Wage Fixing[*]

Axel Gottfries[†] Gregor Jarosch[‡]

February 13, 2025

Abstract

We model and analyze employer cartels that fix wages by committing to a wage ceiling. The setting is a frictional labor market with large employers that compete for workers via posted wages. Wage fixing reduces competition both inside and outside the cartel, leading to market-wide wage depression. Competition from outside employers disciplines the cartel and hence governs its wage impact and profitability. Consequently, wage-fixing schemes are more likely to emerge and remain stable when the labor market has slack, concentration is high, the span of control is small, product demand is elastic, and firms also collude in the product market. We describe a simple sufficient statistic to gauge the harm caused by a wage-fixing cartel.

[*]Thanks to David Berger, Laura Pilossoph, and Isaac Sorkin for useful conversations. Thanks to Leqing Huang for research assistance.

[†]University of Edinburgh, axel.gottfries@gmail.com

[‡]Duke University and NBER, gregorjarosch@gmail.com

"We rarely hear, it has been said, of the combinations of masters, though frequently of those of workmen. But whoever imagines, upon this account, that masters rarely combine, is as ignorant of the world as of the subject. Masters are always and everywhere in a sort of tacit, but constant and uniform combination, not to raise the wages of labour above their actual rate. [...] We seldom, indeed, hear of this combination, because it is the usual, and one may say, the natural state of things, which nobody ever hears of."
(Adam Smith, *Wealth of Nations, Book I, Chapter VIII, Of the Wages of Labour*, 1776)

1 Introduction

Concerns that employers collude to suppress wages date back centuries but have recently drawn renewed interest from competition authorities in the US and Europe. For example, a 2021 US presidential executive order explicitly encouraged the Federal Trade Commission (FTC) and the Department of Justice (DOJ) to prevent employers from collaborating to suppress wages (The President (2021)). Similar concerns have been expressed by European competition authorities (Joint Nordic Report (2024), Aresu et al. (2024)), following a broader shift in modern antitrust research and regulation towards the labor market. This policy shift has coincided with a growing number of legal cases related to employer collusion (see, e.g., Kariel et al. (2024)).

The DOJ and FTC antitrust guidelines emphasize two distinct forms of collusion in the labor market: wage-fixing agreements, where employers coordinate on salary levels or ranges, and no-poaching agreements, where they agree not to hire each other's employees (DOJ and FTC (2016)). Both practices have seen little attention from economists studying labor markets and antitrust.

This paper focuses on wage-fixing agreements. We show how to model employer *cartels* that fix wages in a canonical model of employment and wages in a frictional labor market. The cartel colludes on a wage ceiling ("going rate"), curbing wage competition for workers. We obtain a sharp characterization of the resulting equilibrium pay distribution which we complement with a rich set of qualitative and quantitative numerical observations.

Our approach allows us to i) understand the economic mechanisms through which wage fixing might depress wages, ii) capture its full general equilibrium effect, iii) identify settings in which collusion is particularly likely to be stable, profitable, or damaging to workers, and iv) assess its impact without the demanding information required by a production-function based approach in the industrial organization tradition. In addition, our emphasis on firm granularity allows us to better understand the interplay between wage fixing schemes and market structure.

Our theory builds on the dynamic monopsony model in Gottfries and Jarosch (2023). This is a wage posting model in the spirit of the Burdett and Mortensen (1998) (BM) textbook environment which lends itself naturally to wage-fixing schemes because it is an explicit model of competition in the labor market via wage posting, with employers poaching each other's workers via higher pay along the job ladder. An explicit and descriptively appealing notion of wage fixing thus arises naturally: A subset of the employers can form a cartel that agrees not to offer pay above some ceiling.

Compared with the textbook setting, our framework has granular firms, decreasing returns to scale in production, market power in the product market, and a linear hiring technology. This makes the environment richer, more flexible, and useful for quantitative analysis in a broad range of settings.

After laying out the environment, the main theoretical results establish key properties of the equilibrium wage offer distribution in the presence of a wage-fixing cartel. Absent wage fixing, employers post dispersed, continuously distributed wages. With wage-fixing, there exist several different cases, depending on how aggressively the cartel restricts pay (which we take to be exogenous to begin with). When the wage ceiling is tight, then all cartel employers offer the ceiling, while the remaining, outside employers offer dispersed wages that begin just above it.

We discuss how the shape of the equilibrium changes as the cartel relaxes its wage ceiling. In this case, cartel firms first begin posting some jobs below the ceiling and, eventually, the wage offer distributions between cartel and outside firms begin overlapping. In proving the main existence and uniqueness result (Proposition 1), we establish this characterization, along with several intermediate results that illustrate the key economics at play. We show, for instance, that there cannot be any mass or any gaps in the wage offer distribution away from the wage ceiling and that all firms of the same type act symmetrically.

We next examine the impact of a wage fixing cartel on wages, employment, and profits. There are spillovers to the outside employers, who disproportionately benefit from wage fixing. This is not surprising because the outsiders can always mimic the cartel, so they must fare at least weakly better, but it has strong implications.

First, it means that the outsiders have no incentive to "blow the whistle" on the cartel.

Second, it means that a wage-fixing cartel is fragile, since profits are higher outside. We argue that the temptation to leave the cartel, a common force in collusion settings (Asker and Nocke, 2021), is muted in our granular environment. A large cartel member contemplating exit recognizes that such a move would shift the overall equilibrium, resulting in a jump in pay and drop in profits everywhere. Consequently, a cartel is more likely to emerge and be stable in a concentrated labor market.

Third, it means that the outside firms' labor demand response acts as a constraint on the cartel. If the cartel is too aggressive in curbing wages, the outside competitors—which benefits heavily from cartel activity—expands employment and drives the cartel firms out of the market.

We investigate the disciplining role of the outside employers further and ask how a cartel optimally sets its wage ceiling. We find that an optimizing cartel generally pays well above the reservation wage. A key prediction is thus that harm and profits caused by a wage fixing cartel are large whenever the response of the outside competition is muted. For similar reasons, wage-fixing cartels are only profitable when a sufficiently large portion of the market participates.[1]

We then turn quantitative with the model, which is straightforward to calibrate. We use a benchmark US calibration for a (local) labor market and then introduce a cartel similar in size to the one that has been documented in the US poultry processing industry (DOJ Antitrust Division (2023)).[2] Instead of focusing on a single, quantitative benchmark, we explore a range of cases with a focus on comparative statics, asking which features of the environment make wage fixing cartels particularly harmful to workers.

Our baseline calibration suggests that the overall labor market impact of wage fixing is quantitatively modest. We find that removing the cartel results in market-wide wage gains of just over 2%. Varying key model parameters, however, underscores the critical role of outside firms in disciplining a wage-fixing cartel. For instance, cartel-driven wage losses quickly rise to 6-8% when outside competition is low, say due to labor market slack,

[1] We do not endogenize participation or selection into the wage fixing cartel. Instead, we characterize equilibrium in the labor market given the presence of a cartel.

[2] Multiple US meat and poultry processing employers recently settled in a string of wage fixing cases, see, e.g., Reuters and Reuters (Scarcella, 2024a,b).

or unresponsive because labor and other inputs are strong complements in production. To the same point, we find a key role for the span of control in determining the wage losses from wage fixing. If the outside firms ramp up employment in response to their cost advantage, the cartel can only do limited damage.

The quantitative section next explores how the product market affects the impact of wage-fixing schemes. When demand is inelastic, the outside firms pass through their cost gains from wage fixing lowering prices, a force that moderates a wage fixing cartel. Further, we show that a cartel that also colludes in the product market optimally sets a lower wage ceiling.

We speculate that these findings may explain why wage fixing has frequently been documented in sectors where labor market is concentrated and competition is weak (such as the US food processing industry) or in sectors where one might suspect that collusion easily extends into the product market (e.g. health, sports, education).[3]

We conclude the paper with a final section that offers a sufficient statistic to calculate the (wage) harm caused to workers by a wage-fixing cartel. These calculations use endogenous objects, such as employment inside and outside the cartel, along with model primitives and require industry-specific information that can straightforwardly be obtained on a case-by-case basis.

Related literature

This paper is related to a large and growing literature on market power and barriers to competition in the labor market. That literature frequently takes a static, frictionless, "new classical" (Manning, 2021) approach to competition in the labor market (Robinson, 1933; Card et al., 2018; Berger et al., 2022). In contrast, we take a "modern" or "dynamic" (Manning, 2021) approach that roots market power of employers in search frictions in the BM tradition. In the same vein, Gouin-Bonenfant (2022) explores the implications of increasingly dispersed firm productivity in a BM setting for competition in the labor market. Heise and Porzio (2023) apply an extended BM framework to study spatial inequality. Our paper also relates to recent work that introduces decreasing returns to scale produc-

[3] See, e.g., various DOJ indictments for nurses (U.S. v. Ryan Hee and VDA OC, LLC, formerly Advantage on Call, LLC, 2021), physiotherapist (United States v. Jindal, 2020), healthcare workers (United States v. Manahe, 2022) or recent case in Lithuania for basketball players.

tion into random on-the-job search models (Bilal et al., 2022; Bilal and Lhuillier, 2022; Elsby and Gottfries, 2022; Lentz and Mortensen, 2012). Jarosch et al. (2024) is a related framework that has large firms in a frictional labor market and wage bargaining. In our setting, wages are posted.

