Inequality and Current Account Imbalances in a Monetary Union

Heterogeneous heterogeneities in the Eurozone

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A Master's Thesis presented for the degree of
Master's in Economics

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May 2020

Acknowledgements

First of all, I would like to thank my supervisor Prof. Xavier Ragot for his invaluable guidance, support and availability all along this work. This paper would not have come to fruition without him and I am immensely grateful for that. I would also like to express my gratitude to the pedagogical team of the Master's in economics, Claudine Lamaze, Prof. Emeric Henry and Prof. Sergei Guriev, who have provided an ideal study environment during those two years. This Master's Thesis does not only mark the end of two wonderful years at the École Doctorale, but is also the culmination of five years at Sciences Po. The entirety of my curriculum, from two amazing years at the Reims campus, to an exciting third year at Tulane University, and finally two unforgettable years of coursework in the Master's in Economics in Paris, has been dreamlike. The academic development, the friendships made along the way and the fascinating people I met, all allowed me to blossom as a person, as a future researcher and hope-to-be professor. I would like to express my deepest thanks to my friends for their invaluable support during these years. Alexandre, Alexis, Aliette, Charles, Héloïse, Juliette, Léon, Loris, Margaux, Maxime, Nina, Pierre, Taliza, Thomas and Yann, thank you for being such amazing friends. Although this unusual times have not allowed us to spend as much time together as we could have hoped for, my classmates and future brilliant economists, were beyond incredible during those two years of graduate work. I would like to thank each one of them for always being so kindhearted and inspiring. Alessandro, Ananay, Simeon, Tancrède and Valentin, I am especially grateful for the marvellous friendships we developed, which brought a lot of joy to those two years. Last but certainly not least, I would like to thank my family for their unconditional love and support. My Grandma for her unwavering kindness and care, Tom, for being the most supportive brother someone could ever hope for, and my parents, for always believing in me and doing all they can to help me follow my dreams.

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June 2020

Abstract

This paper presents a theoretical and computational framework of idiosyncratic risks and aggregate shocks in a monetary union. The model allows us to design a monetary authority that sets an invariant policy rate, which implements real rigidities in the country-specific interest rates, that in turn creates regional imbalances. This method is applied to the Eurozone and we study the effects of technological and monetary shocks on the aggregate economy. The results indicate that current account imbalances and wealth inequality both stem from the same private savings behavior, which depend on each country's frequency and magnitude of idiosyncratic shocks. Moreover, we find that Eurozone countries do not share the same tools to face aggregate shocks, which causes a heterogeneity of responses to those perturbations. Finally, this paper identifies the benefits of a common monetary policy for the member states and finds that a central bank helps homogenize the effects of aggregate shocks across the region, for better or worse.

Keywords: Heterogeneous agents, inequality, monetary union, current account.
JEL classification: E21, E42, D14, D31, F32.

1 Introduction

The ideology of the European Union is embedded in its motto: "United in Diversity". 27 member states, with as many different social and economic profiles, all agreed to join forces to form a single political and economic union. 19 of those states entered the Eurozone, a currency union designed to centralize monetary policy while coordinating decentralized fiscal authorities. The implications of such a union have been under the

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microscope of many researchers since the seminal work of Mundell (1961). The recent Eurozone debt crisis has taught us that a one-size-fits-all approach to monetary policy is not always suitable for a set of countries with intrinsic characteristics that are too disparate. This feature can be observed through the lens of numerous variables, such as sovereign debt, inequality or current account imbalances. The debt management of each country within the Eurozone ranges from very low debt-to-GDP ratios, like Germany, hovering around 60%, to countries such as Greece with staggering ratios, over 180%. In the same currency union, we can also find the country with the highest current account balance in the world, Germany, and the sixth$^{\text{th}}$ lowest, France (as of 2017). The split does not end here, within-country inequality also has a wide spread across Eurozone countries, with Gini coefficients for wealth inequality ranging from 0.49 (Slovakia) to over 0.79 (Latvia)[1]. Those variables do not seem to be inter-connected at first, but I argue that they all stem from the disparity of savings behaviors across countries. I focus on European imbalances and wealth inequality as two sides of the same coin, both taking roots in micro-level choices regarding wealth management.

The particularity of a currency union is the delegation of the monetary power to a supranational entity that needs to optimize its decisions with respect to every member state's sets of parameters. Evidently, a Central Bank's policy rate is not the only component of each country's risk-free interest rate. In fact, as detailed by Burton and Inoue (1985), the interest rate is affected by country-specific risk. Moreover, the stability of prices, itself ever-changing, yields heterogeneous inflation rates across the Eurozone. The policy rate is the common trunk of risk-free real interest rates, but each country branches out in different directions according to their specificities. Hence, the monetary authority can drive the individual interest rates but cannot uproot their country heterogeneity components. This idiosyncrasy of currency unions, and especially the Eurozone as it is the object of our analysis, translates into a policy of dealing with aggregate crises with a unified monetary response and coordinated fiscal policies. I use this specificity to analyze the effects of aggregate shocks on the Eurozone.

The evident differences between each country of the Eurozone is usually envisioned as macroeconomic heterogeneity: aggregate variables such as the current account, inequalities, debt-to-GDP ratios etc. are compared across countries to account for their disparities. Nonetheless, this aforementioned manifoldness, is birthed by a "heterogeneity of heterogeneities". In fact, if we consider that each country displays disparate macroeconomic variables, we must acknowledge the microeconomic origin of each difference. As I mentioned in the first paragraph, the savings behavior of each country, at a micro-level, is the cradle of inter-country differences. Hence, one must account for within-country

[1] HFCS 2$^{\text{nd}}$ wave survey data, detailed in Section 2.

heterogeneity in order to study the between group. This heterogeneity of heterogeneities can be described as a multiplicity of inherent characteristics for each member state; in this paper, I study how each economy can have a different relationship with income stability and idiosyncratic productivity. I proceed with a country-by-country application of heterogeneous agents modelling, based on the works of Bewley (1986), Huggett (1993) and even more closely, Aiyagari (1994). These models of heterogeneous agents and incomplete markets allow us to reproduce uninsurable risk, as the models display precautionary savings and the number of Arrow-Debreu securities is less than the number of idiosyncratic states. This structure allows us to feature inequalities in our model and I believe that the study of current account imbalances goes hand in hand with this framework.

2 Empirical Evidence

The idea behind heterogeneity in Europe stems from both the study of aggregate variables and how different they are from one country to another, and the analysis of granular micro-level data that feature contrasting behaviors between income groups inside of each country. In order to get a grasp of core and periphery heterogeneity, I study specifically France and Germany (core), Italy and Spain (periphery) and a Eurozone average across all 19 countries. First, the after-tax Gini coefficient is computed using the expression:

$$\mathcal{G}^W = \frac{2}{n}\left(\sum_{i=1}^{n} iW_i\right)\left(\sum_{i=1}^{n} W_i\right)^{-1} - \frac{n+1}{n} \tag{1}$$

where n is the number of households in the economy, W_i is the wealth of each agent, sorted in ascending order. Hence, we have $\mathcal{G}^W \in [0,1]$ with 0 indicating perfect equality (everybody owns the same amount of wealth) and 1 perfect inequality (one person holds the total wealth). I use the 2^{nd} wave Household Finance and Consumption Survey to estimate the Gini coefficient of after-tax wealth. The data were collected between 2012 and 2015 depending on the country.

Then, I study the set of flat tax rates $\{\lambda, \tau\}$, respectively the capital revenue and wage taxes. For the sake of simplicity of the model, I am interested in taxes net of redistribution that correspond to the financing of public goods. The data I use to obtain these measures come from Trabandt and Uhlig (2011).The financial markets are also a key feature of this paper and must be empirically grounded. I use data from the European Central Bank Statistical Data Warehouse to get $\mathcal{R}$, the average from 2010 to 2016 of nominal annualized interest rate on 10-year maturity government bonds. I also use the OECD IPC inflation indicator to compute p, the average from 2010 to 2016 of inflation rates. I obtain the real quarterly risk-free interest rate R with the following identity:

$$R = \left(\frac{\mathcal{R}}{p}\right)^{\frac{1}{4}} \tag{2}$$

Finally, the net foreign assets, from which we derive the current account, are also a central element of this work, and it comes from World Bank net foreign assets balance indicator (in current LCU), averaged from 2010 to 2016 to obtain NFA, defined as the net foreign assets over quarterly GDP[2].

Table 1: Empirical Values for the Eurozone

	$\mathcal{G}^W$	λ	τ	$\mathcal{R}$	p	R	NFA
France	0.6766	0.35	0.46	1.005	1.0026	1.0024	1.087
Germany	0.7594	0.23	0.41	1.0036	1.0031	1.0005	2.288
Italy	0.6025	0.34	0.47	1.009	1.0031	1.0058	-0.099
Spain	0.5975	0.30	0.36	1.009	1.0028	1.0063	-0.491

Note that in Table 1, $\mathcal{R}$ and p are also expressed in quarterly values. Here, France and Germany are the core countries, Italy and Spain represent the periphery. First, we can observe that France and Germany, although pertaining to the same group, have very different characteristics. Germany features more wealth inequality than France, has a lighter fiscal regime, a lower real interest rate and a higher current account balance. France having one of the heaviest fiscal system in the Eurozone and Germany being at the opposite, especially concerning taxes on capital gains, can account for many disparities in the data. When we look at Spain and Italy, both are similar in terms of inequalities, capital tax regimes and interest rates. The only significant differences lie within the labor tax, which is much higher in Italy, and the net foreign assets are lower in Spain. From these data, there seems to be a negative correlation between real interest rate and the Gini coefficient for wealth, where one could expect a positive relationship. In fact, a higher interest rate means less access to borrowing, which intuitively points to an increase in inequality, as the richer households benefit from a high interest rate and the poorer income groups become more easily credit constrained. Nonetheless, the micro-based inequality is a complex object and can only be studied through an in-depth analysis of low-level savings behaviors. The net foreign assets over GDP, on the other hand, seem to be negatively correlated with interest rates, as the core countries display positive values of NFA, while the periphery is in a negative territory. Once again, this variable seems to emerge from savings choices in each country and might depend on a multitude of parameters, such as

[2] Here the GDP is obtained with the World Bank GDP in current LCU, averaged from 2010 to 2016, and dividing the nominal NFA by the nominal GDP therefore yields a real NFA/GDP.

the distribution of agents' wealth in the economy, the cornerstone of inequalities. We can also note that these country all display different current account balances in the data, as it is defined by the change in net foreign assets over time, which has a high variance across these countries.

3 Literature Review

Micro-level Inequality has a large spectrum of applications in macroeconomic models. In order to implement it, one must diverge from Representative Agents (RA) model and build a Heterogeneous Agents (HA) model. HA models have the benefit of capturing household-specific characteristics and can output distributions of assets in the economy, which is required when dealing with micro-level inequality. This household heterogeneity can be modelled as stochastic endowments as in Bewley (1986), as different preferences across households as in Chan and Kogan (2002) or shocks on the employment status as in Hagedorn et al. (2016). There are evidently numerous ways to model between-agents differences but does it do a better job at matching aggregate fluctuations than RA models? Den Haan (2001) addresses this question and proves that the number of idiosyncratic states matters for the macroeconomy, at the expense of simplicity and computational speed. Hence, in order to conduct theoretical work that can mimic empirical features, we must consider the use of a large number of states. Moreover, an important component of the study of inequality in microfounded models is the availability of granular data on the savings behavior of each agent. In this paper, as detailed in Section 2, I use the HFCS (2^{nd} wave) to analyze the Eurozone households and extract Gini coefficients for wealth inequalities. The use of low-level data is crucial to the accurate study of aggregate variables in a framework of HA models, as precise individual heterogeneity is a paramount feature to analyze macroeconomic differences.

Current Account Imbalances have been extensively covered in the literature of international macroeconomics but this paper does not aim at improving our understanding of the current account in multi-country models. In fact, the model used in this paper has closed economy features. In an autarky model, the concept of current account is irrelevant when modelled as the net exports, but it can also be envisioned as the difference between capital supply (household savings) and capital demand (factors of production of a firm) in the economy. An endogenous interest rate usually guarantees a financial markets equilibrium and effectively sets the demand for bonds equal to the supply of bonds. Nonetheless, in a monetary union, as the interest rate is not freely set by the economy, frictions arise and the bonds market equilibrium cannot occur if the risk-free rate is higher or lower than the market clearing one. I exploit this feature in a closed economy by setting an exogenous and a semi-exogenous interest rate to generate nonzero net foreign assets and therefore current account imbalances. This approach is detailed in Section 4.1. Thus, this method does not build on the international macroeconomics modelling literature but rather mimics empirical data in a closed economy context and produces a financial market that clears internationally but not domestically. The purpose of featuring a current account in this closed economy model is to put its variations under the microscope and try to find its roots in binned savings behavior, that should also be the origin of aggregate inequalities. The link between inequality and the current account balance has been studied in the context of HA models, namely by Al-Hussami and Remesal (2012), who find that an increase in income inequality is negatively correlated with the current account balance. Ranciere et al. (2012) also investigate the relationship between inequality and current account imbalances and find that when an increase in income inequality arises, loans to workers from domestic and foreign investor boosts aggregate demand and results in an increase in current account deficits. These findings support the theory that the current account balance and inequality display negative comovements and might share a common origin.

Monetary Unions are often analyzed as having a core and a periphery. The core is composed of country of high financial stability and less financially stable member states populate the periphery. The findings of Gilchrist et al. (2018) indicate that core countries can face adverse financial shocks by lowering markups and undercutting financially constrained foreign competitors while the periphery has to raise markups to maintain current cashflows at the expense of market shares. The difference in behavior here lies in the degree of financial capacity of the firms in each country. This consolidates the idea that core and periphery heterogeneity is a result of low-level savings management. The relationship between inequality and European imbalances in a monetary union is also investigated in Marzinotto (2016). Their results show that the increase in money supply in Europe since the 1990s relaxed the collateral constraints for illiquid households and had no effect on the other income groups, which resulted in worse current account balances for the most unequal countries, here the periphery. The core countries have a lesser fraction of credit constrained households, and therefore mitigate the impact of this increase in money supply and end up with better external positions than the periphery, as the private debt burdens are lighter for them. This result corroborates the idea that there are "losers" and "winners" in a monetary union, in the sense that not all countries benefit from being in a currency union when common shocks affect the whole region.

4 Model

Here I build the model from the ground up, set-up and solve the dynamic problem and describe the equilibrium structure. The model is based on the workhorse heterogeneous agents model with capital described by Aiyagari (1994). Time is indexed by $t \in [0, \infty)$, there is a continuum of agents of measure 1 distributed on the interval $\mathcal{L}$ according to a measure $\ell(\cdot)$. The variables pertaining to idiosyncratic risks feature a superscript i.

4.1 Model Environment

Preferences. Households derive utility each period from private consumption c_t. The utility function, denoted by $U(c_t)$ is of the CRRA isoelastic type:

$$U(c_t) = \frac{c_t^{1-\gamma} - 1}{1-\gamma} \tag{3}$$

where $\gamma > 0$ is the coefficient of relative risk aversion. Note that for the special case of $\gamma = 1$, l'Hôspital's Rule allows us to define $U(c_t) = \log(c_t)$. This utility structure allows us to derive an analytical solution to the endogenous grid method quickly, which improves computational performances, but this model does not rely specifically on the CRRA utility. The agents do not derive disutility from working in this model as the labor supply is constant and exogenous, it is denoted $\bar{l}$.
The intertemporal utility of agents is given by:

$$\sum_{t=0}^{\infty} \beta^t U(c_t) \tag{4}$$

with $\beta \in (0, 1)$ the constant discount factor, relating utility derived today to future streams of utility.

Idiosyncratic risk. In this economy, the heterogeneity of agents stems from different productivity endowments. Their idiosyncratic productivity takes values $Y_t \in \mathcal{Y}$, where $\mathcal{Y} \subset \mathbb{R}_+$ is a finite set. The stochastic productivity follows an AR(1) process:

$$y_t = \rho y_{t-1} + \varepsilon_t \tag{5}$$

where $y_t = \log(Y_t), \rho \in (0, 1)$ and $\varepsilon_t \overset{iid}{\sim} \mathcal{N}(0, \sigma_\varepsilon^2)$. We can interpret ρ as the persistence of productivity and ε_t its dispersion. This law of motion for y_t is covariance stationary, has mean zero and variance:

$$\sigma_y^2 = \frac{\sigma_\varepsilon^2}{1-\rho^2} \tag{6}$$

For the sake of computational performance, I discretize this continuous stochastic process using the procedure described in Baxter et al. (1990). This method has the advantage of being highly accurate when dealing with high values of ρ, near the unit root, which is a feature this model displays. The output of this process is a symmetric evenly-spaced state space $Y = \{y_1, \dots, y_N\}$ where $-y_1 = y_N$ and N is the chosen number of idiosyncratic states, and a Markov chain defined recursively by:

$$\Theta_2 = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix} \tag{7}$$

$$\Theta_N = p \begin{bmatrix} \Theta_{N-1} & \mathbf{0} \\ \mathbf{0'} & 0 \end{bmatrix} + (1-p) \begin{bmatrix} \mathbf{0} & \Theta_{N-1} \\ 0 & \mathbf{0'} \end{bmatrix} + (1-q) \begin{bmatrix} \mathbf{0'} & 0 \\ \Theta_{N-1} & \mathbf{0} \end{bmatrix} + q \begin{bmatrix} 0 & \mathbf{0'} \\ \mathbf{0} & \Theta_{N-1} \end{bmatrix} \tag{8}$$

Then we divide all interior rows of Θ_N by two, so that all rows sum to one. The result is the following Markov transition matrix:

$$\underset{N \times N}{\Pi} = \begin{bmatrix} \pi_{11} & \dots & \pi_{1N} \\ \vdots & \ddots & \vdots \\ \pi_{N1} & \dots & \pi_{NN} \end{bmatrix} \tag{9}$$

where each element π_{ij} represents the probability of transitioning to state j when current state is i.

