i}}{w_{u,i}}\right)^{-\rho} \\ l_{s,i} = B_i\left(\frac{w_{s,i}}{P_i}\right)^{\epsilon}\sum_{n\in N}\left[\frac{l_{s,n,0}}{\kappa_{s,i,n}^{\epsilon}\sum_{t\in N}B_t\left(\frac{w_{s,t}}{\kappa_{s,t,n}P_t}\right)^{\epsilon}}\right] \end{cases} \tag{30}$$

Where the second equation is a consequence of (19). We can then combine the two equations to obtain the following equality

$$w_{s,i}^{\rho+\epsilon}\left(\frac{1-\eta_i}{\eta_i}\frac{1}{w_{u,i}}\right)^{\rho}\frac{B_i}{l_{u,i}P_i^{\epsilon}}\sum_{n\in N}\left[\frac{l_{s,n,0}}{\kappa_{s,i,n}^{\epsilon}\sum_{t\in N}B_t\left(\frac{w_{s,t}}{\kappa_{s,t,n}P_t}\right)^{\epsilon}}\right] = 1 \tag{31}$$

If we take the left hand side of (31) and consider it as a function $\Omega_i(w_{s,i})$ it is trivial to observe that it is a continuous and increasing function for which

$$\lim_{w_{s,i}\to 0}\Omega_i(w_{s,i}) = 0$$

$$\lim_{w_{s,i}\to +\infty}\Omega_i(w_{s,i}) = +\infty$$

We can then apply the Intermediate Value theorem to show that there exist a unique value of $w_{s,i}$ for which (31) holds regardless of the other regions. This reasoning can be applied to all $i \in N$.

Lemma 2. *Given the system of equation* (20)-(21)-(22)-(23)*, there always exists a vector* l_i *such that a partial equilibrium such that proposition 1 is true exists, given a vector* l_{u,i,n^*}*.*

Given the system of equations (26) -(27) - (29) we can define each of the different variables $\{l_{S,i}, l_{S,n,i}, w_i, w_{S,i}, \pi_{i,k}\}$ as functions of the vector l_i. Additionally, we can define the following set of existence for l_i

$$l_i \in [0, l_{\max}]$$

Where $l_{\max}$ is the value of l_i if all the global available population of both high skilled and low skilled moved to work in region i. We can now apply Brouwer Fixed Point theorem to (5) to show that there exist a value of l_i such that the equilibrium holds. We can then repeat this process for each other region by excluding the already allocated population. □

We can now demonstrate that there uniqueness does not hold for the general equilibrium of the system.

Proposition 2 (Existence of multiple general equilibria)**.** *Given the spatial model defined by the system of equations* (20) - (21) - (22) - (23) *and the set of endogenous variables that describe it as listed in subsection 3.5, there exist multiple general equilibria that describe the model.*

Proof. To prove this proposition we need to show that at least two different sets of variables can generate, respectively, a different equilibrium. First of all we can observe that the partial equilibrium for $\{w_i, \pi_{k,i}\}$ for equations $(20)-(21)$, exists and is unique regardless the other variables that define the general equilibrium. Therefore the values assumed by this set of variables is the same regardless of the initial values of $l_{u,n,i}$.

We can now consider equation (24) with the normalization of $\bar{U}_{u,n} = 1$. Proposition 1 holds, so we can affirm that an equilibrium exists and is unique given an initial set $\{l_{u,n,i}\}$ for a given n. We can consider a special case in which the outgoing migration from n is the same regardless of the destination. In other words $l_{u,n,i} = \frac{l_{u,n,0}}{N}$ for all i. Then rearranging equation (24) we obtain

$$w_{u,i}^{-\theta} = \frac{l_{u,n,0}^{-\frac{2\theta}{\epsilon}} \kappa_{u,n,i}^{-\theta} \gamma^{-\theta}}{\delta^{-\theta} B_i^{-\frac{\theta}{\epsilon}}} \Big[\sum_{k \in N} A_k (d_{k,i} w_k)^{-\theta} \Big] N^{\frac{\theta}{\epsilon}} \tag{32}$$

Thanks to this and to the equilibrium of equations $(20) - (21)$ one can then compute the remaining variables.

We can now consider a second special case, for which $l_{u,n,i} = m$ with m being an arbitrarily small but positive value for all $i \neq i^*$. Consequently, for $i = i^*$ we have that $l_{u,n,i} = l_{u,n,0} - (N-1)m$. In other words, almost all outgoing migration from n is directed to a single region i^*. We will maintain the same normalization as above, such that $\bar{U}_{u,n} = 1$. First we need to demonstrate the existence of $w_{u,i}$ for all $i \neq i^*$

$$\frac{l_{u,n,0}^{-\frac{\theta}{\epsilon}}}{\delta^{-\theta}} = \frac{B_i^{-\frac{\theta}{\epsilon}} \left(\frac{w_{u,i}}{\kappa_{u,n,i}} \right)^{-\theta} (m)^{\frac{\theta}{\epsilon}}}{\gamma^{-\theta} \sum_{k \in N} A_k (d_{k,i} w_k)^{-\theta}} \tag{33}$$

Given that the left hand side is a exogenous finite positive value, we need to demonstrate that there always exists a value of $w_{u,i}$ such that the equation is true. To do so we define the following function of $w_{u,i}$

$$f(w_{u,i}) = \frac{B_i^{-\frac{\theta}{\epsilon}} \left(\frac{w_{u,i}}{\kappa_{u,n,i}} \right)^{-\theta} (m)^{\frac{\theta}{\epsilon}}}{\gamma^{-\theta} \sum_{k \in N} A_k (d_{k,i} w_k)^{-\theta}} \tag{34}$$

We can already observe that $f(w_{u,i})$ is a strictly monotonically decreasing function in $w_{u,i} \in \mathbb{R}_+$ for all positive values of m. It is trivial to show that $\lim_{w_{u,i} \to +\infty} f(w_{u,i}) = 0$ and that $\lim_{w_{u,i} \to 0} f(w_{u,i}) = +\infty$. Applying the Intermediate Value theorem, then, we can state that $\exists!\, w_{u,i} \, \forall \, m > 0$. Therefore we can consider the case where $m \to 1$ for $i = i^*$

$$w_{u,i^*}^{-\theta} = \frac{l_{u,n,0}^{-\frac{2\theta}{\epsilon}} \kappa_{u,n,i^*}^{-\theta} \gamma^{-\theta}}{\delta^{-\theta} B_i^{-\frac{\theta}{\epsilon}}} \Big[\sum_{k \in N} A_k (d_{k,i} w_k)^{-\theta} \Big] \tag{35}$$

Given that $N > 1$ as we are assuming multiple locations it easy tho show that the equilibrium implied by equation (28) is different from the equilibrium implied by (33). But for both proposition 1 holds, therefore there are multiple possible equilibria. □

4 Empirical framework

This section adapts empirical tools and frameworks used within the international trade literature for a migration gravity model, in particular those used by Redding and Venables (2004) in order to estimate a wage equation and their market access potential. As expected given the previous literature, the following estimations clash with the absence of good quality migration data (Kerr et al. 2014). The aim of the paper is, additionally, to provide a benchmark for which new data is necessary for a better understanding of the phenomenon of migration and to which other economic data it needs to be attuned.