Despite the attention from regulators and its anecdotal ubiquity, employer collusion in the labor market has seen little attention from academic economists. Recent exceptions studying collusion theoretically include Martins and Thomas (2023) and Bisceglia (2024). An empirical exception is Delabastita and Rubens (2024) whose approach detects collusion in the labor market via otherwise unexplained markdowns. Similarly, Sharma (2024) develops a test in the neoclassical setting that registers labor market collusion whenever firms expand in response to a competitors' positive demand shock, detecting significant collusion in the Indian textile industry. These approaches towards collusion in the labor market in the IO tradition typically lump all forms of employer collusion into reduced-form wedges. In contrast, this is the first paper to model wage fixing explicitly from first principles.

Of course, our paper is also related to a large and mature literature in IO on cartels, collusion, and price fixing. It emphasizes questions around cartel formation and cartel stability that are closely related to some of those we discuss in the context of the labor market. Several differences are worth noting. First, this literature has, to date, almost exclusively focused on product markets.[4] Our focus is on cartels in the labor market. In addition, we also model cartels that operate jointly in the labor and in the product market. As such, our framework jointly determines employment and wages, prices and output and can hence speak to the full general equilibrium effects of a wage fixing scheme. In contrast, the IO literature typically works with partial equilibrium frameworks. Finally, we work with a dynamic monopsony search model—a workhorse in the "macro-labor" approach to study employment, wages, and (imperfect) competition in the labor market—making it a natural starting point for the study of wage fixing cartels.

The arguments in the IO literature often center around the logic in Friedman (1974)'s take on OPEC—that cartels collapse once they curtail output too aggressively because the resulting price makes it too tempting to produce. The key question then is how technol-

[4]"The Economics of Collusion" (Marshall and Marx, 2012) and other books and survey articles such as Asker and Nocke (2021) on "Collusion, Mergers, and Related Antitrust Issues" pay virtually no attention to labor markets.

ogy, patience and market structure mediate this (see, e.g., surveys Ivaldi et al. (2003) and Levenstein and Suslow (2006)). In a similar spirit, we emphasize how technology and market structure mediate the disciplining role of the outside competition on the cartel. A new theme that emerges with no direct counterpart in the product market literature is how aggregate labor market conditions—slack or tight—matter for wage fixing schemes.

2 Model

We extend the model of Gottfries and Jarosch (2023) to incorporate collusion between employers. We first outline the main parts of the baseline model, but refer the reader to Gottfries and Jarosch (2023) for additional details. We then describe how we model collusion in this setting.

The framework features search frictions and wage posting along the lines of Burdett and Mortensen (1998). Time is continuous, and workers and firms both discount the future at a rate r. There is a unit measure of workers. Workers receive flow income b when unemployed, while employed workers receive a contracted wage w and lose their jobs and become unemployed at rate δ. There are M symmetric profit-maximizing firms, all large with respect to the labor market in the sense that they will each employ a strictly positive fraction of the workforce. Firm i operates a decreasing-returns production function, $F_i(N_i) = xN_i^{\alpha}$, with N_i denoting the measure of employees and x denoting firm-level productivity. All firms produce a homogeneous good.

Search is random, and workers search for jobs both on and off the job. Employers hence experience turnover because workers move both to unemployment and to better paying jobs. To hire, firms post wages and decide on a rate at which they make offers to workers, both employed and unemployed. Whenever a firm hires a worker, it needs to pay a hiring cost c. There is hence an upside to higher pay. It leads to less costly turnover of workers.

Because firms are large and search is random, employed workers sometimes receive job offers from their own employers. We assume that if the worker receives such an offer and it comes with a higher wage, she is free to accept that offer. In this event, the firm does not need to pay the hiring cost again. To the extent that firms post dispersed wages, there is hence scope for internal raises. We focus on a stationary equilibrium.

For the exposition and the theoretical analysis, we assume that demand in the output market is perfectly elastic, with price p normalized to 1. In Section 4.1, we endogenize the price, assuming quasi-linear utility over consumption and entertaining several different forms of conduct in the product market.

Wage Fixing in the Labor Market

We are interested in collusion among a subset of employers in the market. Specifically, we consider *wage fixing*, where $k \geq 2$ cartel members agree not to offer wages above some wage ceiling w_f. We denote the set of firms in the cartel by C. For lack of a better term, we call the $M - k$ firms not in the cartel the *outside* employers.

This type of employer collusion is illegal in most countries, which raises the question of who, beyond the k participants, is aware of it. We assume that all employers in the labor market are aware, including those not participating, who we henceforth refer to as the "outside" employers. These $M - k$ outsiders thus respond to the presence (and the choices) of the cartel when making wage offers and choosing employment. We show below that the outside employers benefit from the presence of the cartel, and hence have no incentive to whistle-blow.

In contrast, we assume that workers are not aware of the collusion and do not learn about it as they navigate the labor market. This assumption seems natural given that these practices are illicit and happen in secrecy. It also is meant to capture that, until recently, these practices received little attention. In addition, the resulting collusion and no-collusion equilibria differ in ways that would make it hard for individual workers to quickly detect collusion through an anomaly. While we consider this assumption to be sensible, it also buys tractability, since the reservation wage does not respond to cartel activity. Assuming a binding minimum wage would yield the same property. We briefly discuss how to endogenize the reservation wage at the end of the next subsection.

Workers

Workers in the baseline setting know all primitives and have rational expectations over the equilibrium, unaware of any collusion. They only make one meaningful decision—the reservation wage, which we characterize in this section—and otherwise mechanically move up the wage ladder.

Unemployed and employed workers expect to make contact with some employer j at perceived rates ψ_j^W and $s\psi_j^W$, respectively, where $s \leq 1$. If that happens, they expect to draw from the firm's perceived distribution of posted wages $F_j^W(w)$. We highlight that these three perceived objects do not correspond to their equilibrium counterparts because worker are unaware of the collusion. Instead, they correspond to the contact rates and wage offer distribution in the equilibrium without collusion due to the information structure outlined above. Workers are assumed to move with some small probability κ when they receive an outside offer with identical pay.[5]

Workers' preferences are linear in income. A worker's *perceived* value of unemployment and employment at wage w hence satisfy, respectively,

$$
\begin{aligned}
rU &= b + \sum_{j=1}^{M} \psi_j^W \int_{w_r}^{w_u} \left(W(\tilde{w}) - U\right) dF_j^W(\tilde{w}), && (1)\\
rW(w) &= w + \delta\left(U - W(w)\right) + \sum_{j=1}^{M} s\psi_j^W \int_{w}^{w_u} \max\left\{W(\tilde{w}) - W(w), 0\right\} dF_j^W(\tilde{w}). && (2)
\end{aligned}
$$

The reservation wage satisfies $W(w_r) = U$, which by standard arguments yields

$$
w_r = b + (1-s)\int_{w_r}^{w_u} \frac{\sum_j \psi_j^W \left(1 - F_j^W(\tilde{w})\right)}{r + \delta + \sum_j s\psi_j^W \left(1 - F_j^W(\tilde{w})\right)} d\tilde{w}. \tag{3}
$$

It follows that the reservation wage does not respond to cartel activity. To endogenize the reservation wage, one only needs to replace the no-collusion objects ψ_j^W and F_j^W with their full equilibrium counterparts ψ_j and F_j in equations (1)–(3).

Firms

The firm problem is similar to that in Gottfries and Jarosch (2023), except for one additional constraint: All cartel employers are subject to a self-imposed wage ceiling w_f.

We cast the problem of firm i as one of directly choosing employment N_i and the distribution of wages across employed workers, denoted $G_i(w)$. In actuality, firms do not directly control these. Instead, to implement the optimal time-invariant solution, firms i) initially hire their desired number of workers at the desired distribution of wages; and ii)

[5] We emphasize that κ is small—which seems sensible—since the main uniqueness proof requires a sufficiently small value to rule out asymmetric equilibria; see lemma A.8.

then maintain this via a constant contact rate ψ_i and distribution of posted wages $F_i(w)$. Because the hiring technology is linear, such an instantaneous transition to steady state is feasible.

Firms hence choose total employment and a distribution of pay to maximize the present value of profits. Cartel firms do so under the additional constraint that pay must not exceed w_f. The firm problem is hence,

$$\begin{aligned}\Pi_i &= \max_{N_i, G_i(w)} -cN_i + \int_0^\infty e^{-rt} \times \\ &\left(xN_i^\alpha - N_i \int_{w_r}^\infty \left(w + c \left(\delta + \sum_{j \neq i} s\psi_j \left(1 - F_j\left(w\right)\right)\right)\right) dG_i(w) \right) dt \\ &\text{subject to } G_i(w_f) = 1 \text{ if } i \in \mathcal{C}.\end{aligned} \tag{4}$$

The firm hires its workforce up front and then sustains it by replacing those it loses to unemployment and higher paying jobs elsewhere at cost c. Flow profits are given by gross revenue net of the wage bill and turnover costs because workers to unemployment and better paying jobs at the competition need to be replaced at cost c per worker. The wage-fixing constraint is the key new piece. A cartel member cannot pay above the wage ceiling w_f.