Firms. The good consumed by the agents is produced by a single representative firm that transforms inputs K_t, the capital and L_t the labor supply into output Y_t. As labor supply is exogenous in this model, we assume the idiosyncratic productivity endowment of agents does not affect the production of the representative firm. Moreover, the technology parameter Z_t is also considered exogenous and constant in this model[3]. The firm is endowed with Cobb-Douglas technology, with elasticity of capital $\alpha \in [0, 1]$:

$$Y_t = F(K_t, \bar{L}) = Z K_t^{\alpha} \bar{L}^{1-\alpha} \tag{10}$$

The before-tax factor prices $\{w_t, r_t\}$, respectively the wage rate and the interest rate, are given by the marginal productivities of capital and labor:

$$w_t = Z(1-\alpha)\left(\frac{K_t}{\bar{L}}\right)^{\alpha} \tag{11}$$

$$r_t = Z\alpha\left(\frac{K_t}{\bar{L}}\right)^{\alpha-1} \tag{12}$$

Fiscal authority. The government finances a public good by levying capital and labor taxes. The taxes are denoted λ for the capital tax, τ for the wage tax and are exogenous

[3]In Section 5 we relax the assumption of Z_t being constant by producing unanticipated shocks to technology.

and constant over time. a_t^i is the number of assets saved by agent i at the beginning of period t and $R_t = 1 + r_t$ is the gross interest rate on the asset. The government budget is the following:

$$T_t = \underbrace{(R_t - 1)\lambda \int_{i \in \mathcal{L}} a_t^i \ell(\mathrm{d}i)}_{\text{tax revenue from capital gains}} + \underbrace{w_t \bar{l} \tau \int_{i \in \mathcal{L}} y_t^i \ell(\mathrm{d}i)}_{\text{tax revenue from wages}} \tag{13}$$

There is no redistribution in this model but these taxes are fundamental to our understanding of savings behavior across the wealth distribution. In fact, the way agents pass on the impact of the taxes on their savings decision is crucial to the structure of inequalities in this model.

Monetary authority. There is no monetary authority *per se* in this framework, as there is no choice of a nominal interest rate nor inflation. Nonetheless, to model a monetary union, we set a common, exogenous R_t that is constant for all periods, but can change during an unanticipated aggregate shock in one of three ways:

1. It can follow a technology shock and then return to its steady-state value following an AR(1) process with no noise;
2. It can stay invariant for the duration of the shock;
3. It can become endogenous after a shock, which represents exiting the currency union.

These three scenarios provide an understanding of the possible transitional dynamics of risk-free interest rates in a monetary union when a shock occurs. These deviations from the stationary equilibrium are explored in details in Section 5, until then, R_t can be considered constant.

Flow budget constraints. The agents can save a chosen amount $a_{t+1} \in [\phi, a_{\max}] \equiv A$ in a risk-free bond, where ϕ is the credit constraint, that prevents the agents from borrowing too much and $a_{\max}$ is the maximum amount of assets held by one household, guaranteeing that the maximum amount of savings an individual holds is not too high. The bond purchased at the end of period $t-1$ pays a gross interest rate $R_t = 1 + r_t$. The agent also chooses a consumption $c_t > 0$ and faces taxes $\{\lambda, \tau\}$. The flow budget constraints are thus:

$$\begin{cases} c_t^i + a_{t+1}^i & \leq ((R_t - 1)(1 - \lambda) + 1)a_t^i + w_t \bar{l}(1 - \tau) y_t^i \\ a_{t+1}^i & \geq \phi \end{cases} \tag{14}$$

Evidently, the first budget constraint binds as the agent will want to either save or consume as the utility function is strictly increasing in c. The second constraint will bind for credit constrained household and will be slacked for the rest of the agents. From this system of constraints, we can deduce the following set of equations:

$$\mathcal{I}^i_{t,0} = (R_t - 1)a^i_t + w_t y^i_t \bar{l} \tag{15}$$
$$\mathcal{I}^i_{t,1} = (R_t - 1)(1-\lambda)a^i_t + w_t(1-\tau) y^i_t \bar{l} \tag{16}$$
$$\mathcal{W}^i_{t,0} = R_t a^i_t + w_t \bar{l} y^i_t \tag{17}$$
$$\mathcal{W}^i_{t,1} = ((R_t - 1)(1-\lambda)+1)a^i_t + w_t \bar{l}(1-\tau) y^i_t \tag{18}$$

where a subscript 0 indicates before-tax and 1, after-tax. $\mathcal{I}^i_{t,0}$ is the before-tax income flow of agent i, $\mathcal{I}^i_{t,1}$ is the after-tax income, $\mathcal{W}^i_{t,0}$ is the before-tax end-of-period wealth and $\mathcal{W}^i_{t,1}$ is the after-tax end-of-period wealth. The aggregate values corresponding to these variables are as follows:

$$\mathcal{I}_{t,0} = \underbrace{(R_t - 1)\int_{i\in\mathcal{L}} a^i_t \ell(\mathrm{d}i)}_{\text{before-tax capital gains}} + \underbrace{w_t \bar{l} \int_{i\in\mathcal{L}} y^i_t \ell(\mathrm{d}i)}_{\text{before-tax labor income}} \tag{19}$$

$$\mathcal{I}_{t,1} = \underbrace{(R_t - 1)(1-\lambda)\int_{i\in\mathcal{L}} a^i_t \ell(\mathrm{d}i)}_{\text{after-tax capital gains}} + \underbrace{w_t \bar{l}(1-\tau) \int_{i\in\mathcal{L}} y^i_t \ell(\mathrm{d}i)}_{\text{after-tax labor income}} \tag{20}$$

$$\mathcal{W}_{t,0} = \underbrace{R_t \int_{i\in\mathcal{L}} a^i_t \ell(\mathrm{d}i)}_{\text{before-tax wealth}} + \underbrace{w_t \bar{l} \int_{i\in\mathcal{L}} y^i_t \ell(\mathrm{d}i)}_{\text{before-tax labor income}} - \underbrace{\int_{i\in\mathcal{L}} c^i_t \ell(\mathrm{d}i)}_{\text{consumption}} \tag{21}$$

$$\mathcal{W}_{t,1} = \underbrace{((R_t - 1)(1-\lambda)+1)\int_{i\in\mathcal{L}} a^i_t \ell(\mathrm{d}i)}_{\text{after-tax wealth}} + \underbrace{w_t \bar{l}(1-\tau) \int_{i\in\mathcal{L}} y^i_t \ell(\mathrm{d}i)}_{\text{after-tax labor income}} - \underbrace{\int_{i\in\mathcal{L}} c^i_t \ell(\mathrm{d}i)}_{\text{consumption}} \tag{22}$$

These variables, at the aggregate and individual levels, are paramount to the analysis of inequality carried out in this paper, as it links low-level optimal savings decisions with distributions of wealth and income and subsequently their aggregate values, summed over the distribution.

4.2 Dynamic Programming

Now that the set-up of the model has been laid out, I solve for the consumption Euler equation through dynamic programming. Here, an agent $i \in \mathcal{L}$ wants to maximize the expected sum of their intertemporal utility, subject to their flow budget constraints:

$$\max_{\{c_t^i, a_{t+1}^i\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i) \tag{23}$$

$$\text{s.t.} \begin{cases} c_t^i + a_{t+1}^i & = ((R_t - 1)(1-\lambda) + 1)a_t^i + w_t \bar{l}(1-\tau) y_t^i \\ a_{t+1}^i & \geq \phi \end{cases} \tag{24}$$

The agent has expectations over the value of y_t^i in the next period, as it is subject to idiosyncratic shocks. We can set-up the Bellman equation with value function $V(a_t^i, y_t^i)$ rewriting the expectation as a sum over probabilities for each possible future state. Let $\mathbb{P}(y_{t+1}^i | y_t^i)$ be the probability $\Pi_{i,j}$ of going from idiosyncratic state i to state j, for each possible value of y_{t+1}^i.

$$V(a_t^i, y_t^i) = \max_{\{c_t^i, a_{t+1}^i\}_{t=0}^{\infty}} u(c_t^i) + \beta \sum_{y_{t+1} \in Y} \mathbb{P}(y_{t+1}^i | y_t^i) V(a_{t+1}^i, y_{t+1}^i) \tag{25}$$

We now define $\nu_t^i > 0$ as the Lagrange multiplier associated with the credit constraint. From the Bellman equation, we can obtain the consumption Euler equation:

$$u'(c_t^i) \leq \beta((R_t - 1)(1-\lambda) + 1) \sum_{y_{t+1} \in Y} \mathbb{P}(y_{t+1}^i | y_t^i) u'(c_{t+1}^i) + \nu_t^i \tag{26}$$

$$u'(c_t^i) \geq \beta((R_t - 1)(1-\lambda) + 1) \sum_{y_{t+1} \in Y} \mathbb{P}(y_{t+1}^i | y_t^i) u'(c_{t+1}^i) \tag{27}$$

This expression relates the marginal utility of consumption today to the marginal utility of consumption tomorrow. The Euler equation in (27) holds with equality when $a_{t+1}^i > \phi$ and inequality holds strictly when the agent is at the credit constraint. This means that the benefit from consuming one additional unit of good today is higher than the discounted benefit from consuming one additional unit of good tomorrow for the poorest households. This fact leads to full consumption of the agent's resources at the constraint. Now that the Euler equation is defined, the next step would be to find policy function for the control variables $c_t^i(a_t^i, y_t^i)$ and $a_{t+1}^i(a_t^i, y_t^i)$, which yield invariant functions of the optimal choice with respect to state variables. These functions are derived computationally using the endogenous grid method.

4.3 Stationary Equilibrium

Now that we have the Euler equation, we need to turn to aggregate variables and check the market clearing conditions.

Firm optimality. As stated in Section 4.1, the good-producing firm chooses K_t and the factor prices correspond to:

$$r_t = F_{K_t}(K_t, \bar{L}) \tag{28}$$
$$w_t = F_{\bar{L}}(K_t, \bar{L}) \tag{29}$$

This condition guarantees that the firm makes optimal decisions with factor prices equal to the their related marginal productivities. This result comes from the firm's profit maximization problem.

Labor market. In this model, the household labor supply $\bar{l}$ is exogenous, hence it is easy to verify:

$$\bar{L} = \int_{i\in\mathcal{L}} \bar{l}\ell(\mathrm{d}i) \tag{30}$$
$$= \bar{l} \tag{31}$$

as the continuum of agents is of measure 1. Therefore, the labor demanded by firms is exactly equal to the household supplied labor. The labor market clears.

Goods market. The total production in the economy $Y_t = F(K_t, \bar{L})$ should be equal to the sum of private consumption, C_t, government tax revenues (in a model with no redistribution), T_t, and investment I_t if the model had all its markets clear domestically. In this model, however, the interest rate is exogenous, which leads to a positive current account balance, as firms seek foreign investments or agents save in foreign bonds. This results in two identities: a naive domestic view that does not account for a nonzero current account balance, and a realistic international one that incorporates CA_t in its equilibrium. Investment is defined as $I_t = \delta K_t$ with δ the capital depreciation and the current account balance is defined as $CA_t = (s_t - I_t)$ with s_t the private savings for the quarter[4]. Thus we have:

$$Y_t \neq \underbrace{\int_{i\in\mathcal{L}} c_t^i(a_t^i, y_t^i)\ell(\mathrm{d}i)}_{C_t} + \underbrace{(R_t - 1)\lambda \int_{i\in\mathcal{L}} a_t^i \ell(\mathrm{d}i) + w_t \bar{l}\tau \int_{i\in\mathcal{L}} y_t^i \ell(\mathrm{d}i)}_{T_t} + \underbrace{\delta K_t}_{I_t} \tag{32}$$
$$Y_t = C_t + T_t + s_t \tag{33}$$

As this model is outside of a purely domestic framework due to the exogenous interest rate, we must have $s_t \neq I_t$. The portion s_t is defined as what is left of the production after

[4]Note that this expression for CA_t is analogous to the change in net foreign assets over the quarter, or the interests paid on the NFA if in equilibrium.

removing the private and public consumption. If $Y_t - C_t - T - t > I_t$, then agents saved more than firms invested during the quarter, leading to an increase in net foreign assets, and therefore a positive current account balance. If $Y_t - C_t - T_t < I_t$, firms invested more than agents saved during the period, inducing a decrease in net foreign assets and a negative current account balance.

Financial Market. The asset market clearing relies on an equilibrium R_t for which the bond demand meets the supply. The demand is represented by K_t as the quantity of risk-free financial instruments emitted by the firm in order to finance its capital. The supply is the sum of agents' savings over the distribution. The financial market clears for a specific price process r_t, but as specified in Section 4.1, the monetary authority sets an exogenous interest rate that could be higher or lower than the optimal endogenous rate denoted $r_t^\star$. If we have $r_t > r_t^\star$, the demand for capital K_D will decrease and the supply K_S (or S_t) will increase compared to the scenario with an endogenous interest rate. A high r_t is associated with more interests to pay for the firms and also more interests to earn for the households. A $r_t < r_t^\star$ yields a higher K_D and a lower K_S. In any case, when $K_S \neq K_D \Rightarrow |NFA| > 0$ with NFA the net foreign assets. In the context of this model, the financial market not clearing is not an obstacle to a stationary equilibrium, as we can interpret it as the financial market not clearing domestically, but the excess demand (supply) meeting foreign supply (demand) and the market clearing in an open-economy context. Hence, in equilibrium, we have $|NFA| > 0$ and the current account balance is defined as the interests paid or received on the exterior position. It is thus given by $CA = -(R-1)NFA$ as a positive (negative) equilibrium NFA leads to inward (outward) capital flows. Hence in this model, we have the following domestic and international identities:

$$K_t \neq \int_{i \in \mathcal{L}} a_{t+1}^i(a_t^i, y_t^i)\ell(\mathrm{d}i) \tag{34}$$

$$K_t = \int_{i \in \mathcal{L}} a_{t+1}^i(a_t^i, y_t^i)\ell(\mathrm{d}i) - NFA_t \tag{35}$$

Here the net foreign assets, defined as $K_S - K_D$, ensure that this expression holds in an international trade context.

Stationary distribution. The distribution of agents in the economy should be invariant in equilibrium. There must be an optimal distribution $\ell^\star(a_t, y_t)$, function of the state variables, such that $\ell_t^\star(a_t, y_t) = \ell_{t+1}^\star(a_t, y_t)$. It is given by:

$$\ell^\star(a_t, y_t) = \sum_{y_t \in Y} \int_{a_t \in \Omega(a_{t+1}, y_t)} \sum_{y_{t+1} \in Y} \mathbb{P}(y_{t+1}|y_t)\ell(\mathrm{d}a_t) \tag{36}$$

where $\Omega(a_{t+1}, y_t) = \{a_t : a_{t+1} = a_{t+1}(a_t, y_t)\}$.

Definition (Open-economy recursive equilibrium). *An open-economy recursive equilibrium is a value function $V : Y \times A \to \mathbb{R}$; policy functions for the agents $a_{t+1} : Y \times A \to \mathbb{R}$, $c_t : Y \times A \to \mathbb{R}_+$; a firm's choice of K_t and L_t (in this case $L_t = \bar{L}$ and is not chosen); price processes $\{r_t, y_t\}$; a stationary distribution $\ell^\star(a_t, y_t)$ such that:*

1. *given prices $\{r_t, w_t\}$, the policy functions solve the agent's optimization problem stated in (23)-(24);*
2. *labor market clears domestically at all dates: $\forall t > 0$, (30)-(31) hold;*
3. *prices $\{r_t, w_t\}$ are optimally chosen by the firm at all dates: $\forall t > 0$, (28)-(29) hold;*
4. *financial and goods markets do not clear domestically, generating a net foreign assets imbalance, but clear internationally at all dates: $\forall t > 0$, (32)-(35) hold;*
5. *the stationary distribution holds for every possible state (assets and idiosyncratic productivity) at all dates: $\forall a_t, y_t \in A \times Y$, and $\forall t > 0$ (36) holds.*

In this paper, the aim is to analyze the distribution of wealth and understand its link to inequality and European imbalances. An open-economy recursive equilibrium is therefore required in order to investigate the dynamics of savings behavior across the stationary distribution. Now, the model is simulated at its equilibrium and the effects of unanticipated aggregate shocks are analyzed in order to investigate different transition dynamics of the core and periphery.