The issue of the multiple equilibria impairs the ability to run simulations and limits us to rely to regression estimations using the available migration flows data. Proposition 1, though, suggests that if we have exact data of one country emigration flows, then we would be able to infer our world unique equilibrium. Contrary to this, the best available data regarding migration relies on immigration data, rather than emigration one, which bars this possible empirical path.

4.1 Composition of the wage ratio

Previous studies related to local productivity shocks such as Akerman et al. (2015) or Autor et al. (2008) fail to include explicitly the effect of a general equilibrium framework in their estimations. Given the model, a change in local productivity has a direct effect on the wage ratio of a country, which in turn implies a spatial reallocation of the agents. Therefore there is an indirect effect on wages through migration. Any estimation of the input labor elasticity would then be biased, because it would also include the elasticity of migration with respect to wages. To show this, let us reconsider equation (31) and compute the analogous equation for $S = u$. We, then, obtain the following system of equations

$$w_{s,i}^{\rho+\epsilon} \Big(\frac{1-\eta_i}{\eta_i} \frac{1}{w_{u,i}} \Big)^{\rho} \frac{B_i}{l_{u,i} P_i^{\epsilon}} \sum_{n \in N} \Big[\frac{l_{s,n,0}}{\kappa_{s,i,n}^{\epsilon} \sum_{t \in N} B_t \big(\frac{w_{s,t}}{\kappa_{s,t,n} P_t} \big)^{\epsilon}} \Big] = 1 \tag{36}$$

$$w_{u,i}^{\rho+\epsilon} \Big(\frac{1-\eta_i}{\eta_i} w_{s,i} \Big)^{-\rho} \frac{B_i}{l_{s,i} P_i^{\epsilon}} \sum_{n \in N} \Big[\frac{l_{u,n,0}}{\kappa_{u,i,n}^{\epsilon} \sum_{t \in N} B_t \big(\frac{w_{u,t}}{\kappa_{u,t,n} P_t} \big)^{\epsilon}} \Big] = 1 \tag{37}$$

Equations (36) - (37) are reminiscent of the wage equations within the trade economics literature. We can already draw some insights from this system of equations. Let us consider equation (36). A higher local population of low skilled agents ($l_{u,i}$), higher low-skilled wages ($w_{u,i}$), a better productivity ratio ($\frac{\eta_i}{1-\eta_i}$) and a higher price index drive up high-skilled wages. On the contrary, higher local amenities reduce high-skilled wages. The last term, that we are going to define as *high-skilled migration access*, captures how easy it is for location i to import high-skilled agents and it is reminiscent of the market access for the trade gravity equations. To simplify the notation we are going to redefine them as

$$\mathrm{MA}_{S,i} = \sum_{n \in N} \Big[\frac{l_{S,n,0}}{\kappa_{S,i,n}^{\epsilon} \sum_{t \in N} B_t \big(\frac{w_{S,t}}{\kappa_{S,t,n} P_t} \big)^{\epsilon}} \Big]$$

In a world without migration, the *high-skilled migration access* is comparable to the local high-skilled population. In other words the *high-skilled migration access* can be conceptualized as the pool of high-skilled workers region i has access to, deflated by the specific bilateral migration frictions. This factor, then, takes into consideration the difference between a small foreign pool of skill-specific workers which do not encounter significant roadblocks when migrating (what is today Austria to Germany, for example), versus a large foreign pool of skill-specific workers which, instead, face a number of difficulties in migrating (what is today Indonesia to Germany). It is then easy to understand why, in equation (36), it drives the high-skilled wages down.

Analogously, for equation (37), the relationships are similar. The main difference is the effect of the productivity ratio as we have defined it, which is opposite in (37) compared to (36). The difference can be easily dropped when we consider the productivity ratio by keeping as the numerator the productivity of the type of agent under consideration.

If we then combine the two equations we obtain

$$\frac{w_{s,i}}{w_{u,i}} = \Big(\frac{\eta_i}{1-\eta_i} \Big)^{\frac{2\rho}{2\rho+\epsilon}} \Big(\frac{l_{s,i}}{l_{u,i}} \Big)^{-\frac{1}{2\rho+\epsilon}} \Big[\frac{\mathrm{MA}_{s,i}}{\mathrm{MA}_{u,i}} \Big]^{-\frac{1}{2\rho+\epsilon}} \tag{38}$$

Equation (38) can be used to observe the true determinants of the wage ratio. We can observe then that, in line with previous literature, the wage ratio is determined by the local availability of skill-specific labor and the productivity ratio. In addition, our model enablas us to identify a further factor, which is the ratio of the *skill-specific migration access*. In other words, the ratio of the global pool of skill-specific workers deflated by the skill-specific bilateral frictions.

4.2 Special cases

Before proceeding with a full empirical analysis, it can be enlightening to observe the effects of two special cases on the wage ratio.

The first one can be seen as "population autarky", in other words, migration frictions are so high that there is no migration across regions, $\kappa_{S,n,i} \to +\infty$. For this specific case, then, the wage ratio is defined exclusively by equation (23) for each region, while $l_{S,n} = l_{S,n,0}$. This is the specific case for previous estimations. In this world, production is fixed by the availability of local factors and trade shares are a simple consequence of the local productivity factors for each region and of the bilateral trade costs.

In the second case, we remove migration frictions $\kappa_{S,i,n} = 1$. Both agents are free now to relocate, so that equation (22) now refers to the share of agents of skill S that decide to locate in region i. Then equations (36) - (37) can be rewritten as

$$w_{s,i}^{\rho+\epsilon}\Big(\frac{1-\eta_i}{\eta_i}\frac{1}{w_{u,i}}\Big)^{\rho}\frac{B_i}{l_{u,i}P_i^{\epsilon}}\Big[\frac{\bar{l}_s}{\sum_{t\in N}B_t\big(\frac{w_{s,t}}{P_t}\big)^{\epsilon}}\Big] = 1 \tag{39}$$

$$w_{u,i}^{\rho+\epsilon}\Big(\frac{1-\eta_i}{\eta_i}w_{s,i}\Big)^{-\rho}\frac{B_i}{l_{s,i}P_i^{\epsilon}}\Big[\frac{\bar{l}_u}{\sum_{t\in N}B_t\big(\frac{w_{u,t}}{P_t}\big)^{\epsilon}}\Big] = 1 \tag{40}$$

Where $\bar{l}_s$ and $\bar{l}_u$ indicate the global availability of agents of a specific skill level. Then it is possible to rewrite equation (38) as

$$\Big(\frac{w_{s,i}}{w_{u,i}}\Big) = \Big(\frac{\eta_i}{1-\eta_i}\Big)^{\frac{2\rho}{2\rho+\epsilon}}\Big(\frac{l_{s,i}}{l_{u,i}}\Big)^{-\frac{1}{2\rho+\epsilon}}\left[\frac{\frac{\bar{l}_s}{\sum_{t\in N}B_t\left(\frac{w_{s,t}}{P_t}\right)^{\epsilon}}}{\frac{\bar{l}_u}{\sum_{t\in N}B_t\left(\frac{w_{u,t}}{P_t}\right)^{\epsilon}}}\right]^{-\frac{1}{2\rho+\epsilon}} \tag{41}$$

In this case, on the right hand side of equation (41), we observe a global factor of skill availability. This can be captured in a regression by the intercept factor, or, if we have a panel data by the year fixed effect.