As is implicit in the statement of the firm problem, firms take the reservation wage w_r and their competitors' actions—the contact rates ψ_j and offer distributions $F_j(w)$—as given when making their choices. In that sense, we consider a Nash equilibrium where the agents, despite being large, take each other's actions as given.[6]

Because the allocation instantaneously jumps to its steady state, an equivalent statement of the firm problem is to maximize flow profits,

$$\begin{aligned}r\Pi_i &= \max_{N_i, G_i(w)} xN_i^\alpha - N_i \int_{w_r}^\infty \left(w + c \left(r + \delta + \sum_{j \neq i} s\psi_j \left(1 - F_j\left(w\right)\right)\right)\right) dG_i(w). \\ &\text{subject to } G_i(w_f) = 1 \text{ if } i \in \mathcal{C}.\end{aligned} \tag{5}$$

The term under the integral is the *user cost of labor*. It consists of the wage paid and the turnover cost, which is wage-specific because higher wages might come with a lower quit

[6] This can be micro-founded, analogously to the textbook BM model, assuming that firms simultaneously commit to their policies at time zero.

rate. The turnover cost includes, in annuitized form, the cost of hiring the initial workforce.

3 Theoretical results

This section offers a theoretical analysis of the equilibrium effects of employer collusion. As a backdrop, we briefly characterize the equilibrium without collusion. We then study a setting with wage fixing.

3.1 Equilibrium without collusion

The equilibrium in this case is a special case of Gottfries and Jarosch (2023). The difference is that in Gottfries and Jarosch (2023), we assume firms are strategic about an endogenous output price. Here, we instead assume that demand is perfectly elastic so that firms take a fixed output price as given.

The unique equilibrium then takes the following form. All M firms post a mix of wage offers uniformly on $[w_r, w_u]$,

$$F(w) = \frac{w - w_r}{w_u - w_r}, \tag{6}$$

where w_u denotes the highest wage posted. Workers contact a firm at endogenous rate ψ, and the highest wage solves

$$w_u = w_r + s\psi(M-1)c. \tag{7}$$

This implies that the user cost of labor is equated across and within firms. The reservation wage is given by

$$w_r = b + (1-s)c\left(1 - \frac{1}{M}\right)\left(M\psi - \frac{r+\delta}{s}\log\left(\frac{r+\delta+sM\psi}{r+\delta}\right)\right). \tag{8}$$

Because the marginal revenue product equals the user cost of labor under optimal hiring, the contact rate ψ satisfies

$$\alpha x \left(\frac{\psi}{\delta + M\psi}\right)^{\alpha-1} \quad = \quad w_r + (r + \delta + s(M-1)\psi)\, c. \tag{9}$$

The left hand side is just labor's marginal revenue product, with firm-level employment given by $N_i = \frac{\psi}{\delta+M\psi}$ as follows from a standard flow balance relationship. The right hand side is the user cost of labor at the reservation wage which is, under optimality, equal to the user cost of labor everywhere in the distribution of posted wages. These four equations fully characterize the equilibrium.

3.2 Equilibrium with wage fixing

We next study a case in which k out of the M employers collude to not raise the wage above some threshold w_f. We do not impose on the equilibrium that w_f is set optimally or that forming or joining the cartel is rational. Instead, we first characterize equilibria given some k and w_f and then study how user cost, wages, and profits for cartel members and outsiders depend on both these variables in Section 3.4.

Equilibrium Characterization and Existence

This section characterizes equilibrium with wage fixing and establishes existence and uniqueness. We relegate most formal details and derivations to Appendix 1 and here just describe key properties and offer intuition.

Characterizing the Equilibrium Wage Offer Distribution The proof of the main existence and uniqueness result below establishes that the equilibrium can take four distinct shapes. We formally characterize these cases in the proof in appendix A and here just describe the corresponding wage offer distributions.

The cases are ordered by how aggressively the cartel sets the wage ceiling w_f, relative to the reservation wage w_r. We denote by ψ_c the rate at which a cartel firm makes offers to unemployed workers.

Case I. All cartel firms offer all their jobs at w_f. The outside firms offer jobs uniformly distributed over $(w_f, w_u]$ at a common rate.

The following condition guarantees that this is an equilibrium: $w_f \leq w_r + s(k-1)\psi_c c(1-\kappa)$. This condition guarantees that the cartel members prefer paying the wage ceiling instead of the reservation wage (or any wage in between) that comes with the additional turnover from losing workers to other cartel employers posting at the mass point.

Case II. The cartel posts a mass of jobs at w_f, but additionally posts uniformly distributed wages at a common rate on $[w_r, w_1]$, with a gap such that $w_1 < w_f$. The outside firms post uniformly distributed wages on $(w_f, w_u]$ at a common rate, as in Case I.

The following condition guarantees that this is an equilibrium: $w_r + s(k-1)\psi_c c \geq w_f > w_r + s(k-1)\psi_c c(1-\kappa)$. The second equality simply states that we are not in Case I, while the first states that the cartel must prefer paying the ceiling over the reservation wage if all cartel activity is below the ceiling.

It is easy to see why the equilibrium changes from Case I to II as w_f rises sufficiently far above w_r. In this case, posting wages below w_f becomes attractive despite the strictly larger turnover cost generated by the mass point above. Wages are uniform below w_1 because firms trade off wages and retention in the usual fashion, which also gives rise to uniformly distributed wage offers in the no-collusion equilibrium. As the cartel posts these additional jobs below the ceiling, it pulls back on hiring from the mass point because it would otherwise grow in size. Eventually, the gap closes and $w_1 \to w_f$, while simultaneously the mass at w_f vanishes. This is exactly where the first weak inequality above holds with equality.

Case III. There is overlap in the wages posted by cartel members and outside employers. Cartel members post uniformly and with uniform rate on $[w_r, w_1]$, while cartel members and outside employers act identically on the next interval, all posting uniformly and with identical rate on $(w_1, w_f]$. Above w_f, only outside employers post, again uniformly distributed and with the same rate. The rates across these three intervals generally differ (and are presented in the appendix). There is no longer mass at the wage ceiling but the ceiling is still binding for the cartel.

The following condition guarantees that this is an equilibrium: $w_r + s(M-1)\psi_c c \geq w_f > w_r + s(k-1)\psi_c c$. The second inequality simply states that we are not in Case II. If the first equality does not hold, then there is a fourth case described in the appendix. We henceforth rule this case out, which can be done via a parameter restriction that guarantees that the first inequality holds. The restriction is cumbersome, and we describe it in the course of the proof of Proposition 1 below (see Assumption 1 and Lemma A.12 in the appendix). The restriction guarantees that employment in the cartel does not contract too much as a result of the wage ceiling. We will see below that the cartel suffers a cost disadvantage relative to the outside firms. The restriction is violated if, due to their cost advantage, the outside firms drive the cartel largely out of the market. In this scenario, the cartel can then satisfy its diminished employment demand by hiring from only a small segment of the wage support $[w_1, w_f]$ with $w_1 > w_r$. The assumption is therefore more likely to be violated in settings where the production function is close to linear.[7]

Figure 1 plots these three cases separately for the cartel (Panel A) and the outside employers (Panel B). It plots the wage offer distribution for a tight wage ceiling w_f (Case I) in blue. The cartel exclusively posts at w_f, while the outside firms post uniformly right above the ceiling.[8]

We also plot the wage offer distribution for an intermediate wage ceiling (Case II), indicated by the vertical dashed red line. The cartel still posts a mass of jobs at the ceiling but, in addition, it posts uniformly distributed wages starting from the reservation wage, with a gap in the support below the ceiling w_f. The outside firms again post uniformly above the ceiling.

Case (III) has the most moderate wage ceiling, indicated by the vertical purple line. There is no mass anywhere, and the cartel exclusively posts uniformly distributed wages up to some intermediate wage level. Above that, both cartel and outsiders post uniformly distributed wages up to the ceiling. Above that, only the outsiders post uniformly distributed wages.

Existence and Uniqueness

PROPOSITION 1. There exists a unique symmetric equilibrium with wage fixing.