5 Model Simulations

The model is solved using the endogenous grid method in Julia, where the output is a set of policy functions $\{c_t(a_t, y_t), a_{t+1}(a_t, y_t)\}$, a stationary distribution $\ell^\star(a_t, y_t)$, an optimal firm choice of K_t and price processes $\{r_t, y_t\}$ that satisfy an open-economy recursive equilibrium $\forall a_t, y_t \in A \times Y$ and $\forall t > 0$.

5.1 Calibration

The calibration is a set of parameters:

$$\mathcal{P} = \{R, \beta, \alpha, \delta, \gamma, \rho, \sigma_\varepsilon, \bar{l}, \lambda, \tau, \phi, a_{\max}, na, ns\} \tag{37}$$

where na is the number of possible values of individual savings exponentially spaced and ns is the number of idiosyncratic states. A subset of $\mathcal{P}$ is heterogeneously calibrated for each country, while the rest is common to all calibrations. The invariant parameters take

Table 2: Homogeneous Parameters

β	α	δ	γ	$\bar{l}$	ϕ	$a_{\max}$	na	ns
0.99	0.36	0.025	1.5	1	10^{-9}	1000	200	5

the following values:
The model is estimated in quarterly format, so β is chosen to match $\beta R < 1, \forall R$ across countries, α is the elasticity of capital, δ is the capital depreciation, chosen to match 10% annually, γ is the coefficient of relative risk aversion, which yields the standard CRRA utility function and not its log form, $\bar{l}$ is the exogenous supply of labor, set to 1 to be equal to the measure of agents in the economy, ϕ is the number of assets associated to the credit constraint, which is slightly higher than zero for computational purposes, $a_{\max}$ is the maximum number of assets possible corresponding to the last grid point, na is the number of asset grid points, chosen for both accuracy and speed and ns is the number of states, which is high enough so that the disparities between idiosyncratic states are significant but do not hinder computational speed.

The rest of $\mathcal{P}$ is calibrated heterogeneously for each country:

Table 3: Heterogeneous Parameters

	R	ρ	σ_ε	λ	τ
France	1.0024	0.991	0.174	0.35	0.46
Germany	1.0005	0.996	0.168	0.23	0.41
Italy	1.0058	0.985	0.159	0.34	0.47
Spain	1.0063	0.983	0.139	0.30	0.36

As detailed in Section 2, R is the quarterly risk-free real interest rate extracted from OECD and ECB data; λ and τ are respectively the capital gains tax and the wage tax, net of redistribution, as estimated in Trabandt and Uhlig (2011). ρ is the persistence of idiosyncratic productivity defined in Section 4 as the autocorrelation of y_t. σ_ε is the standard deviation of the idiosyncratic shock, interpreted as the dispersion of states, also mentioned in Section 4. It describes the spacing of each discrete value of Y. Both those parameters are used to match the after-tax wealth Gini coefficient and the net foreign assets of each country, using the data already discussed in Section 2. The matching algorithm is presented in Appendix A.

We can notice that the persistence of productivity is higher in core countries than in periphery countries, which can lead to higher inequalities, as individuals experience more dependency on their past values of y_t and a low value is thus more punishing in the future. The dispersion of productivity is notably higher in France and Germany, highlighting the differences between each income group, and considerably lower in the periphery and especially Spain, showing more homogeneity between idiosyncratic states. Those parameters are used in the simulation to generate a discrete set of possible values of $y_t \in Y$, and a transition matrix Π as described in Section 4.

5.2 Equilibrium Values

After running the simulation, the model is solved in an open-economy recursive equilibrium. The model is estimated for each of our five countries of interest and the results presented are for the core and periphery for the equilibrium values.

Policy functions. The consumption and savings policy functions relate current asset holdings and idiosyncratic productivity to optimal future consumption and savings decisions. Here the consumption policy function is expressed as an optimal choice $c_t(a_t, y_t)$ for all values of $a_t \in A$, on the x-axis as values of assets available on the grid, and all $y_t \in Y$, as different curves each representing one of ns states.

In Figure 1, we can see from these functions that the heterogeneity of the consumption of each idiosyncratic group is much more apparent in core countries than in the periphery and especially in Germany. The households with asset holdings close to zero are at the credit constraint. Due to the expression of the after-tax wealth as defined in (22), the households with a high idiosyncratic productivity still earn high labor income and can still have a relatively high consumption at the credit constraint. In fact, these agents earn high labor income but do not have any asset holdings. On the other hand, low productivity households at the credit constraint can only enjoy consumption close to zero, due to their labor income being minimal. The curves also appear to be flatter for core countries than for the periphery. The additional consumption optimally chosen for a marginally higher value of asset holdings is lower in France and Germany than in Spain or Italy. We can also witness a higher curvature of the policy functions near the credit constraint for the periphery countries. This relates to the marginal propensity to consume of each household, given their state and beginning of period savings.

Figure 2 shows the relationship between marginal propensity to consume and asset grid points[5]. The MPC is defined as $\frac{\partial c_t}{\partial a_t}(a_t, y_y)$, the partial derivative of the consumption

[5] For the sake of clarity, the grid points are presented instead of the asset values.

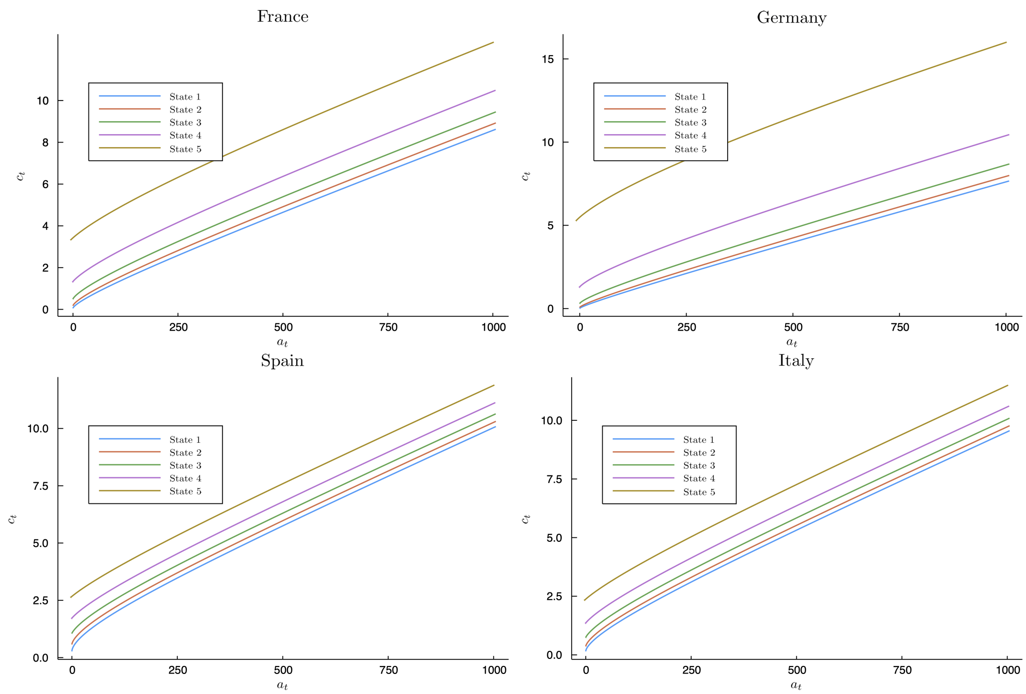


Figure 1: Policy functions for Consumption

policy rule with respect to the assets held at t. A value of 1 means that the entirety of the additional wealth is consumed, and a value of 0 means that an increase in income does not trigger an increase in consumption. Values above 1 are purely computational inaccuracies and should be regarded as equal, or close, to 1. We can see that in core countries, only the households with the lowest idiosyncratic productivity have a marginal propensity to consume close to 1, whereas in Spain, the two lowest productivity levels consume all their increase in wealth when they are at or near the credit constraint. Moreover, the MPC for the second lowest level of productivity in Italy is also higher than in core countries. The rest of the states generally display a low and similar MPC. A clear result drawn from these graphs is that individuals with the lowest idiosyncratic productivity generally consume all their wealth when near or at the credit constraint. This constitutes a poverty trap as they are living hand-to-mouth and cannot save in order to escape it as their income is too low to both consume and save and therefore, consuming becomes the priority. The fact that this is also the case for the second lowest idiosyncratic level in Spain shows the evident disparity in micro-level consumption (and consequently savings) decisions between the core and periphery of the Eurozone.

Now, the savings policy function relates to the optimal choice of $a_{t+1}(a_t, y_t)$ for all values of $a_t \in A$ on the x-axis, and for all states $y_t \in Y$ represented, as before, as differently

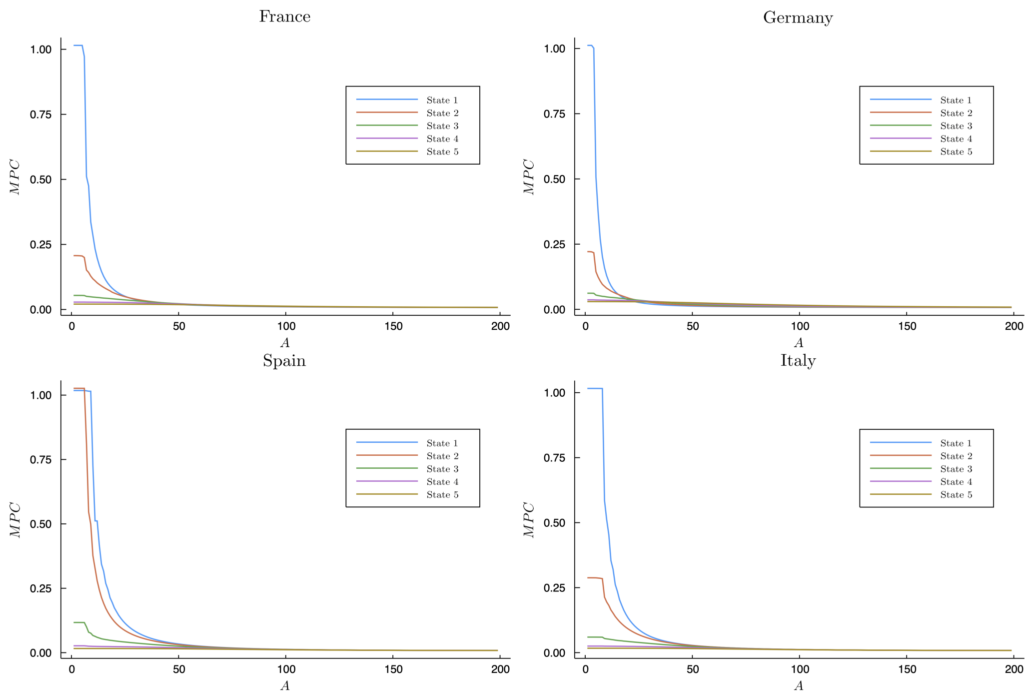


Figure 2: Marginal Propensity to Consume

colored curves. The y-axis in the graph represents the difference between end-of-period assets and beginning of period assets. A positive value means that the agent saved more today than yesterday, whereas a negative value represents dissavings between yesterday and tomorrow which means that a greater proportion of the available wealth goes to consumption. When agents have a positive value of the difference between savings today and yesterday, they will accumulate wealth until they reach the point where it becomes optimal to stop saving more each time and dissave. This saddle point is the maximum amount of savings a household with their idiosyncratic productivity can hold. Following this logic, the highest productivity group should meet this saddle point for $a_t = \bar{a}$, where $\bar{a}$ is the extremum of the ergodic set and no higher value of a_t is attainable in equilibrium.

In Figure 3, the dashed line represents the saddle point, defining the limit of the ergodic set. The households with the lowest level of idiosyncratic productivity never save and are thus trapped in at the saddle point. For the states higher than one but lower than four, the agents start dissaving very quickly and get stuck at a low level of wealth, as consumption becomes more important than savings after this point. The two most productive states can accumulate wealth to a substantially higher amount than the other states. The fifth state, especially, holds a considerably higher wealth before starting to dissave than the other states. This means that only a transition to a very high idiosyn-

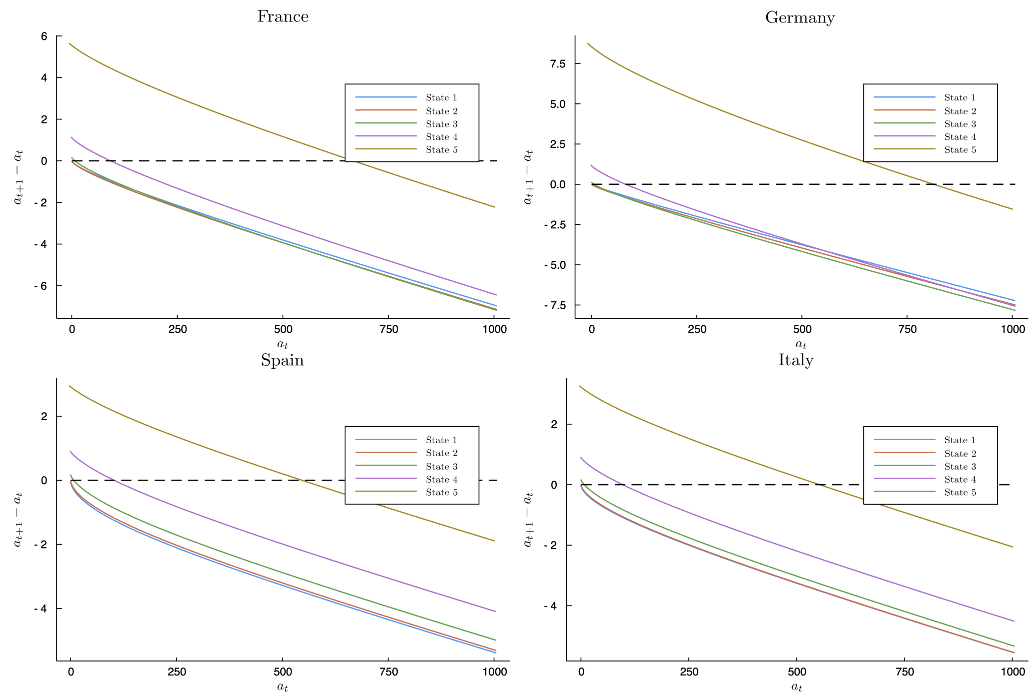


Figure 3: Policy functions for Savings

cratic productivity can translate to a path of wealth accumulation. As the persistence of income is near the unit root, the matrix Π has very high values throughout its diagonal, which means that the probability of going from a bad state to the closest better state is low and the probability of moving from one bad state to a further better state is not significant. Hence, a household in a bad state has very little chance of getting on a path of wealth accumulation and will most likely be constrained by a very small optimal number of assets. The core countries have much higher values of optimal savings for each level of beginning-of-period assets for the highest productivity state than the periphery. Once again, the savings behavior of France and Germany are very different than those of Spain an Italy, and this is also observed in the MPC: the cost of saving compared to consuming is higher in the periphery so the optimal decision rule is to consume more and save less comparatively. Moreover, the four lower states are much more stacked in the core than in the periphery. This is due to inequalities being higher in the core than in Spain or Italy. The highest state individuals own proportionally more assets compared to the other states in Germany and France.

Stationary distribution. The agents are distributed on the state space $A \times Y$ such that from one period to another, they keep the same dispersal. Now, we represent the distribution of agents according to their wealth uniquely and not their idiosyncratic productivity as it is a model feature but is not observed empirically.