In both special cases, estimating either (23) or (41) we should not observe any migration access bias. On the contrary, if equation (38) is the true model describing the local wage ratio, then the above mentioned ratio would be affected by an unobserved bias.

4.3 Estimation procedure

The next objective of our analysis would, then, be to test the framework empirically. If we take the logarithm of (38), we obtain the following equation

$$\begin{aligned}\ln \textit{Wage ratio}_i = {} & \frac{2\rho}{2\rho+\epsilon}\ln \textit{Productivity ratio}_i \\ & -\frac{1}{2\rho+\epsilon}\ln \textit{Skill ratio}_i - \frac{1}{2\rho+\epsilon}\ln \textit{Migration access ratio}_i\end{aligned} \tag{42}$$

Then it is possible to estimate equation (42), in order to differentiate the two components of the wage ratio. On the one hand, the local productivity ratio, while on the other, the global access ratio to the region.

The last term, the *migration access ratio*, is the most difficult to observe. Given that the *skill-specific migration access* is analogous to the market access of the trade economics literature it is possible to employ similar estimation procedures. In particular, if one has bilateral migration data by level of education, then one could employ a simple variation of the estimation procedure developed by Redding and Venables (2004). First one can estimate through equation (16) the following relationship

$$\ln l_{S,i,n} = \ln B_i\Big(\frac{w_{S,i}}{P_i}\Big)^{\epsilon} - \epsilon \ln \kappa_{S,i,n} + \ln l_{S,n,0} - \ln \sum_{t\in N} B_t\Big(\frac{w_{S,t}}{\kappa_{S,t,n}P_t}\Big)^{\epsilon} \tag{43}$$

Equation (43) is a bilateral migration equation, first suggested by Anderson (2011) and then further adopted within the migration literature (Beine et al. 2016). It is easy to observe that from equation (43) it is possible to compute the *skill-specific migration access* through the origin fixed effects. In fact, the first term of (43) can be captured through a destination fixed effects, while the third and the fourth can be captured by a origin fixed effect. Lastly, one can proxy the value of the bilateral migration friction. If the migration dataset is rich enough, then one can recover the value of the migration access for a specific skill level for country i by summing the origin fixed effects and the specific bilateral friction for every origin country. For local country access we assume that $\kappa_{S,n,n} = 1$.

$$\mathrm{MA}_{S,i} = \sum_{n\in N} \exp(\textit{Origin fixed effect}_n + \ln \kappa_{S,n,i})$$

The results can, then, be used to estimate the respective skill wage equation (36) or (37) or combined in order to obtain a simple estimation of the wage ratio with equation (38). We will now proceed to estimate the *skill-specific migration access* as outlined above.

4.4 Bilateral migration estimation

As reported by Kerr et al. (2017) the state of the available data with respect to migration is mediocre at best. The literature in recent years, following Carrington and Detragiache (1998, 1999) has concentrated in collecting immigration data from census and other OECD statistics on international migration data. If we have enough of these country records, then we can assemble a dataset rich enough for us to estimate equation (43). I use the dataset built by Docquier et al. (2009), henceforth DLM09, which is an extension of the Docquier and Marfouk (2006) dataset. The dataset provides immigration stocks for 31 countries for two years, 1991 and 2001. The choice of the DLM09 dataset over the more recent Brücker et al. (2013), henceforth BCM13, which provides similar data for a smaller set of receiving countries but has an extended time frame, is a choice of coverage. The DLM09 provides at least a receiving country per macro geographic area, excluding South America. On the contrary, the BCM13 by excluding data, among others, for Mexico, South Africa, Japan and Korea, leaves uncovered a large part of the world, mainly Africa and Asia, which can easily lead to statistical biases. The summary statistics of the dataset are reported in

	High-skilled	Low-skilled
Number of observations	6411	6510
Number of countries of origin	185	185
Number of countries of destination	31	31
Average number of observations per destination	206.8	210
Average number of observations per origin	34.7	35.2
Average number of observations per year	3205.5	3255
Minimum value	1	1
Maximum value	919139	5455687
Average value	9582.792	4929.044
Standard deviation	29775.95	84082.99

Table 1: Summary statistics

table 1. The complete list of receiving and origin countries is reported in the appendix.

We consider high-skilled agents those with at least tertiary education, while everyone else are considered low-skilled. Equation (43) can be estimated analogously to a gravity equation. We will proceed by including some basic proxies for the migration frictions. The final regression equation will be

$$\ln l_{S,i,n,t} = \gamma_{S,i,n} \, X_{i,n,t} + \alpha_{S,i} + \beta_{S,n} + \delta_{S,t} \tag{44}$$

Regression (44) regresses the log of the skill-specific migration on a vector $X_{i,n,t}$ of variables. In the notation I preserve the usage of the subscripts as in the previous sections, where n indicates the origin countries and i the destination ones. The subscript t indicates, instead, the year of the observation. $\alpha_{S,i}$ indicates the destination fixed effects, $\beta_{S,n}$ the origin fixed effects and $\delta_{S,t}$ the time fixed effects. The regressions, and therefore the fixed effects, are skill specific.

$X_{i,n,t}$ includes several variables that are possibly related to the difficulty of moving across countries. It includes the distance between two countries, the presence of any shared official language and currency (which, additionally, are a good proxy for colonial ties), whether the two countries have a geographical border in common, the volume of trade between the two and, finally, the GDP ratio between the two. Excluding the GDP ratio variable, all factors contributing to the migration friction are symmetrical, therefore $\kappa_{S,i,n} = \kappa_{S,n,i}$. As specified in the theoretical analysis this is not an underlying assumption and we will delve into the details of the implications of this in the analysis of the results.

The results for low-skilled migration and high-skilled one are written, respectively in table 2 and 3 and in both cases I will refer to the results of column (2).