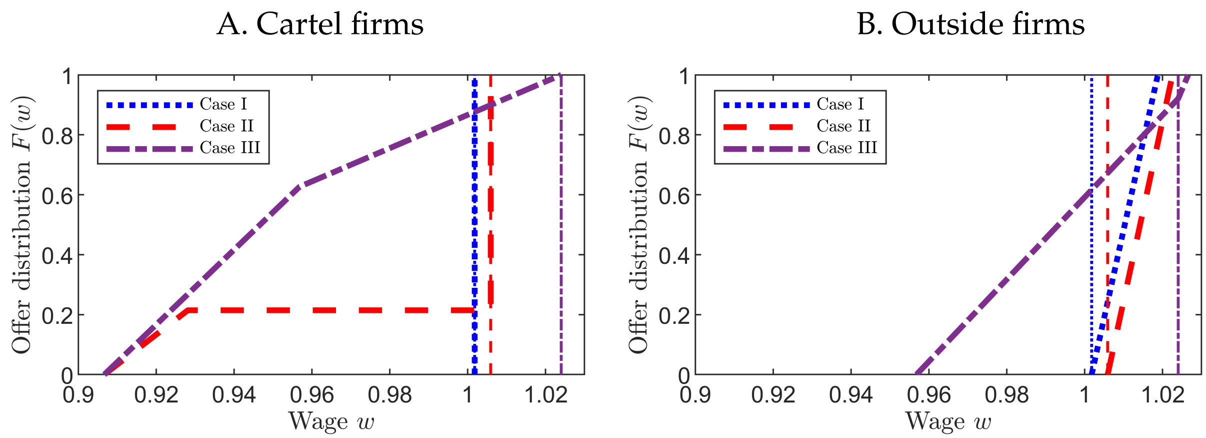
Proof. See Appendix A. □

In the course of proving the result, we establish that the equilibrium has one of the three shapes that we discuss in the previous section. In addition, the proof establishes several intermediate results, some of which we will discuss next, since they convey some of the key economics of the framework with wage fixing.

[7] The reason we do not consider this last case is primarily that we have been unable to rule out equilibrium multiplicity in that range. In the numerical exercises below, we consistently find that a profit-maximizing cartel sets an optimal w_f that falls into Case I, far from this fourth case that we do not consider.

[8] The figure shows that in Case I, our assumption that workers are unaware of cartel activity is inconsequential because the reservation wage is not "binding", as opposed to Cases II and III. Case I is the relevant case in all our quantitative exercises below whenever the cartel optimally picks w_f.

Figure 1: Equilibrium Wage Offer Distributions for Cartel and Outside Firms



Notes: Cumulative distribution function of posted wages for the cartel members in Panel A and outside firms in Panel B for different values of the wage ceiling w_f. The case with a low w_f (Case I) is depicted in blue, with a medium w_f (Case II) in red, and with a high w_f (Case III) in purple. The wage ceiling w_f for each case is depicted with the dashed vertical lines. See the main text for a description of the three cases.

Discussion of Additional Equilibrium Properties

No Mass Lemma A.1 shows that, as per usual in a BM setting, a deviation argument rules out mass. Because this is a granular setting, it rules out many firms posting at the same mass point, the reason being that then one of these firms could post marginally higher pay with strictly lower turnover. Importantly, this logic breaks with wage fixing, simply because multiple cartel members might offer a mass of jobs at w_f, which by definition shuts down any deviation to higher pay.

No Gaps Lemma A.2 establishes that, generally, there cannot be a gap in the support of the wage offer distribution. The reason is simply that firms at the upper end of the gap could otherwise lower pay with no increase in turnover. This logic, however, breaks below a mass point. Here, the turnover rate discretely jumps below the mass due to a jump in the rate at which strictly preferable outside offers arrive. Jointly with the logic sketched in the previous paragraph, this leaves room for a gap in the distribution below the wage ceiling w_f.

This logic is readily observable in Cases I and II in figure 1. The only difference between the two cases is that in Case I, the gap extends beyond w_r because the wage that comes with user cost equal to that at w_f is below the reservation wage.

Identical User Cost Lemma A.6 establishes that outside firms have identical user costs and, which follows directly, the same level of employment. Lemma A.7 does the same for the cartel.

To see why, suppose two outside firms have differing user costs. The firm with the higher cost can always post a wage marginally above the highest wage posted by the other firm. Since turnover is weakly lower at that point, the user cost differential vanishes as the wage differential vanishes, a contradiction.

A similar logic applies within the cartel but with a difference: The logic we just appealed to does not apply at w_f. It follows that any potential user cost differential across cartel employers must arise from different mass at w_f. Indeed, asymmetric cartels might arise where one cartel employer posts plenty of jobs at w_f and the other posts few. This asymmetry results in a user cost differential because the small cartel member then has more competition and higher turnover. But it is also smaller, and if the difference in user cost equals the difference in marginal products, then this can be an equilibrium.

However, this is not a possibility if κ—the rate at which workers move when indifferent—is sufficiently low. In that event, asymmetric wage posting will lead to small user cost differentials that will not line up with the difference in marginal productivity implied by the size difference. Assuming that κ is small, which seems natural, is thus sufficient to establish that the cartel firms have identical user cost.[9]

Symmetry From here, it follows that all firms of the same type act symmetrically (Lemma A.8). To see why, first note that, for any interval on which two identical firms make offers, they have to offer wages at the same rate and with the same distribution to keep the user cost constant and identical; this is because both firms trade off turnover and pay at the same rate. Next, suppose that two firms of the same type post on an least partially different range of wages. Take the highest interval at which only one of the two firms posts. It then needs to be the case that the other firm posts at some wages below that interval, so as to reach the same level of employment. Take the highest of those wages below. The user cost there is then strictly higher for the firm making fewer offers above because it faces

[9] It is straightforward to construct asymmetric equilibria when they exist, following the discussion in the previous two paragraphs.

more outside competition from better paying jobs, which contradicts that the user cost is equated across firms of the same type (see previous paragraph) .

3.3 The Consequences of Wage Fixing

We next formally characterize the impact of wage fixing on wages and profits for cartel and outside firm employers. We then complement the formal results with additional numerical observations that build on our quantitative model below. We summarize our formal findings in the following proposition.

PROPOSITION 2. A) Cartel employers have lower employment, higher user cost, and lower profits compared with outside employers. The impact of wage fixing on the average wage is ambiguous.

B) A reduction in the wage ceiling w_f reduces the user cost for outside employers. It may or may not reduce the cartel firms' user cost. The impact of a binding wage ceiling on the cartel firms' profits is ambiguous.

Proof. See Appendix B. □

The fact that the user cost of labor is *higher* for cartel members follows directly from the equilibrium characterization. The cartel piles up jobs at w_f, and so turnover drops discreetly right above, where the outsiders post continuously distributed wages and hence have lower user cost. Intuitively, the cartel ties its hands by committing to a ceiling w_f. The outside firms operate in the same labor market and are always free to exactly emulate the cartel; hence, they must always be weakly better off.

Cartel Stability An immediate corollary of part A is that cartel members might be tempted to deviate and pay like an outside employer since profits are strictly larger. A wage fixing scheme might thus appear fragile, requiring a form of enforcement. We emphasize that a granular market structure, such as we consider here, makes this less of a concern. A large member, tempted to deviate, would recognize that even under a unilateral deviation, aggregate competition would rise because the cartel's size would shrink to $k-1$. That rise in competition moderates the incentives to exit—the firm would be an outside firm in a labor market with a cartel of size $k-1$, rather than a member of a cartel of size

k. This consideration might potentially undo any such temptation or at least dampen it, stabilizing the cartel. It is also strongest in markets that are concentrated, which suggests that cartel activity might rise when concentration rises. We revisit this below numerically.

Misallocation We highlight that wage fixing leads to misallocation. Part A of the proposition shows that cartel members have a higher user cost than outside firms. As a consequence, they have lower employment than outside firms because all firms optimally equate the marginal revenue product of labor to the user cost. Given identical decreasing returns to scale technology, this results in a misallocation that constitutes an aggregate downside of wage-fixing agreements.

Wage Impact The final part of part A implies that it might actually be possible for equilibrium wages to rise when a cartel fixes wages. This case may arise for moderate wage ceilings because a cartel that posts a mass of identical pay reduces not only competition but also turnover. The gains from the reduction in turnover are shared with workers when the wage ceiling is sufficiently high. We discuss this in more detail at the end of section 3.4.

Is it rational to form a cartel? Part B of the proposition states that it is not always rational to form a cartel. While outside employers strictly benefit from the introduction of the cartel, the impact on profits for the cartel firms is ambiguous. Why might profits actually fall for the cartel members? One might expect that the cartel enjoys higher profits due to lower turnover from internal and external competition. The reason this might not be the case is competition from the outside employers that can free-ride on the cartel. In response to the fall in their user cost, these outside employers increase hiring and expand in size, which drives up competition and turnover for the cartel.

It follows from this that cartels are more likely to arise in settings where the competitive pushback from the remaining employers is less powerful. When is this the case? When the production function features a lot of curvature (low α). By contrast, when it is close to linear, the outside firms aggressively expand in response to their falling user cost, driving out the cartel. On the flip side, this means that whenever a cartel does emerge in settings where the span of control is large, it needs to be large to be profitable, muting the competitive response of the outside employers.

Whistle-blowing Finally, it follows from the second part of the proposition that we should not expect any "whistle-blowing" from the outside employers, since they unambiguously benefit from the formation and existence of a cartel.

3.4 The Optimal Wage Ceiling w_f

So far, we have treated the wage ceiling w_f as exogenous. It is then natural to ask for the optimal w_f from the perspective of the cartel firms. One might suspect that the profit maximizing w_f equals the lowest possible wage, namely w_r. This, however, is not generally the case.