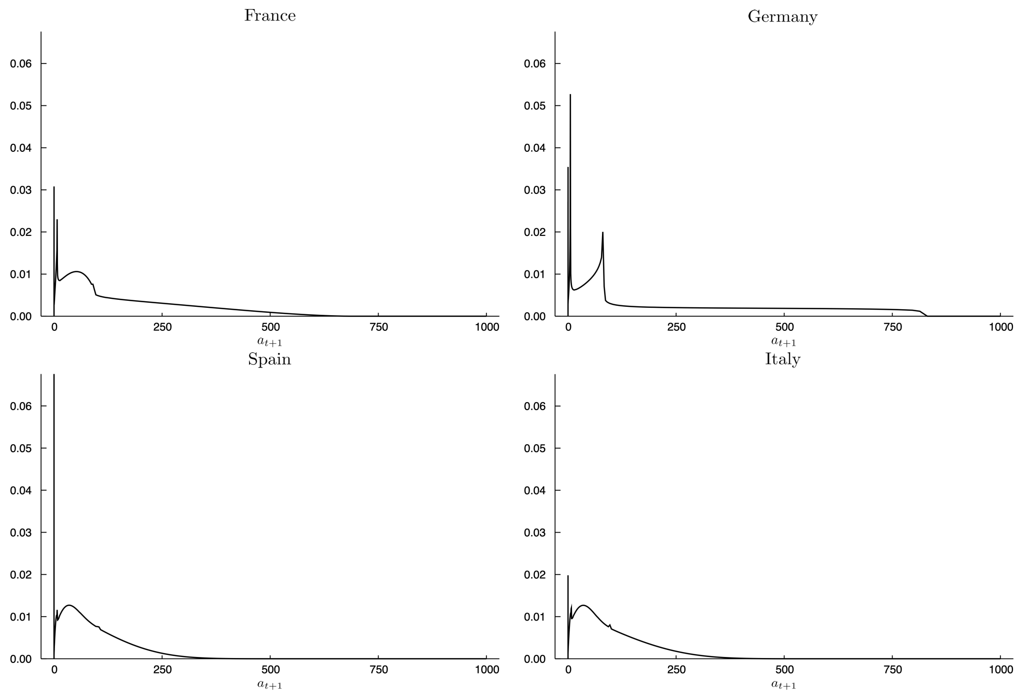


Figure 4: Stationary distribution of Savings

The x-axis represents end-of-period wealth and the y-axis is the frequency. We can see that Germany and Spain have a very high proportion of agents at or close to the credit constraint. Once again we can see a clear imbalance between the distributions of France and Germany and those of Spain and Italy. In the core countries, there is a concentration of agents with low levels of assets but then there is a significant number of households that have a number of assets much higher than the ones near the constraint. In France, agents seem to be distributed in a linear fashion from the end of the concentration to the limit of the ergodic set, in Germany, the trend seems to be flat, while in Spain and Italy, the path to $\bar{a}$ has a much more pronounced slope. This is a direct implication of the high inequalities found in France and Germany: there is a low but positive number of household that holds most of the wealth, while the rest of the economy is near the constraint. In fact, France and Germany seem to peak three times in the distribution, with three concentrations, while Spain and Italy peak once and then slowly converge to $\bar{a}$ with less and less agents for each value.

Aggregate variables. With the stationary distribution and the policy functions, we can compute a large number of aggregate variables that characterize the state of the economy at the economy. These variables provide us with a more complete understanding of the heterogeneity of equilibria between each country.

Table 4: Equilibrium Values

	Y	C/Y	I/Y	T/Y	S	K	CA/Y	R
France	4.25	0.37	0.33	0.31	60.41	55.79	-0.0026	1.0024
Germany	4.43	0.38	0.35	0.26	72.56	62.43	-0.0011	1.0005
Italy	3.99	0.38	0.29	0.32	46.20	46.59	0.0005	1.0058
Spain	3.95	0.46	0.29	0.25	43.52	45.46	0.0029	1.0063

Table 5: Equilibrium Values (cont'd)

	w	$\bar{a}$	% at ϕ	$\mathcal{G}_0^W$	$\mathcal{G}_1^W$	$\mathcal{G}_0^I$	$\mathcal{G}_1^I$
France	2.72	669.39	2.12	0.67249	0.67659	0.59481	0.59456
Germany	2.83	813.64	2.72	0.75751	0.75937	0.71340	0.71331
Italy	2.55	558.09	1.98	0.59500	0.60250	0.45475	0.45496
Spain	2.53	559.09	6.76	0.59045	0.59747	0.39541	0.39595

The GDP is represented by Y, then $C/Y, I/Y, T/Y$ are respectively the private consumption, investment and tax revenue relative to the GDP, S is aggregate private savings defined as the savings supplied by households in the economy by $\int_{i \in \mathcal{L}} a_{t+1}^i \ell(\mathrm{d}i)$. K is the demand for capital in the economy such that $K_t = [F_K]^{-1}(r_t, \bar{L})$. CA/Y is the current account balance over GDP, computed as $(R-1)(K-S)$ (interests paid or received on the NFA in equilibrium as NFA is constant), R is the before tax risk-free real interest rate, w is the before tax wage rate defined by the marginal productivity of labor, $\bar{a}$ is the limit of the ergodic set, % at ϕ represents the percentage of households at the credit constraint, $\mathcal{G}_0^W$ is the before tax Gini coefficient for wealth, $\mathcal{G}_1^W$ is the after tax Gini coefficient for wealth, $\mathcal{G}_0^I$ is the before tax Gini coefficient for income and $\mathcal{G}_1^I$ is the after tax Gini coefficient for income.

First, it is apparent that the composition of GDP is heterogeneous across countries. France displays a high proportion of government consumption and a high aggregate investment while having a low private consumption. France has one of the tightest fiscal regimes in the Eurozone, which implies high revenues from taxes. Italy, also implementing a strict fiscal policy, displays a similar importance of government spending on the composition of GDP. Germany and Spain, having a similar fiscal regime also share an equivalent proportion of tax revenue relative to GDP. Now the part that establishes a salient different between the core and periphery of the Eurozone is the portion of investment in the GDP. In fact, France and Germany display a high I/Y, above the Eurozone average, whereas Italy and Spain have a similar low proportion. As all countries are calibrated with the same capital depreciation, the difference comes from the proportion of capital demand in the economy. K is proportionally higher in the core countries, with a K/Y of respectively 13.13 and 14.10 for France and Germany, and 11.68 and 11.51 for Italy and Spain. The capital stock is proportionally higher, due to more wealth accumulation in the core. Note that the exterior position of the core and the periphery are poles apart, which shows that core countries are more likely to have positive NFA and thus receive interests in equilibrium, while the periphery displays tendencies to have negative NFA, paying interests to the countries that hold their assets. In fact, as seen in Section 2, positive (negative) NFA stem from the excess supply (demand) of capital on the domestic financial markets. This excess supply (demand) means that firms require less (more) capital than what the agents are saving. In turn, the latter save in foreign assets (the former emit bonds on the foreign market). This variable is therefore much more micro-level than aggregate investment for that matter. We can also observe that Spain consumes proportionally more than the other countries, who are close to each other. This high optimal choice of consumption can relate to a high proportion of agents at the credit constraint (6.76%) who, as seen in Figure 3, dissave whenever they have a positive number of assets, which makes them hand-to-mouth households that consume all their wealth in each period.

Now taking a closer look at inequalities in each country, we can see that the core has a higher limit to their ergodic set, which is consistent with a few agents holding most of the wealth. Observing the Gini coefficients we can see that, as before, the core countries are more unequal than the periphery, mostly due to a larger range of attainable wealth in the economy and a different relationship to the tax burden throughout the distribution. This element precisely is also the key to understanding why the Gini after tax can be higher than the Gini before tax: the burden of the tax lies proportionally more on the low-wealth households as their only tangible source of income is their wage and it gets taxed heavily.

In Figure 5, we can see the percentage change between before-tax savings and after-tax savings. This can be interpreted as the weight of the tax burden per wealth decile. The poorest households see their wealth reduced by 40 to 80 percent after tax, while the wealthiest agents experience a net wealth only around 5% less than their gross wealth. This division of the tax burden across the distribution shows how inequalities can increase after the tax kicks in. Note that this is due to the tax being a flat rate and not a progressive tax, as in the data. Hence, the difference between before and after tax inequalities is not accurately depicted in this model but the dynamics provide a rationale

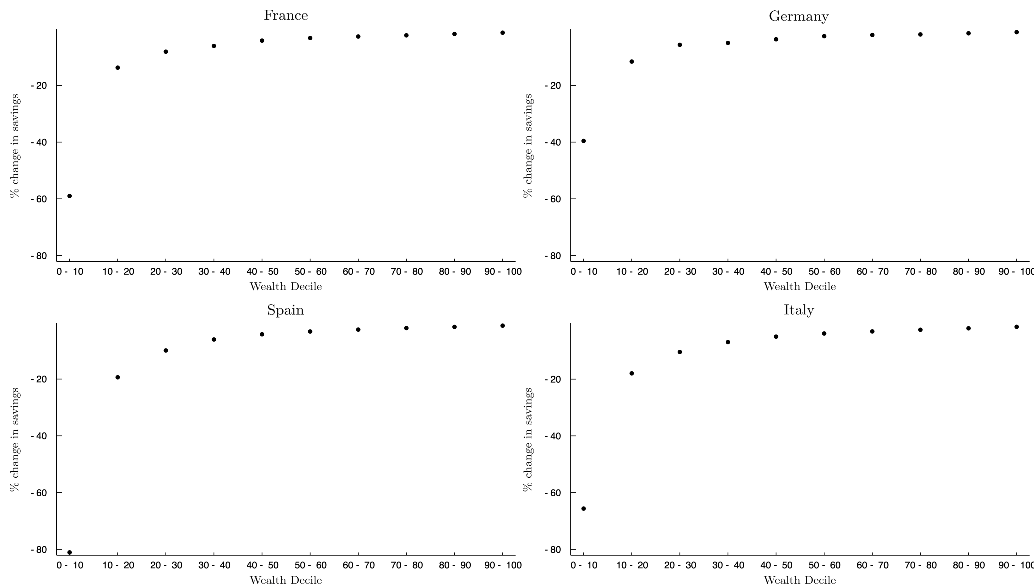


Figure 5: Tax burden per decile

that holds empirically as well: the tax burden is heavier on the middle-class than on the wealthiest households, as the proportion of consumption in the available income is higher in middle-class income groups than in the right tail of the distribution. However, the savings behavior across the distribution is a key feature of this model and the underlying hypothesis of this paper is that the exterior position of a country and the inequalities lie in these decisions.

In Figure 6, the before-tax wealth is in white and the after-tax savings is in black. The graphs all follow the same scale for the sake of simplicity of comparison. We can clearly see the importance of the wealthiest decile in the total savings in France and Germany especially. In fact, the last decile holds more than 50% of the wealth in France, 63% in Germany, 41% in Spain and 42% in Italy. Once again, we clearly see a dichotomy between core and periphery. The periphery sees its wealth more evenly distributed across the deciles, whereas the core, and especially Germany displays a large portion of the wealth being held by the right extremity of the distribution. This disparity of savings behaviors results in higher wealth inequalities and stems from the higher limits of the ergodic set in the core countries. The optimal savings for France and Germany generate higher potential values of wealth than those of Spain and Germany. This result corroborates the idea that those micro-level decisions, that only concern 10% of the population at most, have a great impact on the distribution of wealth of each country.

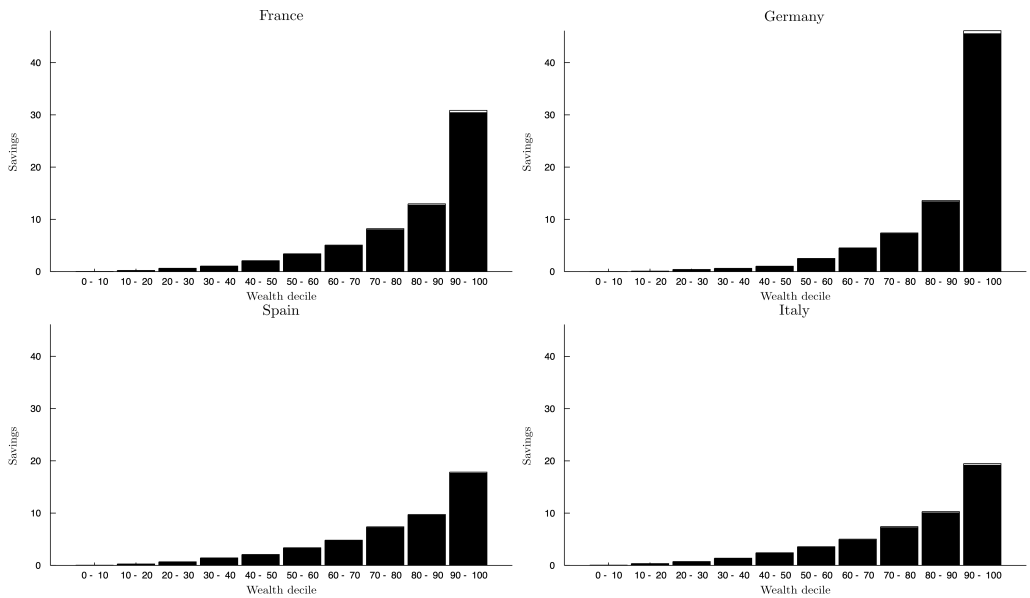


Figure 6: Aggregate savings per decile

5.3 Impact of a Technological Shock

Now that we have explored the equilibrium of this economy, we want to study how the variables described in Section 5.2 respond to an aggregate shock hitting the Eurozone evenly. Here the shocks are unanticipated, happen at $t+1$ and the shocked variables return to their stationary values with a certain autocorrelation.

The core and periphery now receive symmetric technological shocks. The aggregate productivity evolves as follows:

$$Z_{t+i} = Z_t + \mu^{i-1}\zeta, \quad \forall i \geq 1 \tag{38}$$

where Z_t is the equilibrium value such that $Z_t = Z = 1$, Z_{t+i} is the productivity at date $t+i$ with i indexing the number of periods since the shock happened, $\mu \in (0,1)$ is an autocorrelation of the shock and ζ is the shock. For this simulation, the persistence μ will be 0.95 and the shock will be a 1% decrease in productivity. The methodology of the shock follows Boppart et al. (2018), where the aggregate shocks are unanticipated and once they happen, the agents expect it to happen again with probability zero. In this subsection, the interest rate set by the monetary authority will be the same invariant R as before, and will not react to the shock.

In Figure 7, the graphs represent the deviations from equilibrium values of consumption

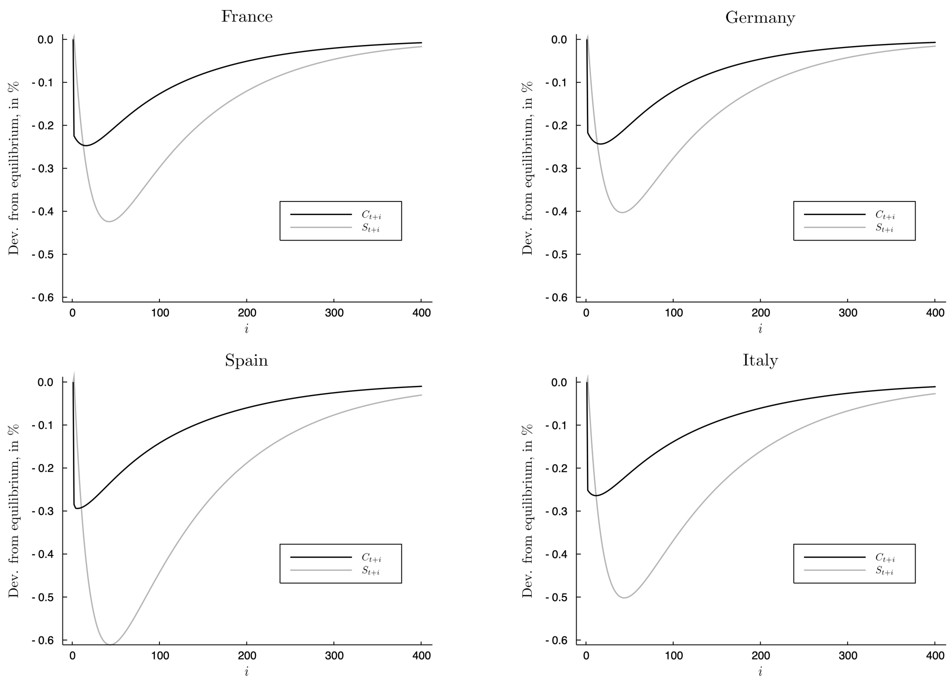
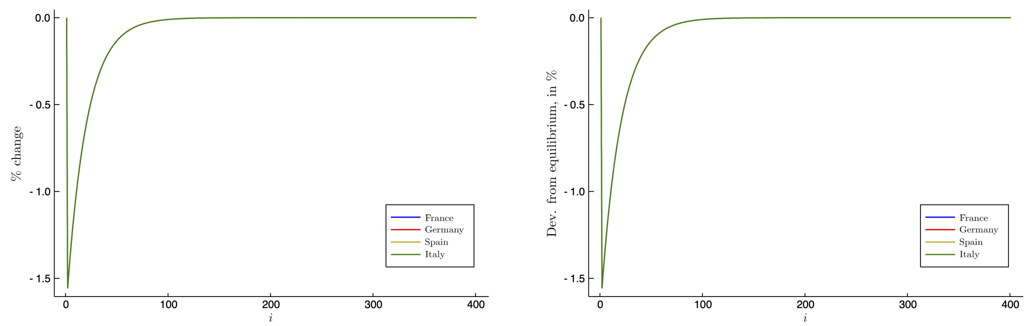


Figure 7: Change in savings and consumption for $\zeta = -1\%$

and private savings after a -1% technological shock. Clearly, the decrease in productivity has a negative effect on those values. The aggregate productivity has a direct impact on the wage rate (and not on the interest rate as it is invariant in this model) and thus on the budget constraint of the households. With their decreased income, they must choose an optimal quantity of assets to sell and of goods to consume. We can see that the core countries deviate less from their equilibrium than the periphery. Spain, being the country that saves the least, sees its aggregate savings reach a lower trough than any other country and also has the highest relative decrease in consumption, although it has a larger equilibrium consumption than the other countries. Moreover we can see that consumption and savings do not evolve in an exactly similar fashion. In fact, consumption decreases first, due to the reduction in available income. Next, the savings decrease as the agents want to smooth their consumption and decide to pass on the effect of the shock to their savings rather than their consumption which is the source of their utility. Then, as the productivity reverts back to its equilibrium value, agents start re-increasing their precautionary savings, which mitigates the re-increase in consumption, and we can see that the savings curve crosses consumption around its apex. We can also see that the periphery displays a higher magnitude of the reaction to the shock, in deviations from the equilibrium, as their stationary savings are lower than in the core. The precautionary savings therefore seem to be more sensitive to an aggregate shock in countries where the starting values of savings are lower. This could be explained by a higher uncertainty in Spain and Italy as they tend to save less than the core, and thus are more sensitive to idiosyncratic shocks.