Both results are highly significant, displaying a R-squared between 0.717 and 0.816 which are comparable to the analogous regression ran by Redding and

Venables (2004) for the trade gravity equation. Additionally, the two different types of agents respond to the same determinants of migration frictions, but differ with respect to the magnitude of their response. In particular migration decreases as the distance between two countries increases and if the countries share a free trade agreement. On the other hand, migration increases if the the countries share a border or a common official language. A comparison of the difference in magnitude between the two types of agents has been summarized in table 4.

Most of these results are not contrary to our intuition with respect to migration. Migration is easier if the countries are closer geographically and if they share a border. Higher economic ties, in particular those related to trade which involves at least a certain amount of human relations, while also reflecting underlying common factors (such as the presence of a large diaspora of the other country) are positively related. The only surprising results are that a free trade agreement would reduce the amount of migration for both types of agents. I do provide two tentative explanation for the first result, one within the framework of the theory I have developed in this paper, while the other is more related to the presence of capital flows.

In the first interpretation, it is possible to consider this negative relationship as a utility trade-off for the agent between the migration friction and the trade frictions, as internalized through the price index. The mechanism of this interpretation is better understandable with an extreme example. Let us imagine an agent which is generated in a region n which has both high trade and migration frictions. Due to the trade frictions the agent faces a high price index which depresses its utility, but can decide to move to a region i which has a better market access (we can imagine the two regions offer the same wages and have the same amenities to further simplify the example). To do so, though, the agent has to pay in utility a migration price due to the migration frictions. In this case a trade agreement between n and any other third country, would improve its market access, thus reducing its price index, which would make the option of staying in region n more attractive to the agent rather than moving to region i and paying the same migration friction cost, comparatively to the initial situation.

The second interpretation involves capital flows which are not included in the model developed in the previous section. If two regions n, where the agents resides, and i sign a free trade agreement, the capital flow between the two would increase, as indicated by di Giovanni (2005) at least in the case of service agreements. If n, before the agreement, was underfunded and over-reliant on labor, and therefore suffering from low wages and high unemployment, it would provide an incentive for the agent to migrate to a different region, thus suffering the migration cost penalty. On the contrary, after the agreement, and a more optimal capital allocation, we would expect to see an increase in wages (due, possibly, to an increase of labor productivity vis-à-vis capital productivity) and in employment opportunities, thus decreasing the incentives of the agent to move to a different region.

Both interpretation can, obviously, co-exist at the same time, but only the first one can be fully incorporated in the theoretical model developed above.

Additionally, as I have already noted previously, the results show that the migration friction is symmetric. This is not an assumption set in the theoretical model and the results would not change whether the migration friction displays or not this characteristic. It is also difficult to imagine that the difficulties one has when moving from n to i are the same then moving in the other direction. Looking for housing might be easier in one direction rather than the other, as the immigration procedures might be simplified in one direction rather then the other. As countries display different procedures for citizens of different nationalities this might be captured by the migration friction. The main missing asymmetric feature, though, is the presence of the diaspora of the origin country. The migration literature (Beine et al., 2015) have highlighted how the presence of a diaspora is an important *pull factor* which helps smoothing the often traumatic experience of migrating to a different country (in particular when the two countries have a different policies, culture and language). It is possible that in the case of particularly diverse countries, such as the US, this factor has been partially captured by the destination fixed effect. But if a destination country displays a few large communities of different countries (such as the Hungarian diaspora in Romania, to cite an example), we might be introducing a negative bias of their migration access.

It is possible now to delve in the specific differences between high-skilled and low-skilled migration frictions, keeping in mind that their differences are entirely a matter of magnitude rather than direction. I will concentrate on the specification of column (3) for both tables 2 and 3. The most noticeable differences lie in the geographical factors of the frictions. In particular high-skilled agents tend to suffer less from the distance between the origin and destination regions. The magnitude of the relationship between distance and migration is greater for low-skilled by a fourth when compared to the same relationship for high-skilled. When considering the presence of a shared border, the magnitude is more than twice for low-skilled than high-skilled. On the contrary, the positive relationship of a common language between origin and destination on migration is 7% higher for high-skilled agents than for low-skilled ones. Finally, high-skilled migration is less affected than the low-skilled counterpart by economic variables. In particular, the magnitude of the negative relationship between migration and free trade agreements is 50% higher for the low-skilled agents. Table 4 reports a summary of the differences in magnitude.

Most of the difference between the high-skilled migration frictions and the low-skilled one is, then, due to geographical factors. Low-skilled agents tend to move to regions that are closer to their origin ones, in particular if they share a border.

An additional result which deserves further analysis, because it will contribute in the construction of the *skill-specific migration access*, are the fixed effects.

The analysis of origin countries - time fixed effects gives us a better picture of assessing the overall 'export' tendency of each country. I provide a snapshot of the results in table 5. Specifically, table 5 reports the top 5 values and the lowest

Variables	**Log migration** (1)	(2)	(3)
Log distance	-1.38***	-1.28***	-1.29***
	(0.0361)	(0.0434)	(0.0431)
Contiguous		1.04***	1.04***
		(0.136)	(0.135)
Common language		1.46***	1.46***
		(0.0626)	(0.0626)
Free trade agreement		-1.12***	-1.12***
		(0.106)	(0.106)
Common currency		-0.0221	
		(0.164)	
Origin country x time fixed effect	YES	YES	YES
Destination country x time fixed effect	YES	YES	YES
N. observations	6510	6510	6510
Adjusted R-squared	0.717	0.748	0.748

Table 2: Low skilled migration

Variables	**Log migration** (1)	(2)	(3)
Log distance	-1.16***	-1.06***	-1.07***
	(0.0311)	(0.0366)	(0.0363)
Contiguous		0.395***	0.410***
		(0.114)	(0.114)
Common language		1.57***	1.57***
		(0.0528)	(0.0528)
Free trade agreement		-0.841***	-0.836***
		(0.0890)	(0.0890)
Common currency		0.222	
		(0.139)	
Origin country x time fixed effect	YES	YES	YES
Destination country x time fixed effect	YES	YES	YES
N. observations	6411	6411	6411
Adjusted R-squared	0.784	0.816	0.816

Table 3: High skilled migration

Variables	Ratios (high-skilled over low-skilled)
Log distance	0.828
Contiguous	0.394
Common language	1.08
Free trade agreement	0.746