Instead, there are two opposing forces. First, by reducing the wage w_f, there is a direct reduction in the user cost of labor. However, a tighter ceiling also benefits the outside employers, who respond by expanding, driving up competition.

This section first studies this situation formally in a simplified case and then offers complementary numerical observations from the quantitative model.

A Simplified Analytical Case

We next show analytically that the tradeoff between the cartel's costs and external competition naturally leads to an interior w_f that lies above w_r. In this sense, a wage fixing cartel does not optimally suppress wages "all the way".[10]

We make analytical progress in the simplified case where the search efficiency of the employed equals that of the unemployed, $s = 1$. Since we are interested in understanding why a cartel would optimally choose $w_f > w_r$ we focus on equilibria described by Cases I and II. These are the cases with a relatively tight wage ceiling (Case I also endogenously emerges when we calibrate the model below).

As shown above, in these cases, the outside firms post wages uniformly from w_f to w_u. Their user cost hence is $\omega \equiv w_f + (r + \delta + (M - k - 1)\psi_u)c$, with ψ_u denoting their individual offer rate. Optimal hiring implies $x\alpha n^{\alpha-1} = \omega$, and so flow balance implies $\left(1 - (M-k)\left(\frac{x\alpha}{\omega}\right)^{\frac{1}{1-\alpha}}\right)\psi_u = \delta\left(\frac{x\alpha}{\omega}\right)^{\frac{1}{1-\alpha}}$. We can use this to obtain a simple (implicit) ex-

[10]Of course, if the cartel includes all employers, $k = M$, then the optimal wage ceiling is trivially just the reservation wage. The following arguments thus apply when $k < M$.

pression for the user cost of outside firms, denoted ω_o, in terms of w_f,

$$\omega_o = w_f + (r+\delta)c + (M-k-1)\frac{\delta\left(\frac{x\alpha}{\omega_o}\right)^{\frac{1}{1-\alpha}}}{1-(M-k)\left(\frac{x\alpha}{\omega_o}\right)^{\frac{1}{1-\alpha}}}c. \tag{10}$$

The left (right) hand side is increasing (decreasing) in ω_o, which implies that ω is strictly increasing in w_f. We can therefore think of the cartel as directly minimizing its own user cost via ω_o.[11] Assuming that $\kappa \approx 0$, the user cost for the cartel members is

$$\omega_c(\omega_o) \equiv w_f(\omega_o) + (r+\delta+(M-k)\psi_u(\omega_o))c = \omega_o + \delta\frac{\left(\frac{x\alpha}{\omega_o}\right)^{\frac{1}{1-\alpha}}}{1-(M-k)\left(\frac{x\alpha}{\omega_o}\right)^{\frac{1}{1-\alpha}}}c. \tag{11}$$

hence

$$\omega_c'(\omega_o) \quad = \quad 1 - \frac{1}{1-\alpha}\frac{c\delta}{\omega_o}\frac{\left(\frac{\omega_o}{x\alpha}\right)^{\frac{1}{1-\alpha}}}{\left(\left(\frac{\omega_o}{x\alpha}\right)^{\frac{1}{1-\alpha}}-(M-k)\right)^2}. \tag{12}$$

The fact that the derivative is less than one reflects that the firms outside the cartel benefit disproportionately from wage fixing. The second term captures the added turnover for the cartel relative to the outside employers. It is this second term that prevents the cartel from generically fixing the wage at its lowest possible value. A low wage ceiling leads outside employers to increase employment, disproportionately increasing turnover at the cartel firms.

To see this most directly, consider what happens when the cartel acts so aggressively that the user cost of the outsiders falls towards $x\alpha(M-k)^{1-\alpha}$. From (11), the outsiders then employ all the workers in the labor market, implying infinitely fast turnover and an infinitely high user cost for the cartel—which cannot be optimal. For slightly higher user costs, the derivative of the cartel's user cost with respect to that of the outside firms is negative, as can be seen from (12). The offsetting competition from outsiders hence restrains the cartel. The derivative turns positive for sufficiently high ω. It is further straightforward to verify that the second derivative is strictly positive. It follows that there exists a

[11] In doing so, the cartel is of course (implicitly) constrained by the reservation wage w_r and an upper bound on the user cost corresponding to the equilibrium without collusion.

unique value for ω that minimizes the cartel's user cost ω_c. Since (11) is independent of the reservation wage w_r, it also follows that there always exists a sufficiently low reservation wage such that the wage ceiling that implements that optimal value is indeed above the reservation wage.

Finally, a special case worth noting is the one where there is a single outside firm, $k = M - 1$. In this case the cartel posts only at w_f and the outsider posts a mass marginally above so the wage distribution is effectively degenerate at w_f. But this may mask that the sole outsider still exerts considerable competitive pressure on the cartel which in turn optimally keeps $w_f > w_r$. Only when $k = M$ does competition fully collapse.

Forces shaping the optimal w_f To further unpack the forces determining the optimal wage ceiling w_f, use that (12) equals zero at the optimum. Use that the level of employment at outside firms, N_o, equals $\left(\frac{x\alpha}{\omega}\right)^{\frac{1}{1-\alpha}}$ to get that, at the optimum,

$$\frac{w_f}{c} = \psi_u \left(\frac{1}{1-\alpha} \frac{1}{1-(M-k)N_o} - (M-k-1) \right) - (r+\delta). \tag{13}$$

This is a measure of how aggressively the cartel optimally sets the wage ceiling w_f that is useful in understanding the forces at play.

A large value of the span of control α means that outsiders increase employment more in response to a fall in their user cost which yields a higher optimal wage ceiling. $1-(M-k)N_o$, on the other hand, is where the outsiders as a whole source their employment from. If this is small then any increase in labor demand by outsiders drives up turnover in the cartel more sharply again yielding a higher optimal wage ceiling. These observations anticipate an important quantitative result below: cartels act much more aggressively when the labor market has slack or when the span of control is small.[12]

[12] The corresponding expression for the general case with $s \leq 1$ is

$$\frac{w_f}{c} = \psi_u \left(\frac{1}{1-\alpha} \frac{1-(1-s)kN_c}{1-(1-s)kN_c-(M-k)N_o} - (M-k-1) \right) - (r+\delta).$$

This follows exactly the same logic but has a generalized expression for labor market slack which takes into account that the employed search with lower efficiency.

Numerical Observations on the Impact of w_f

We complement these results with qualitative numerical observations that build on the calibrated model; we revisit these in the next section.

Profits Figure 2 depicts the profits of cartel members and outside firms as we move the wage ceiling w_f and the size of the cartel k. The black solid line indicates profits when there is no cartel.

The two kinks are where the equilibrium moves from Case I to II and then III, as w_f is relaxed.[13] We focus our discussion primarily on Case I. This is because it follows from panel A that the profit maximizing w_f corresponds to this case.

In addition, it follows from the left panel that the profit maximizing wage ceiling is generally strictly above the reservation wage, as we have shown formally for the simplified setting above. The reason is the strong spillovers to the outside firms, whose employment response disciplines the cartel. Ultimately, as $k \to M$, we have that $w_f \to w_r$, but in general, this is not the case.

We note that profits are generally increasing in the size of the cartel, both for cartel employers and outsiders. In turn, the profit-maximizing wage ceiling is not monotonically related to the size of the cartel. Finally, we depict a case in which the cartel is so small ($k = 5$), that profits are strictly below the no-collusion benchmark.

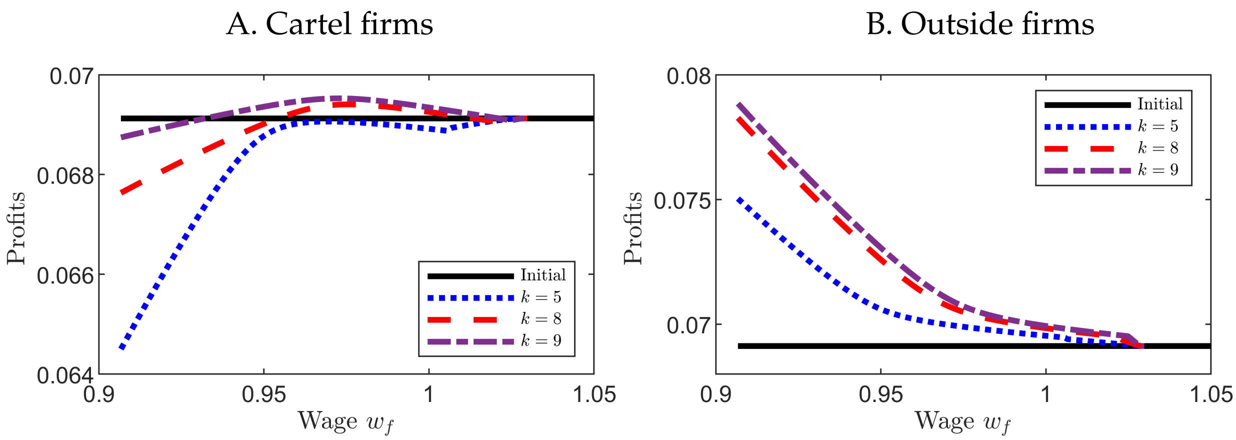
Both panels jointly also clarify that cartel employers, even if the cartel is profitable, should be tempted to leave the cartel because the outsiders fare even better. It further demonstrates that this temptation is muted in a granular setting. A firm leaving the cartel would recognize that the remaining group would be less powerful, resulting in a discrete reduction in profits at outside employers as well.