(a) Demand for capital

(b) GDP

Figure 8: Changes in demand for capital and GDP for $\zeta = -1\%$

In Figure 8, we can see how the demand for capital and the GDP evolve over time after a negative aggregate productivity shock. It is important to note that all countries have a similar response of demand for capital and GDP in terms of deviations from the equilibrium, as the shock hits the production symmetrically across countries. Hence, the absolute value of the deviations are different in each member state, but the whole region follows the same path with a similar relative amplitude. The firms see their productivity decrease, which is a scalar of the production function, thus the GDP decreases and the capital they require follows the same track. The savings we have seen in Figure 7 represent the bonds bought by the households, equivalent to the change in supply of capital in the economy. It is clear that the reaction to the shock is different when comparing the demand and the supply. In fact, this mismatch leads to changes in the net foreign assets, itself creating current account imbalances as the CA balance is the change in NFA. When the firms require less capital than what the households are willing to supply, the agents turn to the foreign market and their capital finances external firms. As foreign investors sell more bonds to domestic agents, this generates a current account surplus. The decrease in aggregate savings is around 0.5% for each country, as seen in Figure 7, while the decrease in demand for capital is around 1.5%, hence it is apparent that each country will experience a current account balance increase for the duration of the shock. The reason why the supply and demand do not match after such a shock is that the decrease in productivity affects directly the firms, translating to a decrease in the wage rate, which in turn decreases the available income of each agent, and they have to optimally decide how to pass it on to their savings or consumption. Hence, a portion of this decrease goes to consumption, which drains the potential decrease in savings and triggers a mismatch.

A higher marginal propensity to consume therefore directly induces a mitigated negative effect of the shock on the current account balance.

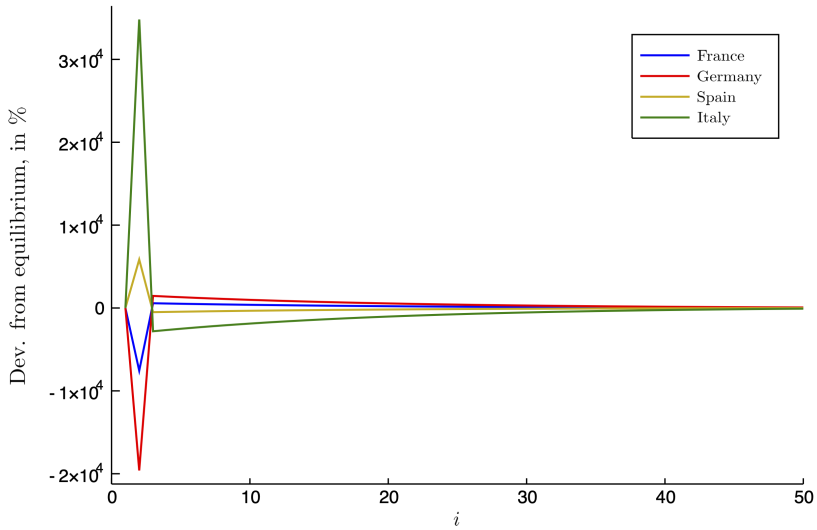


Figure 9: Change in current account over GDP for $\zeta = -1\%$

In Figure 9, we can see how each country behaves differently concerning their exterior position after the technology shock. Note that only 50 periods are graphed instead of the usual 400, as the changes in current account balance revert back to the steady state early. It is also important to note that France and Germany start with a negative equilibrium current account balance over GDP and the graph displays deviations from the equilibrium, hence, a negative value indicates an increase in the current account balance for them. Moreover, we can see that the invariant interest rate generates variations that are too high to be realistic, this issue is addressed in Section 5.5. Although the absolute magnitude of these variations is not informative, their shape and their relative changes are still relevant to the analysis of European imbalances after a shock. All countries seem to experience the shock in three parts: first, an increase in their current account balance, then a decrease compared to their equilibrium values and finally a reversion back to the starting point. As seen in Figure 8, the demand for capital comoves between countries, hence the only between-country variations in the change in current account balance must come from the supply, which is the result of a micro-level trade-off between consumption and savings. Countries that are prone to higher idiosyncratic risks need high precautionary savings to insure themselves. The periphery countries have lower values of ρ (idiosyncratic productivity persistence), thus the diagonal terms in the matrix Π for periphery countries are all less than the diagonal terms for the core countries. This implies that the probability of staying at the same productivity level from one period to another is higher in the core than in the periphery. This leads to more uncertainty and therefore, more precautionary savings required to insure oneself. This explains the deeper peaks in savings for Spain and Italy in Figure 7 and leads to more volatile current account balances after a shock in Figure 9 as the periphery suffers more from changes in aggregate savings than the core, which has high values of $diag(\Pi)$. This result supports the idea that current account balance is intimately linked to idiosyncratic risk, which is also a determinant of inequality. However, in Figure 9, the highest magnitudes are those of Germany and Italy and this can be explained by their equilibrium current current accounts being very close to zero and thus the deviations are very high.

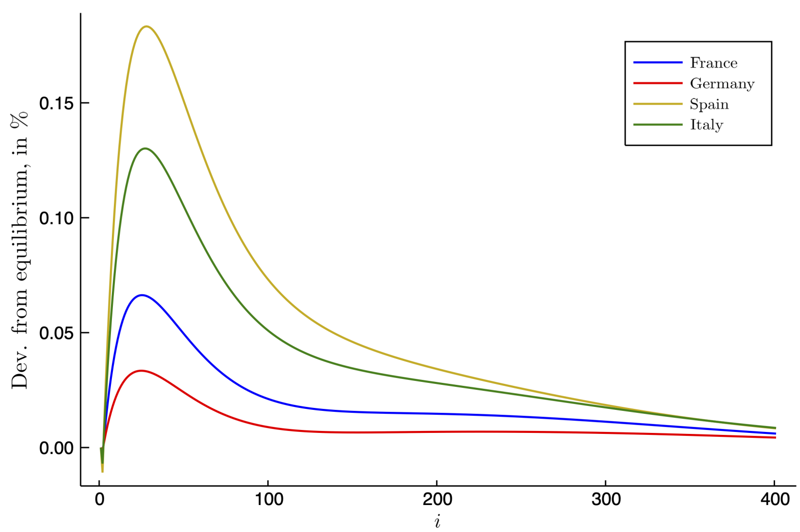


Figure 10: Change in Gini for wealth for $\zeta = -1\%$

In Figure 10, the Gini coefficient for wealth is plotted against time after the productivity shock. First, the decrease in aggregate productivity clearly increases wealth inequality in the economy, as the shock decreases the wage rate, which in turn decreases the available income of households and lets credit constrained households save and consume even less. The increase in inequality is higher in periphery countries than in core countries. We can recall that the persistence of individual productivity ρ relates closely to social mobility, as a high value of ρ indicates a more rigid transition path through the periods as $diag(\Pi)$ will only contain high values. The probability of going from one productivity to another is lower when ρ is higher. This implies that in core countries, where the persistence is high, the idiosyncratic risk is lower, as the transitions to different states are less frequent. Consequently, core countries, who display low precautionary savings due to less idiosyncratic risks do not need to decrease their savings as much as the periphery to face the shock. This, in turn, creates more inequalities for the countries that face a higher idiosyncratic risk as the middle-class households who only hold some wealth have to spend all their savings to face the shock, whereas the high wealth households do not need to decrease their savings by a large amount as they already stop saving if their wealth is at a high enough level. Hence, the magnitude of idiosyncratic risks is also evidently linked to inequality, as well as the current account balance (Figure 9).

Ultimately, a negative technological shock has heterogeneous effects on each country, depending on their respective idiosyncratic risks. Nonetheless, some trends are easily identified: a decrease in demand and supply for capital, although mismatched, a decrease in consumption, an increase in the current account balance, then a small decrease due to the difference in paths of supply and demand, and finally an overall increase of wealth inequalities. A higher idiosyncratic risk is fertile ground for further increases in inequality and higher decreases in precautionary savings, resulting in a higher current account balance.

5.4 Impact of a Monetary Shock

A monetary shock in this economy affects all member states the same way as they all depend on the same monetary authority. The difference from a technological shock, from an empirical standpoint, is that each country has its own ex-ante interest rate and do not share a common R_t, as they all share the same aggregate productivity Z_t. Here, as stated in Section 1, the equilibrium risk-free interest rate is composed of both a common monetary component and a country-specific risk. Therefore, the equilibrium interest rates need to experience an additive shock, otherwise the comparison between countries would be less straightforward. We can imagine that the central bank raises the policy rate by a certain amount ξ and the country-specific component of each interest rate does not change. The process for the shock is then given by:

$$R_{t+i} = R_t + \Gamma^{i-1}\xi, \quad \forall i \geq 1 \tag{39}$$

where $\Gamma = 0.95$ is the autocorrelation of the shock and ξ is the amplitude of the shock. The shock is initially $\xi = 0.1\%$, meaning that 0.1 percentage points are added to the interest rate of each country, and $\xi = 1\%$ is also investigated to see if the magnitude of the shock changes the paths of variations.

In Figure 11, we can observe that an increase in the real interest rate does not affect micro-level savings and consumption decisions the same way a technological shock does. For the core countries, the shock induces a higher variation in consumption than in savings.

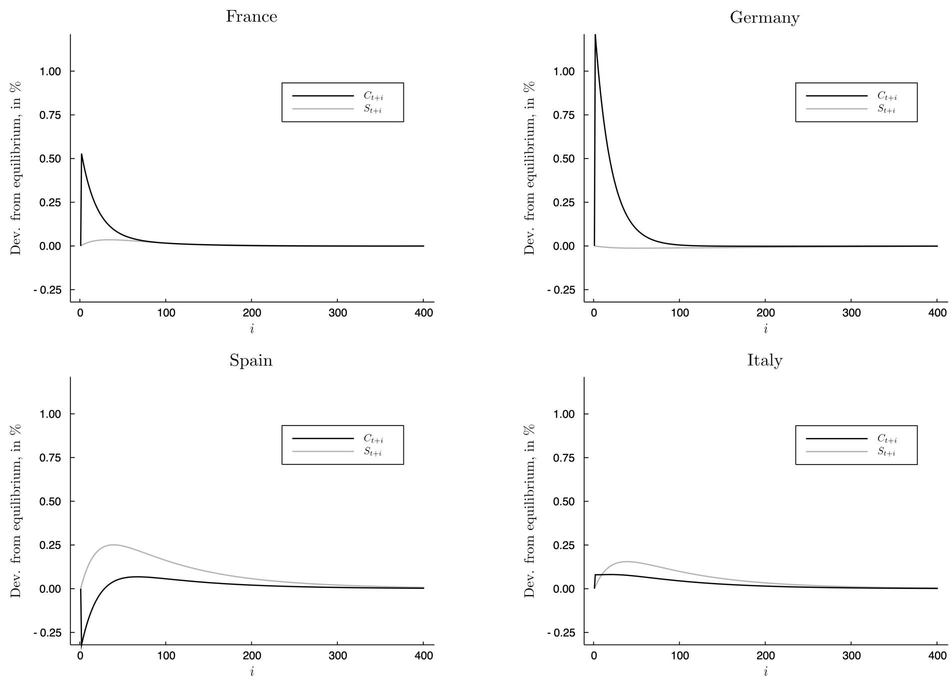


Figure 11: Change in savings and consumption for $\xi = 0.1\%$

France and Germany have a low country-specific risk, which makes their base interest rate lower than the periphery. Hence, a rise in interest rate will be much more apparent in France and Germany than in Spain and Italy, where the equilibrium rate is already high. In the core countries, the aggregate shock increases consumption sharply at the beginning and then reverts back slowly to its stationary value. The aggregate savings follow a smoother path, where they slowly increase in a hump shape, and then revert back to its equilibrium. The increase in R_t translates to a decrease in demand for capital from the firms, for which the higher interest rate is less advantageous, and an increase in supply as the bonds will pay a higher rate, more enticing for the households. In the core, the distribution of wealth is more spread out than in the periphery. Thus, a higher interest rate induces more income from capital holdings, which affects a higher portion of the distribution of agents in the core than in the periphery, where more individuals are close to or at the credit constraint and cannot benefit from this increase in interest rate. The households in the core countries who experience an increase in capital gains are concentrated at the equilibrium around the threshold over which saving becomes less optimal and start dissaving, in favor of higher consumption. Hence, in these countries the increase in wealth is passed on to consumption for the most part. It is important to note that the increase in interest rate, leading to a decrease in capital demand, has a negative effect on the wage rate, which affects households in a heterogeneous fashion.

The ones near or at the credit constraint rely more heavily on their wage than on their savings. Therefore, in an economy where more agents are concentrated at or near the constraint, the shock will induce a smaller increase in consumption overall, as it will be mitigated by those low-wealth households. This is the case for Spain and Italy, where aggregate consumption respectively decreases and increases very slightly. In Italy, the reasoning is that the concentration of agents near or at the constraint is high, therefore the increase in interest rate will have a negative impact on household income for a large portion of individuals. The increases in consumption and savings depend on the higher wealth households, who can afford more consumption after the shock as they hold a lot of savings and do not rely on their wage, and the middle-class, who has a few assets and can afford precautionary savings after a shock. We can see that in Spain, where consumption is the highest at the equilibrium, consumption decreases after the shock. The distribution of wealth in Spain has the fattest left tail of the four countries, with 6.58% of agents at the credit constraint. Hence, the increase in consumption by the high wealth households is cancelled out by the large decrease in consumption by the constrained agents.

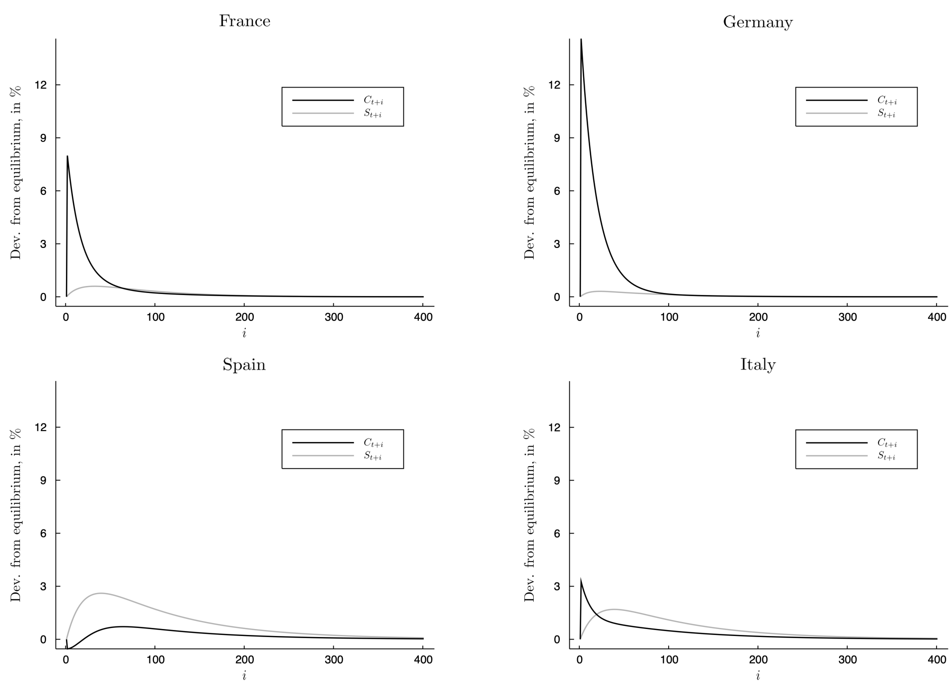
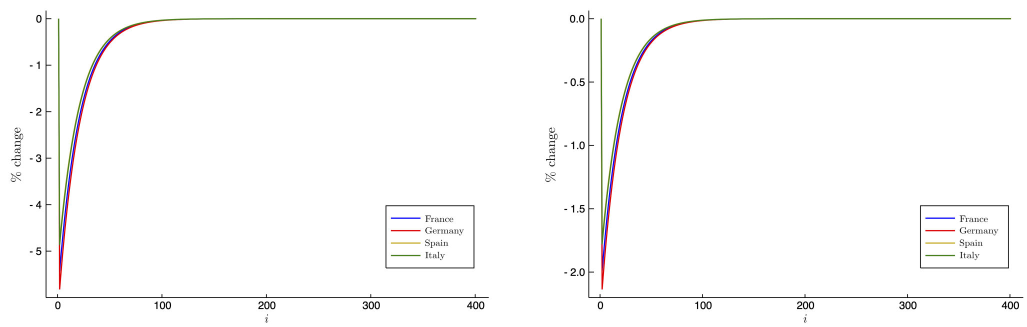


Figure 12: Change in savings and consumption for $\xi = 1\%$

In Figure 12, we investigate the effect of a monetary shock with a higher magnitude than before on consumption and savings. The interest rate is increased by 1%, an amplitude ten times higher than before, in order to exaggerate the effects of the shock and study the change in paths. We can see that the narrative still holds for France, Germany and Italy, but Spain no longer decreases its consumption but now increases it slightly. This can be explained by the fact that the large increase in interest rates constitutes a larger increase in capital gains for wealthy households than the decrease in wage for constrained households. A large monetary shock only generates a scaling effect compared to small one for all the variables we study except consumption, so it is not necessary to continue comparing them and we can keep analyzing the effect of a shock of 0.1% on the economy.