Table 4: Summary of the regressions results

5 for both high-skilled and low-skilled migration. Overall we can see a similar picture when analysing the countries involved: connected and populous nations are at the top, while smaller and isolated ones, such as the pacific or Caribbean island-countries, are closer to the bottom. It is possible that for certain areas the result are undervalued due to the absence of close receiving countries in the same macro geographical areas. The countries most at risk of this are those located in the macro geographical areas of Africa, the MENA region and, to a less extent, South America. To explain this with an example, with the exclusion of the data from South Africa, we do not have a good picture of intra-African migration. Having data only with respect to long distance migration, which is the main factor that discourages cross-country movements, we might be underestimating regional movements. Additionally, with the exclusion of US data, we do not have information regarding informal migration. While this does not affect the global validity of the estimation, it might create downward biases in particular with respect to specific regions and with respect to low-skilled migration. Having said this, we can still infer some interesting results. The first difference to be noticed is the discrepancy with respect to the time factor. the fixed effects for the high-skilled migrants are higher in year 2000, while the contrary is true for low-skilled migrants. From these results it seems that between 1990 and 2000 high skilled migrants have increased, while low-skilled ones have decreased. Running some back-of-the-envelope computations, it is possible to notice that the average of the difference between the fixed effects of 2000 and 1990 for the high-skilled migration is 3.36, while the same value for the low-skilled is $-.0557$. In other words, it is safe to state that high-skilled migration has increased more then the decrease of low-skilled migration. Another way to approach this is to notice that the different values for the fixed effect based on year are less polarized for the low-skilled values than the high skilled ones. The lowest observation for year 2000 in the complete list for the high-skilled is thirty-eighth from the bottom, while the lowest one for year 1990 for the low-skilled is fifth. Analogously, the highest value for year 1990 for the high-skilled is fifty-eighth, while the highest observation for 2000 for the low-skilled is seventh. These results, as I will state later in the next subsection is going to affect the results for the two different periods of observations.

At the margin, though, emerge some interesting differences. Let us consider the cases of Great Britain and Italy, two countries with a similar level of wealth and population. While the two countries are in similar positions for the low-skilled origin fixed effects (respectively third and fifth during year 1990), Italy has a more than 5% lower fixed effect for the high-skilled migration with respect to Great-Britain in the same year. A possible explanation is the lowest number of people with a tertiary degree in Italy with respect to Great Britain (Bolton, 2012; ISTAT, 2020). Overall, these differences can lead to significant variations in the *skill-specific migration access*. Another possible explanation for these differences is structural inequality. Let us consider here the case of two eastern communist countries such as China and Vietnam. If in these countries access to education is, formally or de facto, restricted to a minority, then it is understandable to observe a comparatively lower tendency to migrate for the skilled than the unskilled. Once a person reaches a privileged position within a society, it will have much less interest to migrate in a foreign country where its privilege cannot be exerted in the same way. In other words, the results of table 5 can be used to a wide degree of possible studies.

		High-skilled			Low-skilled
USA	2000	22.4908354	USA	1990	21.8360499
Great Britain	2000	22.0583625	China	1990	21.6574073
Germany	2000	21.9279239	Great Britain	1990	21.6166797
China	2000	21.725447	Germany	1990	21.6039944
Philippines	2000	21.2204838	Italy	1990	21.5497914
...	...	...	...		
Bhutan	1990	11.6029935	Nauru	1990	13.90846
Maldives	1990	11.5034113	Nauru	2000	13.8924209
Equatorial Guinea	1990	11.2058656	Oman	2000	13.8383017
Vanuatu	1990	10.974438	San Marino	2000	12.9797111
Nauru	1990	10.5690748	Palau	2000	12.5920026

Table 5: List of selected country of origin fixed effects

4.5 Pseudo-Poisson maximum likelihood estimation

Following the criticism of Silva and Tenreyro (2006) of any non-linear transformation, I run a pseudo-Poisson maximum likelihood (ppml) estimation. The results are summarised in tables 6 and 7.

Variables	**Log migration** (1)
Log distance	-0.593***
	(0.123)
Contiguous	0.613*
	(0.238)
Common language	1.15***
	(0.137)
Free trade agreement	-0.232
	(0.241)
Common currency	0.0477
	(0.223)
Origin country x time fixed effect	YES
Destination country x time fixed effect	YES
N. observations	6411
Adjusted R-squared	0.784

Table 6: High skilled migration (ppml)

Variables	**Log migration** (1)
Log distance	-0.949***
	(0.148)
Contiguous	0.984***
	(0.294)
Common language	1.48***
	(0.194)
Free trade agreement	-0.356
	(0.371)
Common currency	-0.241
	(0.341)
Origin country x time fixed effect	YES
Destination country x time fixed effect	YES
N. observations	6510
Adjusted Pseudo R-squared	0.796

Table 7: Low skilled migration (ppml)

As with the previous results we can observe that low-skilled agents suffer from higher migration frictions than high-skilled ones. There are two main differences that one can notice when comparing the results. The first one is that it is impossible to reject the null hypothesis of a free trade agreement. Secondly,

Variables	Ratios (high-skilled over low-skilled)
Log distance	0.625
Contiguous	0.623
Common language	0.777

Table 8: Summary of the regressions results

the presence of a common language has a stronger effect on low-skilled migrants than high-skilled ones. A summary of the comparison between the two types of result is shown in table 8. These results warrant some skepticism when dealing with the results in tables 2 and 3, in particular when dealing with the effects of a free trade agreement, but globally do not counter the global interpretation of the previous results.

4.6 Computation of the skill-specific migration access

Having run the auxiliary regressions, one can now write a full function to compute the *skill-specific migration access* for each country i. The formulas adopted are

$$\begin{aligned}\mathrm{MA}_{u,i} = \sum_{n \in N} \big[&\exp(\text{origin fixed effects}_{u,i} - 1.29 \ln \text{distance}_{i,n} \\ &+ 1.04\, \text{contiguous}_{i,n} + 1.46\, \text{common language}_{i,n} \\ &- 1.12\, \text{free trade agreement}_{i,n}\big]\end{aligned} \tag{45}$$

$$\begin{aligned}\mathrm{MA}_{s,i} = \sum_{n \in N} \big[&\exp(\text{origin fixed effects}_{s,i} - 1.07 \ln \text{distance}_{i,n} \\ &+ 0.410\, \text{contiguous}_{i,n} + 1.57\, \text{common language}_{i,n} \\ &- 0.836\, \text{free trade agreement}_{i,n}\big]\end{aligned} \tag{46}$$

I list in table 9 the top five estimation of the migration access and the bottom five similarly to table 5 for both types of *skill-specific migration access*. The results are normalized with respect to the USA of 2000.

In both lists the ordering of the *skill-specific migration access* reflects closely that of the origin country fixed effects. The first conclusion one can draw from these results is that the *skill-specific migration access* is mostly influenced by a country's own labor market. Even taking into consideration the short distance of San Marino from a large group of populous countries, its relative position with respect to rest of the world for the *low-skilled migration access* is similar to that of the pacific islands of Nauru and Palau. The second remark, though, goes in the opposite direction. Despite the similarities between the two lists the distance between the observations of the countries varies a lot. As an example one can see that a difference of 0.5 between the high-skilled fixed effects of the USA and Germany leads to the former *high-skilled migration access* being double than the latter. Most of these are results are due to the exponential function in equations (45) and (46).