Wages Panel A in Figure 4 plots mean wages, which are normalized to one without collusion. We plot them against the wage ceiling w_f and again for different cartel sizes k. The two sets of kinks again indicate the transition from Case I to II and then III. We again focus on Case I.

In Case I, mean wages monotonically fall as the ceiling tightens. They decline as the

[13] The three cases are more easily visually discerned in Figure 3 since, as can be seen there, Case II only arises for a small range of w_f.

Figure 2: Profits



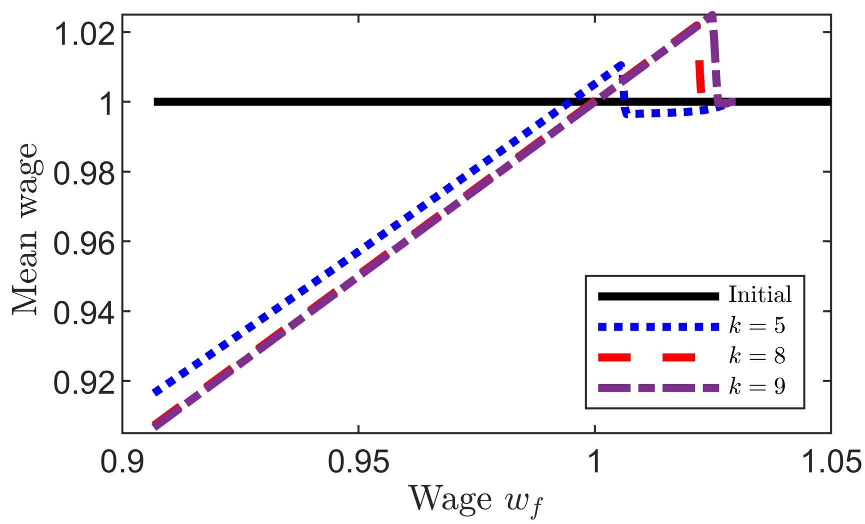
Notes: Firms within the cartel in panel A and outside firms in panel B. The solid black line gives profits without collusion. Model is solved numerically for the baseline calibration in section 4.

cartel grows, given w_f. We note that, for wage ceilings w_f above one and in Case I, a seemingly paradoxical result obtains. Both mean wages (Figure 3) and profits (Figure 2, both panels) are higher than they are under the baseline when the cartel is large enough to be profitable ($k = 8$ and $k = 9$). The reason this happens is that competition is inefficient in this setting because it induces costly worker turnover. The paradoxical case occurs because turnover is much reduced when the cartel firms all post a mass of identical offers and workers transition with sufficiently low probability when receiving an offer with identical pay. If the wage ceiling is sufficiently moderate, then the gains from the drop in turnover brought about by the cartel are shared by both workers and employers. This, however, is not the case here when the cartel optimally sets w_f, which clearly harms workers.

4 Quantitative Evaluation

This section offers a quantification of the wage impact of wage-fixing cartels. We begin by extending the model for quantitative purposes.

Figure 3: Mean Wages



Notes: The figure presents the mean wage for different numbers of cartel members. The solid straight black line gives the mean wage without collusion. The parameters are those underlying the baseline calibration in section 4.

4.1 Model Extensions

This section shows how to add capital to the production function. Then, it shows how to generalize to inelastic product demand and product market power.

Adding Capital

We add capital and assume that labor and capital are imperfect substitutes with an elasticity of substitution σ. The production function is CES,

$$Y_i = x\left(\theta N_i^{\frac{\sigma-1}{\sigma}} + (1-\theta)K_i^{\frac{\sigma-1}{\sigma}}\right)^{\alpha\frac{\sigma}{\sigma-1}}. \tag{14}$$

Capital can freely be adjusted. Denote the rental price of capital by v. Firms make the same choices in the labor market as in the baseline model. In addition, they choose a level of output Y_i. Given firm level employment, capital is then implied by the chosen level of output.[14] Optimality, as usual, implies that

$$\frac{\partial Y_i}{\partial N_i}\left(\frac{\partial Y_i}{\partial K_i}\right)^{-1}=\frac{\theta}{1-\theta}\left(\frac{K_i}{N_i}\right)^{\frac{1}{\sigma}}=\frac{\omega_i}{v}, \tag{15}$$

where ω_i denotes the user cost of labor for firm i.

The reason for this extension is as follows: Cartels ultimately lead to cost savings for both the cartel and the outside employers. As discussed previously, if the outsiders' response is highly elastic, this restrains the cartel and limits its scope for wage suppression. Anything that mutes the response, therefore increases the harm wage fixing does to workers.

In many empirically relevant settings, such as those covering the US food processing industry, labor is arguably a strong complement to other factors of production, in particular capital and intermediates. This should mute the outside employers' labor demand response to a falling user cost of labor. This is a first order issue for quantitative consideration, and the extension allows us to account for it.

Product market power

We next extend the model to endogenize goods prices. This allows us to extend the numerical analysis to cases where large employers arguably also have product market power and where cartels collude in both the labor and the product market.

As in Gottfries and Jarosch (2023), we assume that all firms in a labor market produce an identical composite good and that the boundaries of the product market are the same as those of the labor market. Workers have linear preferences over an outside good ("money"), so their utility is quasi-linear. Employers care only about the outside good. Then, there is transferable utility between workers and firms. Workers choose consumption C to maximize the instantaneous utility function

$$v=\frac{\eta}{\eta-1}\bar{Q}^{\frac{1}{\eta}}C^{\frac{\eta-1}{\eta}}+I-pC,$$

[14]The reason to set up the firm problem in this form is as follows. Firms, as before, choose a contact rate ψ_i as well as distribution $F_i(w)$ taking each others choices as given. But this means that a firm might recognize that, by choosing a higher contact rate, its competitors choices would yield a different amount of employment and hence output. In a setting with product market power this in turn would have price impact. Assuming that firms choose output and let a flexible factor adjust shuts down this considerations.

where $\bar{Q}$ is a positive constant. They do so taking income I, which is either equal to the wage or the flow income of unemployment, as given.

Quasi-linear preferences imply that consumption is equalized across all workers and, since there is a unit measure of them, equilibrium consumption of each worker is simply $C = \sum_j Y_j$. Optimal consumption requires $p = \bar{Q}^{\frac{1}{\eta}} C^{\frac{-1}{\eta}}$, which results in the following iso-elastic market-level inverse demand function,

$$p = \bar{Q}^{\frac{1}{\eta}} \left(\sum_{j=1}^{M} Y_j \right)^{-\frac{1}{\eta}}. \tag{16}$$

We entertain three different forms of conduct in the product market. In all formulations, we consider a Nash equilibrium where each firm treats its competitors' choices as fixed when making its own decisions. Firms, as before, decide on a contact rate ψ_i and a wage offer distribution $F_i(w)$. They also decide on output Y_i, which now depends on both employment and capital. The three different forms of conduct will result in three distinct expressions for the marginal revenue product of labor at firm i, m_i. Denote by $\frac{dY_i}{dN_i}$ the marginal product of labor at firm i (from (14)).

Conduct I: Strategic. We first consider the case in Gottfries and Jarosch (2023) in which firms internalize their price impact. In this case, the marginal revenue product takes the following form

$$m_i = \frac{dY_i}{dN_i} \times p \left(1 - \frac{1}{\eta} \frac{Y_i}{\sum_j Y_j} \right). \tag{17}$$

This will be our baseline scenario.

Conduct II: Nonstrategic. We next consider the case where firms treat the output price as exogenous. This case is similar to the theoretical exposition above where we assumed that the output price is fixed, corresponding to an infinite price elasticity of demand. The difference here is that we actually do allow for a finite price elasticity, but firms do not internalize their price impact.

The reason this setting is worth considering is that it might capture cases where there are many symmetric labor markets that are all in the same product market. With the boundaries of the product market being much wider than those of the labor market, any local employer would not perceive a price impact of their actions although, collectively across all employers, there would be such a price impact. In this case, the marginal revenue product takes the following form,

$$m_i = \frac{dY_i}{dN_i} \times p. \tag{18}$$

Conduct III: Collusion in the product market. As a third alternative, we consider the case where collusion extends to the product market. The collective revenue of the cartel is given by $p \sum_{j \in C} Y_j$. In this third case, the "marginal" revenue for cartel firm i is

$$m_i = \frac{dY_i}{dN_i} \times p \left(1 - \frac{1}{\eta} \frac{\sum_{j \in C} Y_j}{\sum_j Y_j}\right). \tag{19}$$

To understand this expression, compare it with (17). There, the firm recognizes that expanding employment leads to an adverse price effect that reduces its own revenue. Equation (19) is simply the counterpart where the firm recognizes that the adverse price effect reduces revenue everywhere in the cartel. Of course, the firms outside the cartel continue to act according to (17).