(a) Demand for capital

(b) GDP

Figure 13: Changes in demand for capital and GDP for $\xi = 0.1\%$

In Figure 13, as in Section 5.3, all countries see their demand for capital and GDP deviate from equilibrium in a similar fashion, and the countries with a higher equilibrium stock of capital decrease their demand further than those with less capital. As described above, the increase in interest rate translates to a decrease in the demand for savings for the firms. A higher R_t means a higher cost to emitting bonds, hence it follows that firms are less interested in renting capital at such a rate. There is an evident mismatch in demand and supply for capital as we have seen in Figures 11 and 12 that the aggregate savings always increase after the shock, thus the demand decreases and the supply increases. It means that capitals will flow from households to foreign firms in order to make up for this discrepancy, if the resulting current account balance is positive.

In Figure 14, we can observe positive deviations from the equilibrium in the current account balance for Italy and Spain, and negative ones for France and Germany (which translate to an increase in the balance, as they start negative). This increase supports the idea just described above that the mismatch of demand and supply for bonds leads to households investing in foreign capital. As all capital demands move with almost similar amplitudes across countries, the heterogeneity across countries must originate from the supply, the households. As seen in Figure 11, the households increase their savings in a hump-shaped path, where the core does not respond to the shock as much as the

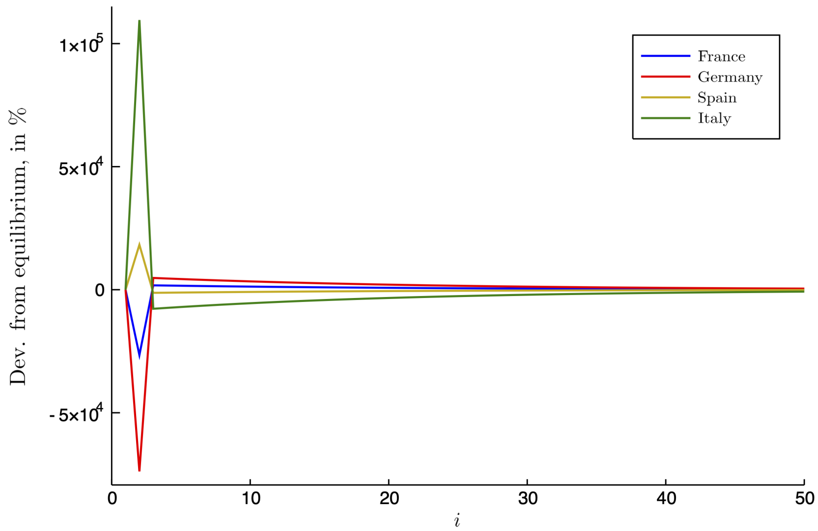


Figure 14: Change in current account over GDP for $\xi = 0.1\%$

periphery. France and Germany display small absolute variations from their stationary current accounts, as the agents do not respond to the shock with much higher savings. This is due to their persistence of idiosyncratic productivity being high, therefore they do not need to insure themselves against risk as much as the periphery. Spain and Italy on the other hand, experience a much larger increase in their exterior position, as their precautionary savings increase after the shock. As in Figure 9, it is important to note that countries with an equilibrium current account balance close to zero, such as Italy or Germany, display very high variations compared to the steady-state. Moreover, the absolute values of the current account are higher in Germany and France.

In Figure 15, we see the evolution of the Gini coefficient for wealth after the monetary shock. The inequalities decrease, as here, the wage decreases and the capital gains increase. The poorest households do not benefit from the increase in R_t and suffer from the decrease in w_t, hence they diminish their consumption but cannot further decrease their savings. The middle-class experiences a sharp increase in capital gains, which allows them to be more spread out across the distribution, even though the wage decreased. The wealthiest households benefit fully from the increase in interest rate as they do not rely solely on their wage, but consume rather than save more as they are close to the limit of the ergodic set and would rather consume than save. Consequently, the left tail of the distribution does not change their wealth, the middle-class gets spread out more evenly and the right tail does not save considerably more. As a result, the decrease in inequality is evident.

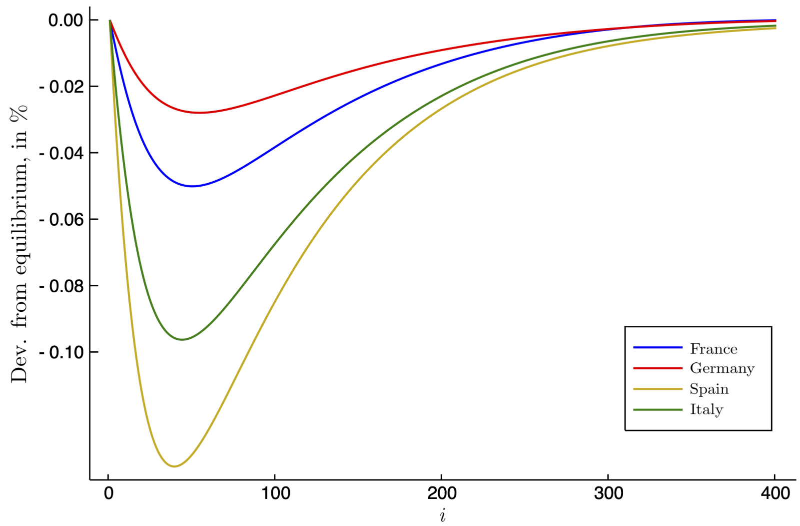


Figure 15: Change in Gini for wealth for $\xi = 0.1\%$

Overall, the effect of the monetary shock is less straightforward than the technological shock: it influences both the wage negatively and the rental rate of capital positively. This benefits the middle-class and the right tail of the distribution, as they get more capital gains, but the left tail is stuck at the credit constraint and sees its consumption being decreased. The magnitude of the shock is crucial to the analysis of its effect: a small shock will mainly be detrimental to aggregate consumption as the decrease in wage will affect more people than the increase in interest rate, while a large shock will be beneficial to the aggregate consumption as the middle-class will rely more on savings than wage and will be able to consume more. This in turn results in a less unequal economy and an improved exterior position due to private savings blowing up and a plummeting demand for capital. Capital will flow outward and it will generate positive European imbalances.

5.5 Currency Union versus Independence

In Section 4.1, the monetary authority is described as having the choice between three scenarios after a technological shock hits the Eurozone symmetrically. It can keep the same invariant interest rate; correlate it with the shock or dissolve (or a country unilaterally leaves the union) and let R_t evolve endogenously. The aggregate shock that hits the economy is the same as in (36) with $\zeta = -1\%$. We now denote R^E_{t+i} the endogenous interest rate, R^C_{t+i} the correlated rate and R^I_{t+i} the invariant one. They are defined as follows:

$$R^I_{t+i} = R_t, \quad \forall i \tag{40}$$

$$R^E_{t+i} = 1 + F_K(K_{t+i}, \bar{L}) - \delta, \quad \forall i \geq 1 \tag{41}$$

$$R^C_{t+i} = R_t + \psi^{i-1}(R^E_{t+1} - R_t), \quad \forall i \geq 1 \tag{42}$$

where the endogenous interest rate is determined by the marginal product of capital and the correlated interest rate takes the value of the endogenous one at the period of the shock and then returns to R_t with an autocorrelation of $\psi = 0.95$. The correlated interest rate serves the purpose of limiting the variations of the current account balance that can be witnessed with an invariant interest rate. It can be interpreted as a shock to country-specific risk that lifts the interest rate to its optimal individual value (R^E_{t+i}) and then dissipates slowly over time.

The goal of this section is to investigate how each country reacts to a common interest rate, compared to its endogenous one, in order to analyze the effects of belonging to a monetary union on individual countries when an aggregate shock hits them, exploring the counterfactual of each member state potentially leaving the Eurozone and letting the financial markets regulate themselves with the R^E_{t+i} scenario. The motivation behind this exercise stems from the growth of eurosceptism when a crisis occurs in the whole Euro area. As discussed in Guiso et al. (2019), constituents tend to blame the euro and thus the policies of the ECB when crises such as the financial crisis of 2008 occur. This section aims at addressing this issue and comparing a common monetary policy (R_t) to its endogenous counterfactual (R^E_{t+i}).

First, in Figure 16, we can see the different paths of the interest rate after the technological shock hits. The endogenous interest rate follows very different paths in each country as it is struggling with two conflicting forces: the negative technological shock; and the financial market seeking a clearing price. In every country, the decrease in productivity induces a decreases in the interest rate, as a direct effect from the marginal productivity of capital. In the countries with a positive (negative) exterior position, the interest rate decreases (increases), in order to lower (raise) private savings and raise (lower) investment. The magnitude of this effect depends on how far the invariant interest rate is from the market clearing one. We can see that in the core, the interest rate unambiguously decreases, due to the shock and to the positive NFA. Then, when the shock dissipates, the interest rate goes up before decreasing again as the market re-adjusts to find the market clearing rate, which is evidently lower than the invariant one. In the periphery, Spain and Italy both have negative NFA and therefore, the market seeks a higher interest rate to bolster savings and hinder investment. The effect of the shock is absorbed by the

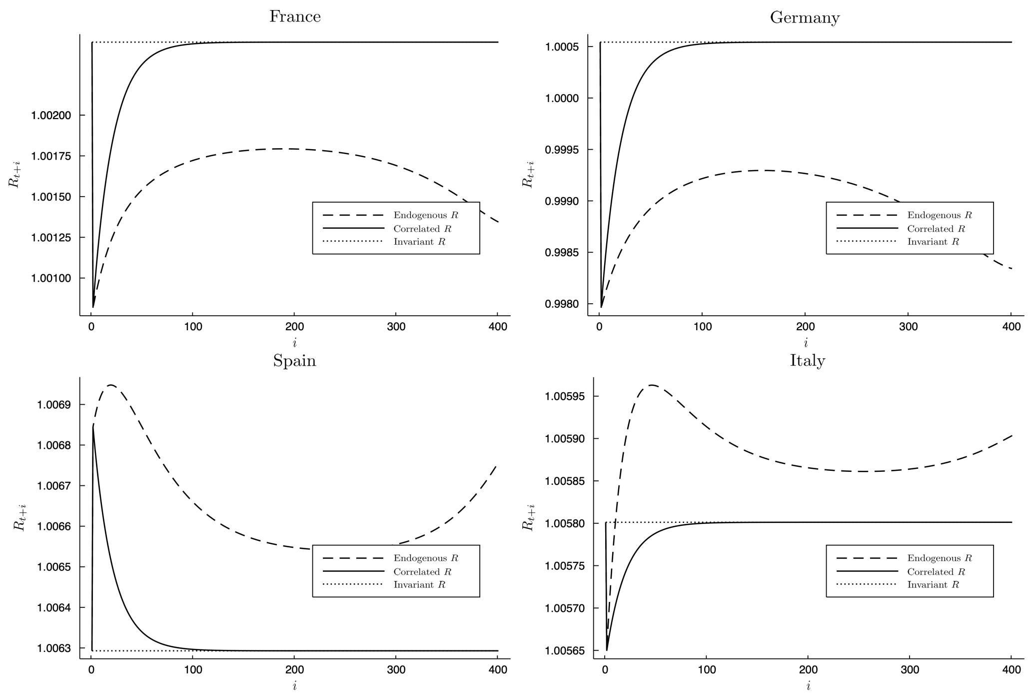


Figure 16: Change in interest rate for $\zeta = -1\%$

clearing rate seeking effect in Spain, as the interest rate does not decrease at first. Then, the interest rate decreases before increasing again, as agents start increasing their savings due to the higher interest rate, which leads to excess savings, creating a decrease in the interest rate, before reaching a higher equilibrium value. In Italy, the magnitude of the shock is higher than the market clearing effect at first, which leads to a lower interest rate, which then increases and follows the same path as Spain. These differences come from the heterogeneity of starting points in NFA for each countries and how the agents respond to these changes in terms of savings. Note that the invariant and correlated interest rates do not require any interpretation.

In Figure 17, the change in consumption after the technological shock can be observed. It is important to note that in the case of the correlated interest rate, and to some extent, the endogenous one, this section cannot be compared to Section 5.3, as the shock on Z_t is also accompanied by variations of the interest rate, which are themselves studied separately in Section 5.4. We can see that the decrease in productivity affects consumption negatively, as it directly decreases the wage rate and the interest rate for the correlated and invariant cases. These increases lead to a lower available income for the households and thus a decrease in consumption. In the endogenous case however, the market clearing

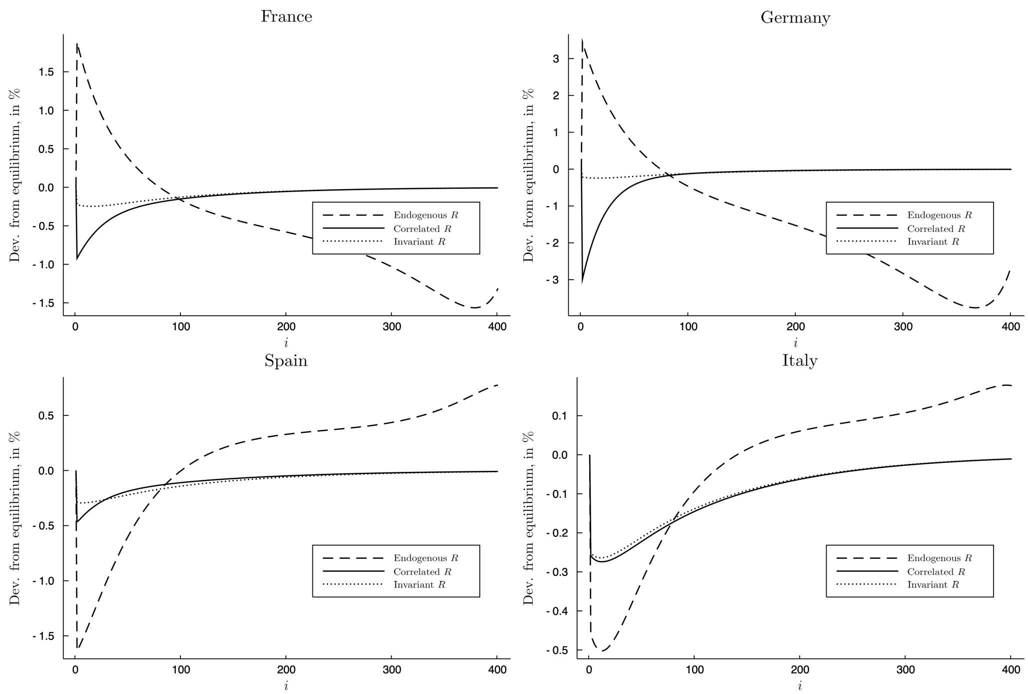


Figure 17: Change in consumption for $\zeta = -1\%$

effect is leading the variations as the interest rate adapts. For the core countries, a lower interest rate means less incentives to save, which leads to an increase in consumption. It is then followed by a decrease, as the market clearing interest rate is lower in the endogenous equilibrium, leading to less savings and thus less wealth in the long run for the households. In the periphery, the productivity shock instantaneously drives the consumption down, as agents need to keep their savings to insure themselves against idiosyncratic risks, and therefore do not decrease their savings as much, which allows them to increase their savings even more when the interest rate increases. This then leads to a higher available income in the future, and thus more consumption. As the market clearing interest rate is higher than the currency union one, the endogenous equilibrium consumption is thus higher than the original steady state.