		High-skilled			Low-skilled
USA	2000	1	USA	1990	2.69
Great Britain	2000	0.650	China	1990	2.25
Germany	2000	0.571	Great Britain	1990	2.16
China	2000	0.466	Germany	1990	2.14
Philippines	2000	0.282	Italy	1990	2.02
...	...	...	...	...	...
Nauru	1990	0.0000475	Nauru	1990	0.00143
Equatorial Guinea	1990	0.0000470	Oman	2000	0.00138
Maldives	1990	0.0000414	San Marino	2000	0.00126
Tuvalu	1990	0.0000367	Nauru	2000	0.00122
Bhutan	1990	0.0000360	Palau	2000	0.000740

Table 9: List of selected migration access values

To account for the effect of the local labor market I provide in table 10 the top five and bottom five foreign migration access values. To compute these I follow equations (45) and (46), without considering the same country fixed effect. As above, the values are normalized with respect to the USA.

The picture is different compared to the one in table 9. At the top of the list we observe European countries such as Belgium, Ireland and Switzerland, which share borders and language with populous neighbours, while at the bottom we island-countries. There are some interesting remarks as well. In particular, the cases of Lebanon and Ireland which have a better access due to their position and language spoken, respectively to low-skill migrants and high-skill ones. It is not surprising that Ireland, despite sharing only one border with another country and being comparatively isolated from mainland Europe, manages to have such a high foreign migration access for high-skill migrants, when considering that two of the countries with the highest proportion of skilled workers are anglophone (the UK and the USA).

		High-skilled			Low-skilled
Belgium	2000	6.89	Macau	1990	37.1
Luxembourg	2000	5.53	Belgium	1990	32.0
Macau	2000	5.22	Switzerland	1990	22.4
Switzerland	2000	4.81	Belgium	2000	14.3
Ireland	2000	3.97	Lebanon	1990	14.1
...	...	...	...	...	...
Sao Tome and Principe	1990	0.0206	Indonesia	2000	0.687
Maldives	1990	0.0192	Kiribati	2000	0.680
Comoros	1990	0.0192	Micronesia	2000	0.343
Bhutan	1990	0.0135	Tuvalu	1990	0.304
Tuvalu	1990	0.00604	Tuvalu	2000	0.292

Table 10: List of selected foreign migration access values

The last table of results concerns what I have referred to as the *migration-access ratio* which appears in equation (38). Analogously to the previous results, I report a selected list of *migration access ratio* in table 11.

It has to be reminded, though, that there might be a significant downward bias in this paper estimation of the *low-skill migration access* for two reasons. The first one is that lower amount of destination countries samples and their unequal geographical distribution, might has lead to a lower estimation of low-skill migration in those areas that are not covered. In particular, I do not have a complete picture of migration within the Sahel region, the MENA region, South America and Central Asia, which have a comparatively higher proportion of low-skill agents than high-skill ones (Max Roser and Esteban Ortiz-Ospina, 2013), while I do have a complete picture of Western migration patterns. The second issue is that, with the exception of the United States, the destination countries sampled do not record irregular migration, which is safe to assume to be composed mostly by low-skill migrants. If part of the migration South-North of the world happens irregularly and this type of migration concerns mostly low-skill migrants, then I are further underestimating low-skill migration, in particular, from those countries that have a comparatively easier access to the sampled destination country. Let us consider the case of Morocco and Spain. If part of the low-skill migration of Morocco happens irregularly at the border of the two Spanish enclaves in Northern African, then I might be underestimating the origin country low-skill fixed effect of the North African country. This in turn might noticeably lower the *migration access ratio* for those country that have their *low-skill migration access* more dependant on Morocco, such as the neighbouring ones, Algeria, Mauritania and Spain itself. A further issue with these results is the significantly higher values of observations for the 2000, when compared to those of the 1990. It might be that migration suffered a significant drift in favour of high-skilled agents between 1990 and 2000, during the years in which the concept of globalization rose. At the same time, the size of the drift and the absence of observations in other periods leads me to treat the comparison between the two with a pinch of skepticism.

In all cases, the value for the *foreign migration access ratio* is higher than the standard *migration access ratio*. Access to migrants from abroad are generally more skewed toward the high-skill than the standard migration access. Additionally most of the countries at the top of the lists are pacific or Caribbean island-states (with the exclusion of Oman), which is expected given that migration between close and confining countries offers less frictions for the low-skilled agents. Countries at the bottom of the lists are influenced by the proximity to large countries with large unskilled populations.

The main implication of these results according to the theoretical model is that countries with larger ratios should have a lower skill premium when compared to other countries. Assuming that the difference between the two time frames is genuine and not a result of skewed underlying data, it could be inferred that while migration in the 1990 drove the skill premium up, the opposite happens in the 2000. Without drawing inter-temporal conclusions, it still possible to conclude that a country like Palau has a lower wage ratio than a world without migration, when compared to a country such as the USA in the same condition.

		Complete			Foreign
Palau	2000	11.5	Micronesia	2000	29.3
Saint Kitts and Nevis	2000	8.94	Samoa	2000	24.4
San Marino	2000	8.77	Kiribati	2000	24.3
Oman	2000	8.72	Bahamas	2000	24.1
Antigua and Barbuda	2000	8.66	Saint Kitts and Nevis	2000	24.1
...	...	...	...	...	...
The Gambia	1990	0.0241	Myanmar	1990	0.214
Vietnam	1990	0.0240	Syria	1990	0.203
Portugal	1990	0.0179	Laos	1990	0.194
Cambodia	1990	0.0162	Macau	1990	0.176
Cabo Verde	1990	0.0125	Cambodia	1990	0.162

Table 11: List of selected migration access ratios

4.7 Wage equation estimation

Migration is a difficult topic to comprehensibly sample. Firstly, most states do not have the state capacity to accurately collect data regarding their emigration rates, and, furthermore, few states have the capacity to do so for immigrant population. This problem is only exacerbated by irregular migration which is systematically sampled only by the United States.

Additionally, national data regarding skill-based productivity and skill-based wages collected in an homogeneous manner is difficult to collect, as it cannot be inferred from macroeconomic data as it is generally done for the most simple and available income inferences. To have skill-based income and productivity data one has to have access to high quality national labour survey which are scantly available before 2000 for most countries, at least in a comparative manner.

The aim of this paper was to show that an estimation of the *skill-specific migration access* is possible. The best way forward for future analysis is to wait the next waves of censuses and labour survey while at the same time extend the data of the DLM09 for more recent years, and without reducing, and if possible extending, the number of sampled destination countries. This would enable the best use of more recent comparative tools such as the OECD surveys or the EU Labor Force Survey.