4.2 Calibration

The type of cartels we study in this paper naturally operate in the shadows. As such, it is hard to offer a standard quantitative macro exercise that aims to gauge the wage losses caused by such practices. We therefore proceed as follows. Wherever possible, we calibrate the model using an approach similar to that in Gottfries and Jarosch (2023), targeting standard empirical moments for the US labor market under the assumption that there is no collusion. The baseline calibration can thus be viewed as a "representative" US labor market.

We then introduce a cartel of the size that has been operative in the US poultry industry and re-compute the labor market equilibrium. We then repeat this exercise for a wide range of modifications to the baseline calibration. The aim is to get a ballpark estimate for the wage consequences of wage fixing and to understand how this estimate depends on various market features.

The discount rate r is set to match an annual discount rate of 5%. The curvature of the production function is set to $\alpha = .64$, in line with the estimates of Cooper et al. (2007) and Cooper et al. (2015). The average wage is normalized to one. The hiring cost c equals one month of wages, in line with Manning (2011). We set the elasticity of industry demand, η, to 1.3, which is in the middle of the range given in Edmond et al. (2023) and comparable to De Loecker et al. (2021). The probability of a move between cartel employers when pay is the same is set to $\kappa = 0.01$, small but non-zero. There are $M = 10$ identical firms within a labor market to rationalize a Herfindahl-Hirschman index (HHI) of approximately 1000 (Berger et al., 2022; Jarosch et al., 2024).

In each calibration, we pick the common level of productivity x to match a monthly job finding rate of 45%. We set the exogenous separation rate δ to 2.9% so that the unemployment rate is 6%. We pick the relative search efficiency s to rationalize a job-to-job rate of 3.2% in a large labor market, in line with Moscarini and Thomsson (2007).

We target these moments under the assumption that there is no cartel and then study the impact of collusion by increasing k from zero to a setting in which the cartel controls approximately 80% of employment under collusion, in line with the DOJ complaint against the poultry industry (DOJ Antitrust Division, 2023). We therefore pick $k = 8$. When we introduce the cartel, we always set w_f to its profit-maximizing level.

4.3 Quantitative Results

Table 1 compares wages and user costs in an equilibrium without collusion with their counterparts in an equilibrium with wage fixing. The first column reports our baseline results. The additional columns then modify an assumption, recalibrate, and redo the exercise. We first discuss the baseline and then describe the remaining columns. All columns correspond to product market conduct I, and this is then varied in the next table.

Baseline Results. A cartel that covers approximately 80% of employment and picks the profit-maximizing wage as its ceiling generates wage losses of approximately 2.4%. There are strong spillovers, with wages falling almost as much for the (few) outside employers, as can be inferred from the fifth row. The user cost (sixth row) is even more depressed than wages because the cartel creates lower turnover. This is more pronounced among the outside employers (seventh row), which indicates that these are actually the primary beneficiaries of the cartel, as we discussed above.

Table 1: Impact of collusion

	Baseline	CES	$\alpha = 0.85$	$\eta = 1.1$	$u = 0.04$	$u = 0.1$
Emp. share	0.797	0.799	0.795	0.798	0.797	0.796
N_o/N_c	1.021	1.008	1.031	1.015	1.018	1.024
$(w_f - w_r)/(w_u - w_r)$	0.563	0.135	0.644	0.614	0.824	0
du	-0.046	-0.05	-0.039	-0.038	-0.026	-0.079
$d\log(\mathrm{E}[w])$	-0.024	-0.08	-0.014	-0.018	-0.003	-0.061
$d\log(w_o)$	-0.023	-0.078	-0.013	-0.017	-0.002	-0.058
$d\log(\mathrm{E}[\omega])$	-0.041	-0.092	-0.033	-0.037	-0.023	-0.072
$d\log(\omega_o)$	-0.048	-0.101	-0.038	-0.042	-0.029	-0.079

Results based on an equilibrium with wage fixing and $k = 8$ versus an equilibrium without collusion. Details are in main text. o and c index outside and cartel employers.

We also note that the cartel firms are almost as large as the outside employers (second row). The outsiders are just 2.1% larger than the cartel firms, which correspondingly control almost 80% of employment. This indicates that the labor misallocation induced by the cartel is limited. The third row shows how tight the profit-maximizing wage ceiling is. In line with the discussion above, the outside competition moderates the cartel, which sets w_f far above the reservation wage w_r and, in fact, closer to the highest wage w_u.

Lastly, consider the impact on unemployment or, equivalently, the employment rate (fourth row). Wage fixing reduces the user cost of labor for all firms which increases aggregate labor demand. The effect is substantial, with the unemployment rate dropping from 6% without a cartel to 1.6% when the cartel is active. To understand the magnitude of this, it is useful to consider the market level elasticity of employment demand. Using that, according to table 1, the user cost gains from wage fixing is similar across firms, we can approximate the market level (un)employment response as follows,

$$\begin{aligned} du &\approx \sum_{i=1}^{M} N_i \exp\left[d\log(\mathrm{E}[\omega]) \times \frac{d\log(N)}{d\log(\omega)}\right] - \sum_{i=1}^{M} N_i \\ &= \sum_{i=1}^{M} N_i \exp\left[d\log(\mathrm{E}[\omega]) \times \frac{1}{1-\alpha\frac{\eta-1}{\eta}}\right] - \sum_{i=1}^{M} N_i \\ &= 0.94 \times (\exp(-0.041 \times 1.17) - 1) \approx -4.4\%. \end{aligned}$$

This approximation is close to the exact results in table 1 which (additionally) captures the heterogeneity in user costs across firms. The second line shows that the span of control α and the demand elasticity η govern the market level employment response to a fall in the user cost.

CES production function. The next column shows that the wage losses can quickly rise to a much higher level when the cost share of labor is low and complementarities between factors are strong. Here, we use the CES production extension discussed in the previous subsection, with an elasticity of substitution $\sigma = .5$ and a weight on labor θ such that the wage bill relative to overall factor payments is just 10%.

In this case, the wage losses more than triple: The cartel can adopt a more aggressive wage ceiling (third row) because the low cost share and the complementarities mute the outsiders' employment response to a fall in the user cost of labor.

Larger span of control. Column three looks at the same forces from a different angle. It shows that if the span of control is larger than in the baseline, then the wage losses from optimal wage fixing are smaller. Again, the reason is the discipline imposed by the outside employers, which is stronger if these can aggressively scale up when costs fall. This is the case when the span of control is large (high α).

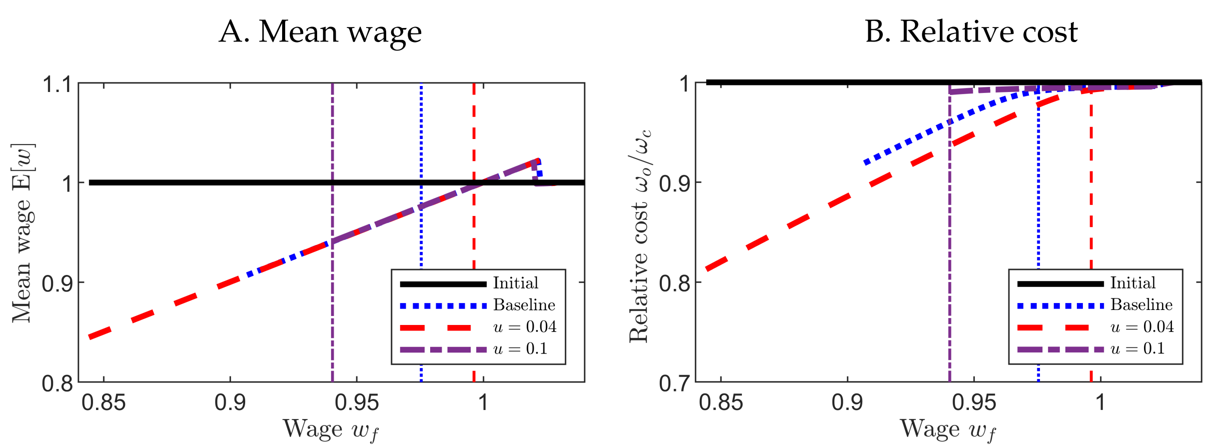
Product Demand The next column lowers the elasticity of market-level product demand from 1.3 (baseline) to 1.1. Here, the wage losses from wage fixing fall below 2%. To see why, note that costs are increasingly passed through into prices when demand is less elastic. This makes the cost reduction from wage fixing less profitable and acts as a constraint on the cartel. Additionally, the cost advantage of the outsiders, which drives the cartel out of the market, becomes increasingly detrimental.

Local unemployment The last two columns consider a decrease (an increase) in the local unemployment rate from its baseline level of 6%. We maintain all other calibration targets at the levels described above and move the unemployment target to 4% and 10%, respectively. The results show that wage-fixing cartels do far more damage when the labor market has slack and competition is loose. In turn, when competition is fierce and wages are high, a cartel has little room to constrain pay. Starkly, even with a 10% unemployment rate, the moderating force from the outside competition vanishes to the point where the cartel optimally sets the wage ceiling at the reservation wage. This leads to wage losses that are more than double those in the baseline scenario.