In Figure 18, the aggregate savings are plotted for each monetary scenario. Note that Figures 22-25 represent the aggregate savings per quartile for each country after the shock and are available in Appendix B. The productivity shock increases aggregate private savings in each country for the correlated and invariant cases. The decrease in productivity directly leads to a decrease in wage, which means that agents have less available income and have to dissave in order to smooth their consumption. In the endogenous case, we

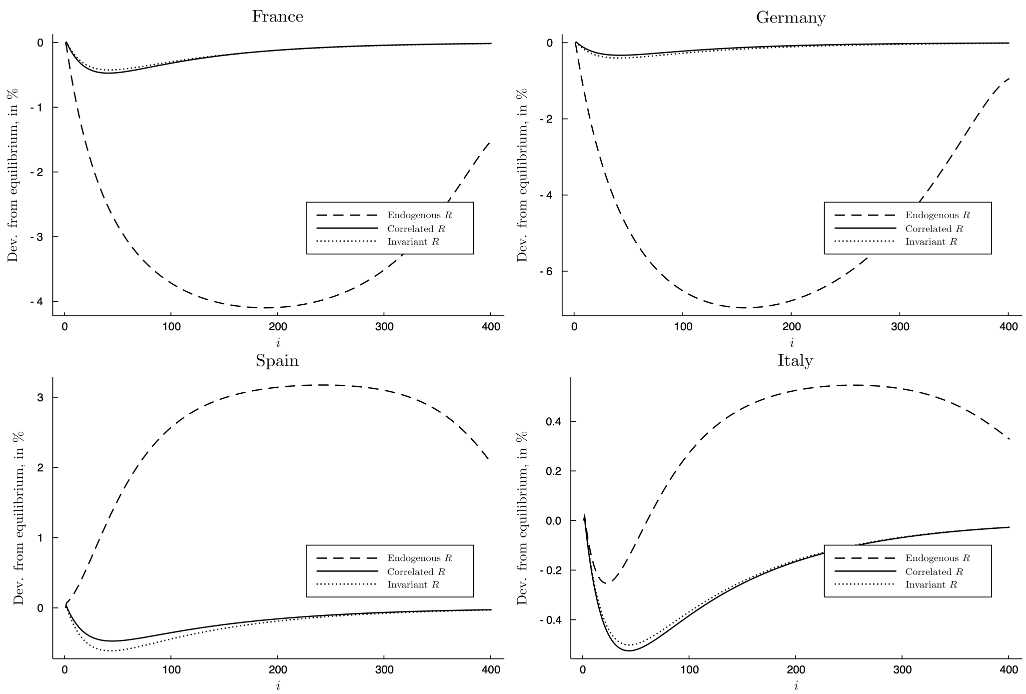
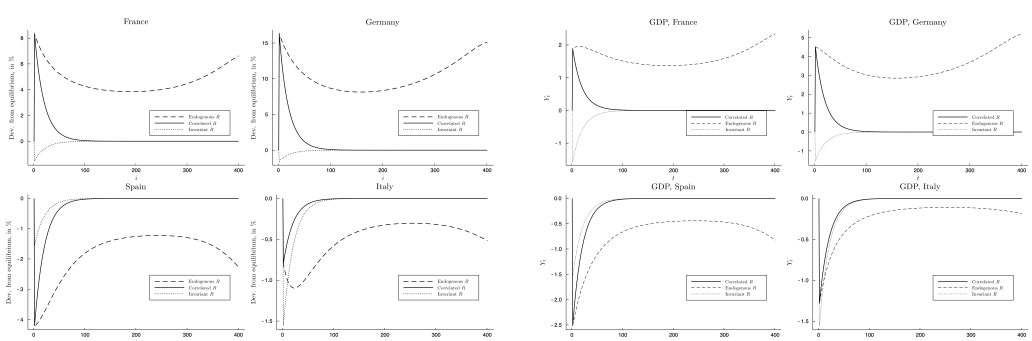


Figure 18: Change in savings for $\zeta = -1\%$

can see that the core countries see their savings decrease much more than in the two other scenarios. In fact, the decrease in interest rates discourages savings and the decrease in the wage also plays a role in the diminishing available income. In Spain, the increase in the interest rate plays a more important role in the variations than the decrease in wage, hence agents save more. In Italy, however, the effect of the decrease in wage resonates more significantly, causing a decrease in savings at first. Then, the increase in interest rates catches up and attracts savings. In any case, the endogenous scenario moves with a larger amplitude than the other two, as multiple forces are at play, instead of just the shock. We can decompose the effects by quartiles that we call Q_1, Q_2, Q_3 and Q_4, sorted in ascending order. In France, in the correlated and invariant cases, Q_2, Q_3 and Q_4 see their savings decrease as the productivity shock lowers the wage and reduces the available income. The agents thus substitute their savings for consumption in order to minimize their decrease in utility. Q_1 however, sees its wealth increase by a small amount, as the households who are not at the credit constraint prefer to save more in order to insure themselves against the risk of a transition to the worst state, where a large share of agents are credit constrained. In the endogenous scenario, the decrease in interest rates leads to multiple effects. As the market seeks a clearing price, capital becomes cheaper for the firms, who then increase the wage, counteracting the decrease in productivity. This leads to an increase in available income, compensating the decrease in interest rates for the households who rely more on labor income than on capital gains. Hence, Q_1 sees its savings increase as agents can escape from the credit constraint, Q_2 and Q_4 lower their savings as they do not benefit as much from the interest rate and Q_3 first increases its savings when the shock hits, as they substitute from consumption to insure themselves against idiosyncratic risks, and then decrease it when the market seeks its clearing rate. In Germany, the effects are the same as in France, except for Q_3 in the correlated and invariant cases who instead of decreasing their savings, increase them slightly as Germany relies more on savings than France, therefore they substitute more from consumption in order to save additional amounts. In the periphery, the reactions to the shock are similar to France in the invariant and correlated cases, except that Italy displays a higher magnitude of each decrease or increase. In fact, in Italy, agents rely more on savings than in Spain, as they save more in equilibrium and consume a lesser portion of their GDP. In the endogenous case, as the interest rate increases in the periphery, rather than decreases for the core, Q_3 and Q_4 increase their savings, as they have enough wealth to substitute from consumption and benefit from the increase in capital gains. Q_1 and Q_2 however, rely more on their wage, and an increase in the interest rate reduces the demand for capital from the firms, thus reducing the wage further. This magnified decrease leads to less and less savings from these households, as they have the necessity to smooth their consumption. Therefore, we can clearly see the divide between core and periphery in the endogenous case: without the central bank freezing or limiting interest rate movements, the countries with a worse exterior position face a decrease in demand for capital that further reduces the wage and leads to less and less available income for those who rely on labor income. On the other hand, the core countries see the interest rate decrease, allowing the wage to go up and counteract the decrease in productivity.



(a) Demand for capital

(b) GDP

Figure 19: Changes in demand for capital and GDP for $\zeta = -1\%$

In Figure 19, the demand for capital and GDP, affected by both R_{t+i} and Z_{t+i}, show two sets of reactions in all scenarios. First, in the case of an invariant R_t, the decrease in the demand is only shaped by the technological shock. A lower productivity translates directly to a lower demand for input factors (in this case, capital). In the case of an endogenous interest rate, the core countries see their capital prices decrease, thus leading to an increase in both the demand for capital and the GDP. The decrease in productivity drives the demand down but the lowering interest rates counteract this effect with a larger amplitude, therefore leading the demand and GDP upward. In the periphery, the negative productivity shock is magnified by a higher interest rate, thus boosting the decrease in both variables as the firms face a less productive technology, as well as higher input prices, hence slowing down production. Once again, the central bank has the power to homogenize the effect of a shock on the whole region, rather than creating more disparities between countries.

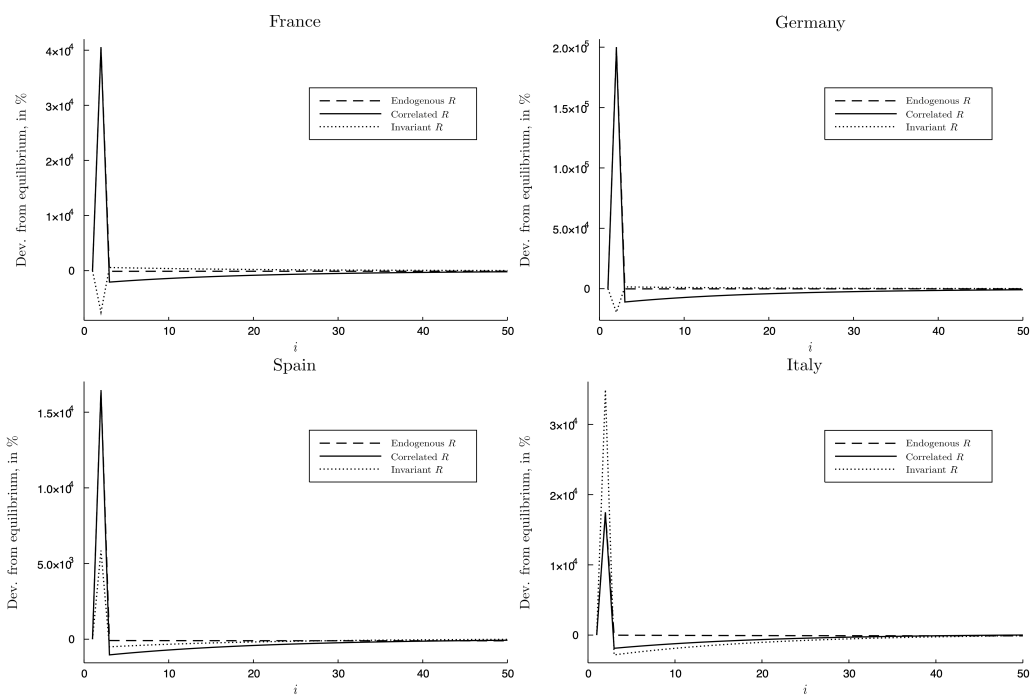


Figure 20: Change in current account over GDP for $\zeta = -1\%$

In Figure 20, we can see the reaction of the current account balance to the negative productivity shock. France and Germany have a negative current account balance at the equilibrium as their NFA are positive and thus receive interests from their foreign asset holdings, whereas Spain and Italy have a positive current account balance as they save less than firms invest. An increase in the core countries therefore translates to a further lowering of the current account balance, and a decrease means that the balance moves towards positive values. In the invariant interest rate case, the decrease in productivity leads to a decrease in demand for capital in each country, as seen in Figure 19 (a). The overall changes in savings are not as large as the decrease in investment, as seen in Figure 18, thus the resulting current account balance has to increase. We can see that core countries experience lower magnitudes than the periphery, as savings are more rigid in the core due to lower idiosyncratic risks. In the endogenous case, the periphery experiences the same direction of variation, as the decrease in productivity is accompanied by an increase in capital prices, but the core sees its current account balances worsen, as the demand for capital rises due to a lower input price for capital. Furthermore, in the periphery, the endogenous interest rate leads to higher savings than the other scenarios, thus increasing the balance even further, whereas the core decreases its savings with a higher amplitude in this case, as seen in Figure 18. The increase in demand for capital and the decrease in supply of savings in the core therefore lead to an unambiguous drop in the current account balance, translated to an positive change (as it is negative in equilibrium). This worsening exterior position of the core is intimately linked with how the agents save less to insure themselves from idiosyncratic risks in these countries and how the interest rate moves in a free market setting.

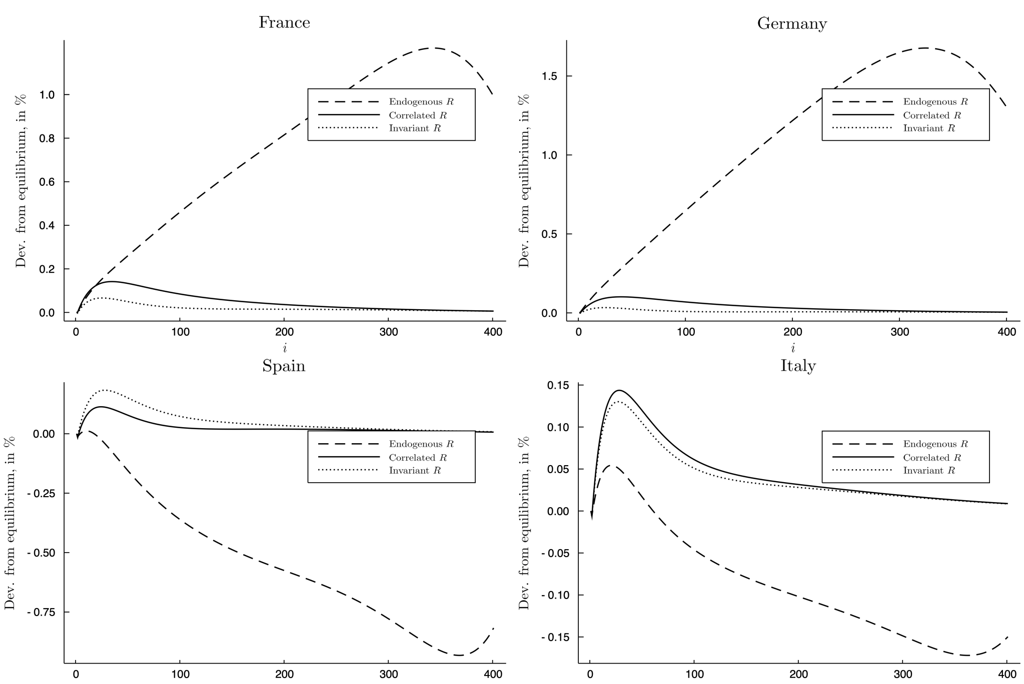


Figure 21: Change in Gini for wealth for $\zeta = -1\%$

In Figure 21, the Gini for wealth globally increases when the shock hits the economy. In fact, in the correlated and invariant cases, the lower productivity leads to lower wages which in turn leads to less disposable income for the middle-class, who can no longer use it to save more and close the gap between them and the wealthiest households. In the endogenous case, the effects are once again at polar opposites for the core and the periphery. In the core, the decrease in wage is mitigated by the decrease in interest rates, which decreases capital gains but increases labor prices. In these countries, the dispersion of idiosyncratic productivities σ_ε^2 is higher, thus leading to higher differences in productive wages. Hence, the increase in the wage due to the change in capital demand drive inequalities upward as high state agents benefit from much higher wages than the middle-class (as values of Y_t are exponentially spaced). In the periphery, the decrease in wage is only accompanied by an increase in interest rate, leading to more capital gains. The wage is also more concentrated in these countries, leading to less salient differences between good and bad idiosyncratic states in terms of wage. The upper middle-class thus closes the gap between them and the wealthiest households, therefore decrease inequalities. In Figures 26-27, we can see the Gini coefficients for wealth within the richest and poorest 20%. This validates our aforementioned hypotheses concerning the movements of income groups within the overall distribution of assets.

In summary, a technological shock hits the economy heterogeneously with regards to the monetary policy and the countries in question. We can clearly distinguish a contrast between a common monetary policy and an individual one when comparing the core and periphery. One clear result is that the preexisting foreign asset positions lead the changes in interest rate, which then exacerbate core and periphery differences. Within the union, the salient disparities between core and periphery lie within the dispersion of idiosyncratic productivities and their associated risks. In fact, the former is more concerning in core countries, where inequalities are higher in equilibrium and the latter is higher in the periphery, as social mobility is less rigid. In monetary independence however, the core benefits from more production and consumption in a crisis, while the periphery suffers from opposite effects. On the other hand, with a free financial market, the core sees its inequalities and current account balance worsen after a crisis, while the periphery improves them. These differences lie within the savings behaviors of each country, linked to their idiosyncratic profiles, and their equilibrium net foreign assets positions. Spain and Italy require more savings to face idiosyncratic shocks, due to higher risks of experiencing a shock transition, while France and Germany can allow themselves to lower their savings during a crisis to benefit from more consumption.

6 Results

This paper sheds light on three main concepts: the relationship current account imbalances and inequalities entertain with micro-level savings behavior; the heterogeneity of tools Eurozone countries have when facing aggregate shocks; and the benefits of a common monetary policy.

First, current account imbalances and wealth inequalities have proven to be intimately linked to savings decisions made at the microeconomic level. In Sections 5.2, 5.3 and 5.4, this concept is covered extensively and the central point supporting this hypothesis is that countries differ in their social mobility, represented by the persistence of their idiosyncratic shocks, and in the dispersion of their wealth distribution, equivalent to the range of idiosyncratic productivities. Social mobility influences the way agents consider precautionary savings, which drive the aggregate bond holdings. This diffusion of capital across the distribution is what constitutes the wealth inequalities, but also what powers the flows of savings to or from foreign markets. This is specifically the case for monetary unions, as their demand for capital is less flexible due to a less volatile, or fixed, interest rate. Hence, financial market clearing depends only on the private savings in equilibrium. Too much precautionary savings and the capitals will flow outward, to foreign markets; too little and the foreign capitals fill the gap with the domestic firms and generate a current account deficit. The dispersion of the wealth distribution, on the other hand, is unrelated to the persistence of individual productivities *per se*, but has a similar effect on the micro-level decisions: a high dispersion means that an idiosyncratic shock can shift your income by a large amount, while a small spread implies that even when a shock occurs, the effect is not large enough to trigger the need for high precautionary savings. Persistence relates to the frequency of the shocks, whereas dispersion is linked to their magnitudes. When the dispersion is high, agents need to save considerable amounts in order to insure themselves against idiosyncratic risks, which directly translates to an increase in the wealth of those who can afford to save. This, in turn, has the same effect as the persistence on inequalities and current account balance.