4.8 Alternative model

An alternative explanation for differences in commuting between high-skilled and low-skilled agents is that they face different elasticities of migration with respect to the real wage ϵ as in Cucu (2020). The question, then, becomes whether the two theories are observationally equivalent. To do so we extend the basic theoretical model, without extending the theoretical section of the paper. The corresponding model would be similar to Cucu (2020) which shows numerically existence, but with the same sets of frictions between high and low skilled workers. The system (20)-(21)-(22)-(23) becomes then

$$w_i l_i = \sum_{k\in N} \pi_{k,i} w_k l_k \tag{47}$$

$$\pi_{k,i} = \frac{A_k (d_{i,k} w_k)^{-\theta}}{\sum_{t\in N} A_t (d_{i,t} w_t)^{-\theta}} \tag{48}$$

$$\frac{l_{S,i,n}}{l_{S,n,0}} = \frac{B_i \left(\frac{w_{S,i}}{P_i^{\alpha} r_i^{1-\alpha} \kappa_{i,n}} \right)^{\epsilon_S}}{\sum_{t\in N} B_t \left(\frac{w_{S,t}}{P_t^{\alpha} r_t^{1-\alpha} \kappa_{t,n}} \right)^{\epsilon_S}} \tag{49}$$

$$\frac{l_{i,u}}{l_{i,s}} = \Big(\frac{1-\eta_i}{\eta_i} \frac{w_{i,s}}{w_{i,u}} \Big)^{\rho} \tag{50}$$

We can then provide the corresponding system (36) -(37)

$$w_{s,i}^{\rho+\epsilon_s} \Big(\frac{1-\eta_i}{\eta_i} \frac{1}{w_{u,i}} \Big)^{\rho} \frac{B_i}{l_{u,i} P_i^{\epsilon_s \alpha} r_i^{\epsilon_s (1-\alpha)}} \sum_{n\in N} \Big[\frac{l_{s,n,0}}{\kappa_{i,n}^{\epsilon_s} \sum_{t\in N} B_t \big(\frac{w_{s,t}}{\kappa_{t,n} P_t^{\alpha} r_t^{1-\alpha}} \big)^{\epsilon_s}} \Big] = 1 \tag{51}$$

$$w_{u,i}^{\rho+\epsilon_u} \Big(\frac{1-\eta_i}{\eta_i} w_{s,i} \Big)^{-\rho} \frac{B_i}{l_{s,i} P_i^{\epsilon_u \alpha} r_i^{\epsilon_u (1-\alpha)}} \sum_{n\in N} \Big[\frac{l_{u,n,0}}{\kappa_{i,n}^{\epsilon_u} \sum_{t\in N} B_t \big(\frac{w_{u,t}}{\kappa_{t,n} P_t^{\alpha} r_t^{1-\alpha}} \big)^{\epsilon_u}} \Big] = 1 \tag{52}$$

For simplicity we isolate the alternative *skill-specific migration access*

$$\text{AMA}_{S,i} = \sum_{n \in N} \Big[\frac{l_{S,n,0}}{\kappa_{i,n}^{\epsilon_S} \sum_{t \in N} B_t \big(\frac{w_{S,t}}{\kappa_{t,n} P_t^{\alpha} r_t^{1-\alpha}} \big)^{\epsilon_S}} \Big]$$

And lastly the estimatable equation

$$\frac{w_{s,i}^{\epsilon_s}}{w_{u,i}^{\epsilon_u}} = \frac{\eta_i}{1-\eta_i} \Big(\frac{l_{s,i}}{l_{u,i}} \Big)^{-\frac{1}{2\rho}} P_i^{-\frac{\alpha(\epsilon_u - \epsilon_s)}{2\rho}} r_i^{-\frac{(1-\alpha)(\epsilon_u-\epsilon_s)}{2\rho}} \left[\frac{AMA_{s,i}}{AMA_{u,i}} \right]^{-\frac{1}{2\rho}} \tag{53}$$

The first clear difference is that trying to estimate equation (53) we would observe that local price indexes and housing costs do matter in determining the wage ratio of a location. This was not the case in equation (41) (even if we included housing costs in the model described in subsection 3.5). Because the agents responded in the same way to the real wage deflators (such as the price index and the housing price) they had no direct influence on the skill premium. In equation (53), instead, due to the fact that the different agents react differently to variations in the real wage, the difference in elasticity of migration is inherited by the local wage premium. Additionally in this case the simple interpretation of equation (41) as a decomposition of the wage ratio does not hold in equation (53) as the factor captured by the wage ratio includes the difference in migration elasticities with respect to the real wage.

5 Conclusion

The aim of this paper was to introduce an initial, but rigorous, general equilibrium approach which includes different types of agents. The immediate result of this paper is a more rigorous approach to the relationship between migration and income inequality. The model suggests that each regional difference in accessibility to different types of worker contributes to the wage ratio. In other words, it suggests that previous estimates of the effect of the local skill availability and the productivity ratio on the income inequality have been overestimated. If different agents display different elasticities, we also demonstrate that nominal wage deflators, such as the local price index and the cost of rent, do contribute to the wage ratio.

Furthermore, the paper exploits a similar structure of the *skill-specific migration access* to the market access to propose an estimation procedure based on a fixed effect estimation similar to the one run by Redding and Venables (2004). Further research can be extended in this direction by the construction of a better skill-based wage comparative dataset, which would enable to test empirically the results listed above. Theoretically, a more in-depth analysis of the rippling effects of local shocks on other regions can help better evaluate local policies and their consequences. Additionally, the introduction of skill-specific agglomeration spillovers would be a necessary addition in order to capture the full extent of the relationship between the distribution of the economy and the skill of different agents.

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Appendix A: Existence and uniqueness theorem

In this appendix we will provide the full text of the Allen et al. (2016) theorem 3. The paper is, at the time of writing, a draft, but has already been adopted in published papers, such as Monte et al. (2018). Given its usage in published papers we feel confident in invoking it for the proofs of this paper.

Let us consider a generic mathematical gravity framework. There are H vectors of endogenous variables $x^h \in \mathbb{R}^N$ with $h = 1, 2, \ldots, N$ and each vector x^h contains N endogenous variables $x_i^h \in \mathbb{R}$ with $i = 1, 2, \ldots, N$. We can define the corresponding sets Ω_H and Ω_N. Let us consider specifically the set of equations

$$\lambda^k \prod_{h=1}^{H} (x_i^h)^{\beta_{k,h}} = \sum_{j=1}^{N} K_{i,j}^k \left[\prod_{h=1}^{H} (x_j)^{\gamma_{k,h}} \right] \quad i \in \Omega_N;\, k, h \in \Omega_H \tag{54}$$

Where λ^k is an endogenous scalar that balances the two sides of the equation. The exogenous parameters of the model are $K_{i,j}^k$ and the two elasticities $\beta_{k,h}$ and $\gamma_{k,h}$. We additionally denote two H x H matrices $\mathbf{\Gamma}$ and $\mathbf{B}$, whose elements are respectively $(\mathbf{\Gamma})_{k,h} = \gamma_{k,h}$ and $(\mathbf{B})_{k,h} = \beta_{k,h}$

Theorem 1. *Consider the system of equations* (54)*. Assume* $\mathbf{\Gamma}$ *is invertible. Define the matrix* $\mathbf{A} = \mathbf{B}\mathbf{\Gamma}^{-1}$*, the matrix* $\mathbf{A}^p$ *to be the matrix composed by the absolute values of* $\mathbf{A}$*, such that* $(\mathbf{A}^p)_{k,h} = |(\mathbf{A})_{k,h}|$*, and define* $\rho(\mathbf{A}^p)$ *as the largest eigenvalue. Then we have:*

1. *If* $K_{i,j}^h > 0$ *for all* i, j, k*, then there exists a strictly positive solution;*
2. *If* $\rho(\mathbf{A}^p) \leq 1$ *and* $K_{i,j}^h > 0$ *then there exists at most one strictly positive solution(to-scale);*
3. *If* $\rho(\mathbf{A}^p) < 1$ *and* $K_{i,j}^h > 0$ *then the solution can be computed by a simple iterative procedure;*
4. *If* $\rho(\mathbf{A}^p) > 1$ *and all elements of each column of* $\mathbf{A}$ *have the same sign, then there exist a kernel* $K_{i,j}^h \geq 0$ *such that there are multiple strictly positive solutions, i.e. for some* $\{i, j\}$ *set of frictions, the uniqueness conditions above are both necessary and sufficient.*

We will provide a sketch of the first two parts of the proof here, which are the ones I rely the most on. The complete demonstration is available in Allen et al. (2016).

The first part is a consequence of the Brouwer fixed point theorem, the second, instead, can be contructed by contradiction. If one defines two possible solutions, which are not up-to-scale, then they proceed to show that there exist a contradiction with the Collatz-Wielandt formula, if the largest eigenvalue of $\mathbf{B\Gamma^{-1}}$ is less than 1.

Appendix B: List of countries included in the regression

Origin: Angola (AGO), Albania (ALB), United Arab Emirates (ARE), Argentina (ARG), Armenia (ARM), Antigua and Barbuda (ATG), Australia (AUS), Austria (AUT), Azerbaijan (AZE), Burundi (BDI), Belgium (BEL), Benin (BEN), Burkina Faso (BFA), Bangladesh (BGD), Bulgaria (BGR), Bahrain (BHR), Bahamas (BHS), Bosnia and Herzegovina (BIH), Belarus (BLR), Belize (BLZ), Bolivia (BOL), Brazil (BRA), Barbados (BRB), Brunei Darussalam (BRN), Bhutan (BTN), Botswana (BWA), Central African Republic (CAF), Canada (CAN), Switzerland (CHE), Chile (CHL), China (CHN), Côte d'Ivoire (CIV), Cameroon (CMR), Congo (COG), Colombia (COL), Comoros (COM), Cabo Verde (CPV), Costa Rica (CRI), Cuba (CUB), Cyprus (CYP), Czech Republic (CZE), Germany (DEU), Djibouti (DJI), Dominica (DMA), Denmark (DNK), Dominican Republic (DOM), Algeria (DZA), Ecuador (ECU), Egypt (EGY), Eritrea (ERI), Spain (ESP), Estonia (EST), Ethiopia (ETH), Finland (FIN), Fiji (FJI), France (FRA), Micronesia (FSM), Gabon (GAB), Great Britain (GRB), Georgia (GEO), Ghana (GHA), Guinea (GIN), The Gambia (GMB), Guinea-Bissau (GNB), Equatorial Guinea (GNQ), Greece (GRC), Grenada (GRD), Guatemala (GTM), Guyana (GUY), Hong Kong (HKG), Honduras (HND), Croatia (HRV), Haiti (HTI), Hungary (HUN), Indonesia (IDN), India (IND), Ireland (IRL), Iran (IRN), Iraq (IRQ), Iceland (ISL), Israel (ISR), Italy (ITA), Jamaica (JAM), Jordan (JOR), Japan (JPN), Kazakhstan (KAZ), Kenya (KEN), Kyrgyzstan (KGZ), Cambodia (KHM), Kiribati (KIR), Saint Kitts and Nevis (KNA), South Korea (KOR), Kuwait (KWT), Laos (LAO), Lebanon (LBN), Liberia (LBR), Libya (LBY), Saint Lucia (LCA), Sri Lanka (LKA), Lesotho (LSO), Lithuania (LTU), Luxembourg (LUX), Latvia (LVA), Macao (MAC), Morocco (MAR), Moldova (MDA), Madagascar(MDG), Maldives (MDV), Mexico (MEX), North Macedonia (MKD), Mali (MLI), Malta (MLT), Myanmar (MMR), Mongolia (MNG), Mozambique (MOZ), Mauritania (MRT), Mauritius (MUS), Malawi (MWI), Malaysia (MYS), Namibia (NAM), Niger (NER), Nigeria (NGA), Nicaragua (NIC), Netherlands (NLD), Norway (NOR), Nepal (NPL), New Zealand (NZL), Oman (OMN), Pakistan (PAK), Panama (PAN), Peru (PER), Philippines (PHL), Palau (PLW), Papua New Guinea (PNG), Poland (POL), Portugal (PRT), Paraguay (PRY), Qatar (QAT), Russia (RUS), Rwanda (RWA), Saudi Arabia (SAU), Sudan (SDN), Senegal (SEN), Singapore (SGP), Solomon Islands (SLB), Sierra Leone (SLE), El Salvador (SLV), San Marino (SMR), Somalia (SOM), Sao Tome and Principe (STP), Suriname (SUR), Slovakia (SVK), Slovenia (SVN), Sweden (SWE), Eswatini (SWZ), Seychelles (SYC), Syria (SYR), Chad (TCD), Togo (TGO), Thailand (THA), Tajikistan (TJK), Turkmenistan (TKM), Tonga (TON), Trinidad and Tobago (TTO), Tunisia (TUN), Turkey (TUR), Tuvalu (TUV), Taiwan (TWN), Tanzania (TZA), Uganda (UGA), Ukraine (UKR), Uruguay (URY), United States of America (USA), Uzbekistan (UZB), Saint Vincent and Grenadines (VCT), Venezuela (VEN), Vietnam (VNM), Vanuatu (VUT), Samoa (WSM), Yemen

(YEM), South Africa (ZAF), Zambia (ZMB), Zimbabwe (ZWE).

Destination: Australia (AUS), Austria (AUT), Belgium (BEL), Canada (CAN), Switzerland (CHE), Czech Republic (CZE), Germany (DEU), Denmark (DNK), Spain (ESP), Finland (FIN), France (FRA), Great Britain (GRB), Greece (GRC), Hungary (HUN), Ireland (IRL), Iceland (ISL), Italy (ITA), Japan (JPN), South Korea (KOR), Luxembourg (LUX), Mexico (MEX), Netherlands (NDL), Norway (NOR), New Zealand (NZL), Poland (POL), Portugal (PRT), Slovakia (SVK), Sweden (SWE), Turkey (TUR), United States of America (USA), South Africa (ZAF).