Then, the level of crisis preparedness of each Eurozone country is widely heterogeneous. This idea is covered in Sections 5.2, 5.3 and 5.4 as well. The periphery is characterized by high interest rates and idiosyncratic risks, while the core displays low rental rates and less social mobility. The characteristics of the periphery are fertile ground for more volatile savings when an aggregate shock hits the economy. The nature of the risks and the equilibrium exterior positions of these countries contribute to high precautionary savings in equilibrium, which are then largely decreased by a negative productivity shock, and increased by a positive monetary shock. This high volatility leads to a higher sensitivity to aggregate shocks regardless of the monetary scenario. Core countries are intrinsically more stable, for better or worse, as crisis will not be too difficult to handle for them, but they will not reap the rewards of expansions as much as the periphery.

Finally, the common monetary policy enforced by the Eurozone seem to have heterogeneous effects across member states, and we try to identify its benefits in Section 5.5. The most salient result is that the constrained movements of the interest rates hinder the effects of an aggregate shock in the Eurozone, regardless of its sign. This implies that an economic boom as well as a crisis are attenuated in the common currency paradigm, whereas an independent monetary policy bolsters the effects of each shock. Of course, this paper does not model the coordination of the European Union in times of crisis, which are detailed in Aguiar et al. (2015). Without the European Union to actively attenuate the consequences of crises, this analysis cannot fully encompass the effect of negative aggregate shocks. Furthermore, it seems that core countries would experience much different variations, compared to the periphery, if it had monetary independence. Crises in the periphery can translate to expansions in the core. These findings seem to indicate that, at a time of shocks, the common monetary policy constrains the deviations of each variable, but also forces each country to follow the same path after a crisis, thus homogenizing the member states and allowing for coordinated responses to a shock. The cost of an endogenous rate, however, is unclear as the realm of policy implications of leaving the Eurozone extends further than the scope of this paper.

7 Conclusion

This paper presents a model of heterogeneous agents applied to a monetary union and investigates the implications of region-wide shocks on aggregate variables. The main results of this model are that the current account balance and wealth inequalities both stem from micro-level savings behaviors, which implies that net exports take their roots in microeconomic decisions; the Eurozone member states are equipped in a heterogeneous fashion to face aggregate shocks, as the frequency and magnitude of their idiosyncratic shocks, as well as exterior positions, are disparate, which creates differences in each country's response to a crisis or an economic boom; and finally that a common interest rate helps homogenizing the responses of the member states.

Evidently, this paper has its caveats. First, the model is built around flat taxes on capital gains and labor, while an appropriate structure should have progressive taxes and a VAT tax. Moreover, the fiscal authority's budget is only used to finance a public good, whereas redistribution would be more fitting in the context of studying inequalities. Furthermore, the monetary authority does not optimize an overall interest rate but is just modelled with a simplification of the reality: an invariant policy rate. Although this allows us to analyze the paths of aggregate variables with respect to an endogenous counterfactual, its accuracy is limited. Next, the model is in closed economy, where the current account balance is generated by a disequilibrium of the financial markets, while this representation is convenient and allows us to sketch results concerning imbalances, a small open economy would be better suited to represent this paradigm.

Ultimately, this paper builds ground for further research in the field of heterogeneity in monetary unions and could benefit from a more accurate representation of reality through model improvements. An interesting feature that would shed light on the inherent dynamics of each country when facing a shock is time-varying idiosyncratic risks, as described in McKay (2017). These comovements between individual risk and the business cycle could help us understand the link between skill premium and wealth inequality in a currency union, where member states must showcase interesting disparities in their intrinsic skill prices. Moreover, in the context of monetary unions, the idea of countries pouring out their savings in other member states and attracting investments from abroad is an interesting feature that could also help explain the intricacies of the distribution of savings in a currency union.

Appendices

A Calibration Details

This appendix details the matching algorithm used to find the pair of parameters $\{\rho, \sigma_\varepsilon^2\}$ for each country.

First, we define empirical measures of the Gini coefficient for after-tax wealth and the net foreign assets over GDP, via the data described in Section 2, as $\mathcal{G}_{data}$ and NFA_{data}. The goal of the matching is to minimize the distance between the data and the model, and as inequalities and net foreign assets are at the center of this paper, we use these variables to anchor our algorithm. The minimization problem then becomes:

$$\min_{\{\rho, \sigma_\varepsilon^2\}} \alpha \|\mathcal{G}_{data} - \mathcal{G}_{model}(\mathcal{P})\| + \beta \|NFA_{data} - NFA_{model}(\mathcal{P})\| \tag{43}$$

where $\alpha = 100$ and $\beta = 1$ are weights found through trial and error and $\mathcal{P}$ is the vector of parameters defined in (37). In order to perform this minimization, we set starting values chosen again through trial and error, we set lower and upper bounds for each variable.

The optimization algorithm used is Nelder-Mead which is convenient for this problem as it does not require a gradient and instead uses an iterative simplex that searches for local minima. The resulting matching reports errors of the following scales:

Table 6: Matching errors

	$\mathcal{G}$	NFA
France	1.14×10^{-11}	6.10×10^{-10}
Germany	1.40×10^{-11}	3.40×10^{-9}
Italy	5.03×10^{-11}	3.97×10^{-10}
Spain	6.53×10^{-11}	8.77×10^{-10}

We can see that the matching errors are not significant on the scale of our model simulations and the inequalities and net foreign assets are thus accurately matched.

B Additional Figures

This appendix contains Figures 22-27.

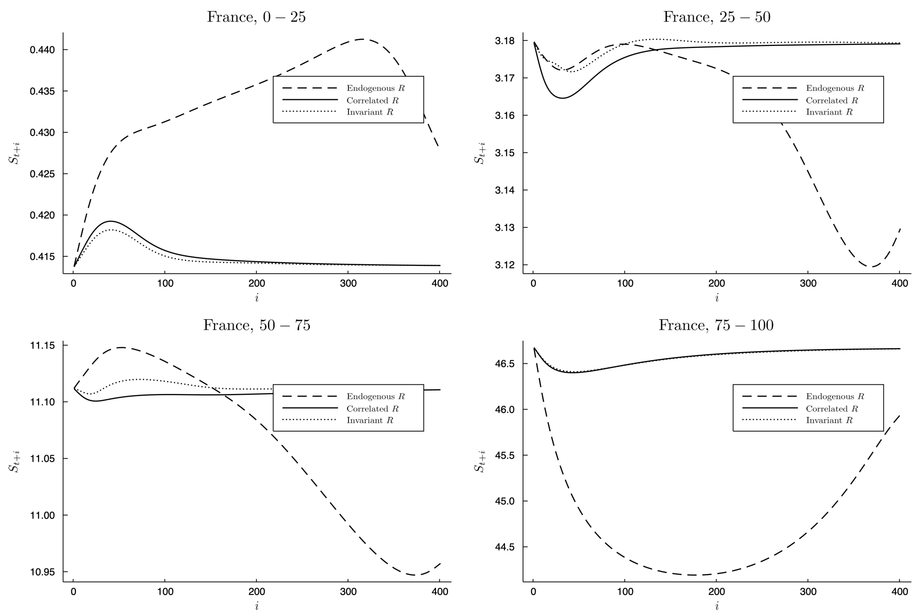


Figure 22: Savings per quartile for France for $\zeta = -1\%$

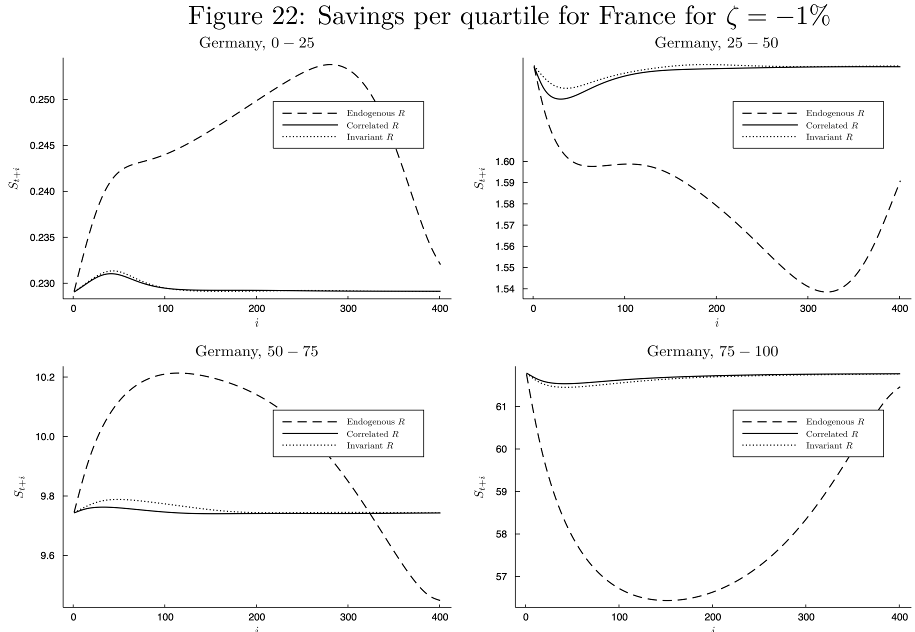


Figure 23: Savings per quartile for Germany for $\zeta = -1\%$

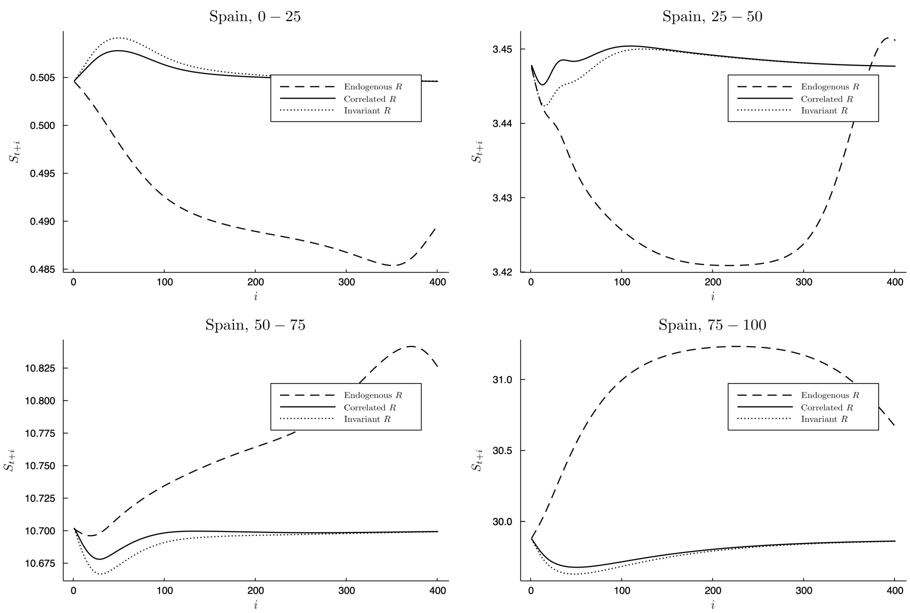


Figure 24: Savings per quartile for Spain for $\zeta = -1\%$

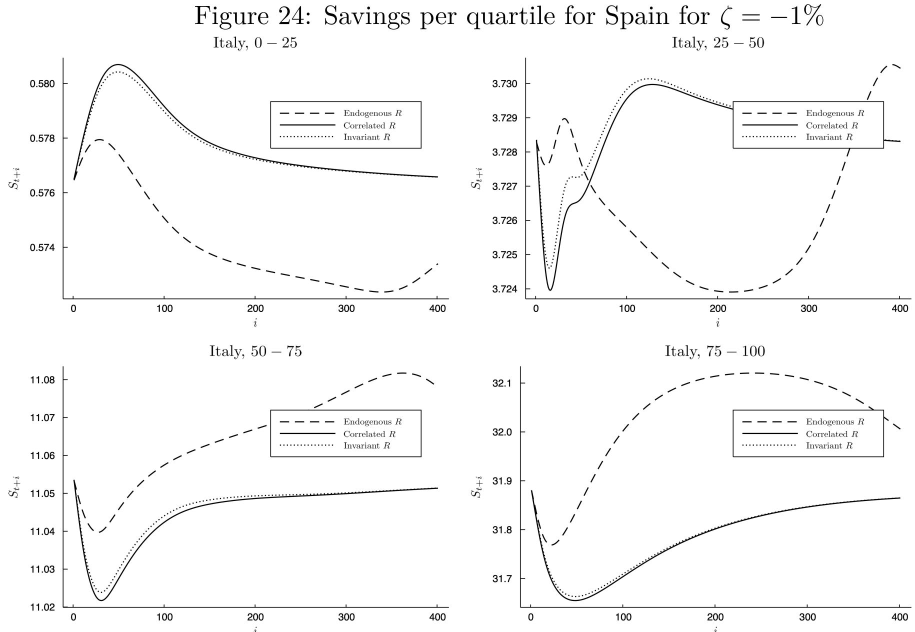


Figure 25: Savings per quartile for Italy for $\zeta = -1\%$

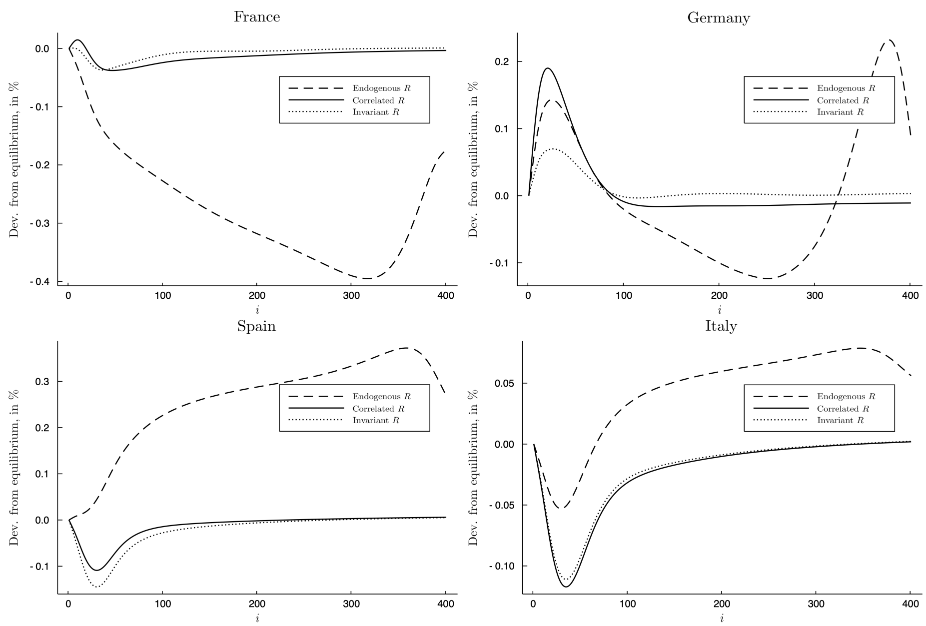


Figure 26: Change in wealth Gini for the poorest 20% for $\zeta = -1\%$

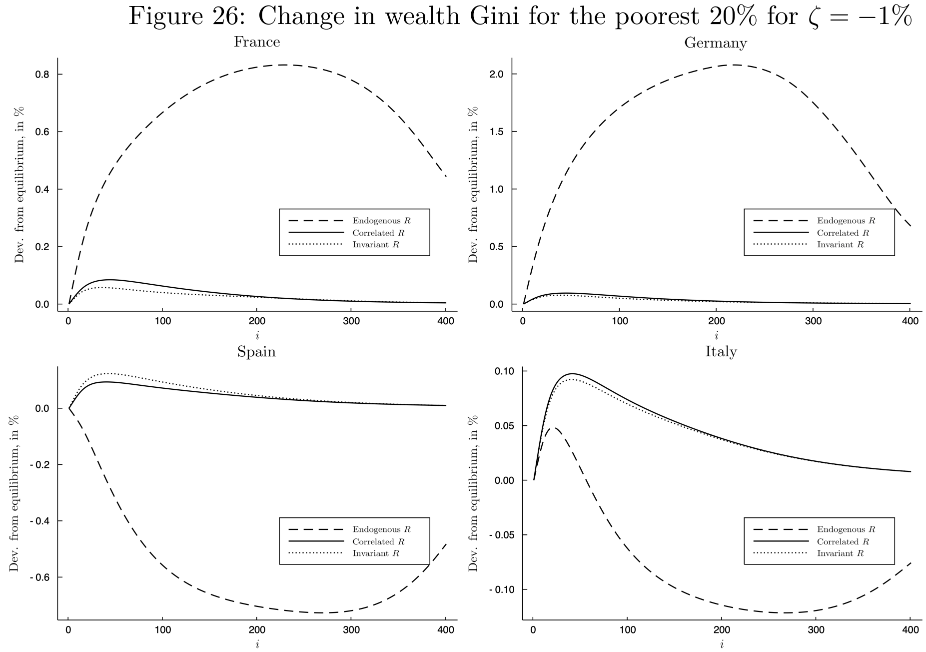


Figure 27: Change in wealth Gini for the richest 20% for $\zeta = -1\%$

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