It is instructive to revisit mean wages as a function of w_f, which we now plot for the baseline, as well as the high and low unemployment setting, in Panel A of Figure 4. We also indicate the profit-maximizing w_f (horizontal lines) as well as, implicitly, the support for w_f that begins at w_r, the value of which depends on the case. The figure shows that variation in wage losses across the three settings is driven by differences in the profit-maximizing ceiling. With slack, the cartel optimally sets the wage ceiling at reservation. With a tight labor market, it has little room and sets the wage ceiling far higher.

To further unpack this, panel B plots the relative user cost between outside and cartel employers. It shows that the cost differential is mild before starting to sharply drop off at some point, with the outsiders gaining substantial cost advantages when the cartel becomes too aggressive. The cartel then optimally picks a wage ceiling right above this point. The tighter the labor market, the earlier this point is reached. With sufficient slack, this point is never reached, and the cartel then sets $w_f = w_r$.

Figure 4: Wages and Relative User Cost



Notes: The figure presents the mean wage for the different calibrations as a function of the wage ceiling w_f. The vertical lines indicate the optimal choice of w_f. The solid straight black line gives the mean wage without collusion. The parameters are those underlying the baseline calibration in Table 1. The vertical lines indicate the cartel's profit-maximizing wage ceiling.

Together, these exercises show that cartel activity in our baseline setting causes relatively modest harm to workers. When the cartel is less constrained by competition, how-

Table 2: Product market conduct

	Strategic (I)	Price takers (II)	Collusion (III)
Emp. share	0.797	0.796	0.496
N_o/N_c	1.021	1.024	4.069
$(w_f - w_r)/(w_u - w_r)$	0.563	0.566	0.332
du	-0.046	-0.046	-0.039
$d\log(\mathrm{E}[w])$	-0.024	-0.024	-0.026
$d\log(w_o)$	-0.023	-0.023	-0.023
$d\log(\mathrm{E}[\omega])$	-0.041	-0.041	-0.032
$d\log(\omega_o)$	-0.048	-0.048	-0.039

Counterfactual results based on recalculating the equilibrium without collusion for the three different forms of conduct in the product market introduced in section 4.1. o and c index outside and cartel employers.

ever, this changes quickly, and the harm caused to workers can be large.

The Role of Product Market Conduct

Table 2 investigates the role of product market conduct for the labor market consequences of an employer cartel. We consider the three different types of conduct laid out in Section 4.1, with Conduct I referring to the baseline results we have already discussed. Table 2 contrasts these with Conduct II (Nonstrategic) and Conduct III, where the cartel also colludes in the product market. As before, we recalibrate to meet the calibration targets under the three different settings (but we keep the number of cartel members fixed at $k = 8$).

A downward sloping product demand curve generally limits the value of wage fixing. The lower user cost of labor is partially passed through to consumers via a lower product price. Furthermore, as already described, wage fixing benefits the outside firms, whose labor demand response both drives up wages and drives down the output price. This response, however, is somewhat muted when everyone, including the outsiders, is strategic in the product market. Hence, the wage gains from eliminating a cartel are a little larger in the strategic case.

Comparing columns one and two shows that these additional considerations are quantitatively negligible, since the effects of removing the cartel are indistinguishable across the two cases. This changes, however, if the cartel also actively colludes in the product market, as shown in the last column.

In this case, the cartel strongly cuts back on employment and output, to the point where employment at each of the two outside employers is over four times larger than that at a cartel firm (second row). Correspondingly, the cartel employs less than 50% of the total workforce. Despite this, removing the cartel results in market-wide wage gains that are even larger when compared with the baseline in which the cartel employs 80% of the total workforce. This demonstrates a strong interaction between collusion in the product and labor markets. When wage-fixing employers collude in the product market as well, they suppress wages and employment more aggressively because the desired markup in the product market is far larger when the cartel acts as a block there as well.

5 Gauging Harm in Practice

The above calculations accurately report the harm caused by a cartel according to the model, given model primitives. But, of course, they require fully solving the model and then solving for a counterfactual which, while straightforward, is time-consuming and requires some coding expertise.

This section therefore offers a simple way to gauge the (wage) harm caused by a cartel in the spirit of a sufficient statistic. The aim is to express harm in terms of easily observable endogenous quantities along with several exogenous primitives.

We first cover the simplest case, in which the cartel covers most of the market, $k \geq M - 1$. Worker harm $\mathcal{H}$ is defined by mean wages with the cartel relative to mean wages without it. Appendix C shows that this can be expressed as

$$\mathcal{H} = \frac{M-1}{M}\left(\frac{s\bar{N}}{1-\bar{N}} - \log\left(\frac{1-(1-s)\bar{N}}{1-\bar{N}}\right)\right)\frac{1-(1-s)\bar{N}}{s\bar{N}}\delta c - (w_f - w_r)$$

where $\bar{N}$ denotes the total level of employment without collusion.

Starting with the first term, one needs to know the number of employers in the market M. Appendix C.1 shows how the relative search efficiency of the employed s can be constructed from data on worker flows between firms.

The next input required is $\bar{N}$ which is counterfactual. The results for the change in the unemployment rate reported in table 1 suggest that $\bar{N}$ is typically around 5 percent lower than the actual employment rate under the cartel.[15] The average rate of job loss δ and the hiring cost c can be determined in a case-specific fashion based on industry characteristics.

w_f and w_r remain. We presume that, whenever a wage fixing cartel is discovered in practice, information on w_f is revealed. w_r, in turn, is well approximated via the minimum wage, in particular in the United States. Of course, $w_f - w_r = 0$ in any case if $k = M$.

When $k > M - 1$ this is exact. It also works well for our baseline model calibration where we have a cartel of size $k = 8$ among a total of $M = 10$ firms. To see this, set $M = 10$, $\bar{N} = .94$, $s = .3$, $\delta = .03$ and $c = 1E[w]_{k=0}$ As can be seen in the left panel in Figure 2, for $k = 8$ and the profit-maximizing w_f, $w_f - w_r \approx .07$. Jointly, this gives wage harm of approximately 2.5%, just slightly above the 2.4 log points reported in Table 1, Column 1.

The above equation is for large cartels. The following expressions cover the general case where $k < M - 1$,

$$
\begin{aligned}
\mathrm{E}[w]_{k=0} &= w_r + \frac{M-1}{M}\left(\frac{s\bar{N}}{1-\bar{N}} - \log\left(\frac{1-(1-s)\bar{N}}{1-\bar{N}}\right)\right)\frac{1-(1-s)\bar{N}}{s\bar{N}}\delta c, \\
\mathrm{E}[w]_{k>0} &= w_f + \frac{M-k-1}{M-k}\frac{N-N_c}{\bar{N}}\left(\frac{1-(1-s)N}{1-N+sN_c} - \frac{1-(1-s)N}{s(N-N_c)}\log\left(\frac{1-(1-s)N}{1-N+sN_c}\right)\right)\delta c.
\end{aligned}
$$

In addition to the above, these require knowledge of the number of cartel firms k, total employment by the cartel N_c, and overall employment N (while the cartel is active). Plugging in the values corresponding to our quantitative baseline calibration recovers the results we compute above where we fully solve the model.

These equations are useful because they allow to calculate the harm created by a wage fixing cartel, according to our model, without having to solve the model. Instead, they only require information that can relatively straightforwardly be obtained on a case-by-case basis.

6 Conclusion

We model and analyze wage fixing cartels in a canonical dynamic monopsony environment with large firms. Wage fixing depresses labor market competition and employers

[15] Harm is increasing in $\bar{N}$ so a conservative estimate applies a large value for the employment losses when a cartel is removed.

outside the cartel benefit from it, even disproportionately. It follows that competition by outside employers curbs a wage fixing cartel. Cartels are thus more likely to arise, be stable, and do substantial damage where the competitive response from outsiders is limited either due to technological reasons, product market conduct, or because of the local market structure. These observations can guide future research, in particular empirical work, on wage fixing in the labor market.

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APPENDIX

A Proof of Proposition 1

Denote by w_u^* the highest wage when there is no collusion. Any $w_f \geq w_u^*$ is not binding in equilibrium (and the equilibrium therefore corresponds to the baseline without wage fixing). Similarly, if $w_f < w_r$, no worker is employed at the colluding firms and the equilibrium corresponds to the benchmark equilibrium with only the outside $M - k$ firms. The following proof therefore presumes that $w_f \in [w_r, w_u^*)$.

A.1 Preliminaries

We begin by deriving some properties of the equilibrium.

Lemma A.1. *At any mass point, there can at most be one firm, except at w_f where there can be more firms that are all in the cartel.*

To prove this, assume there are multiple employers all posting jobs at a mass point. Then, a firm at the mass point would find it profitable to post jobs with marginally higher pay and discretely lower turnover. The reason is that workers move with strictly positive probability when indifferent.

Next, we establish the following.

Lemma A.2. *There cannot be any gap in the support, except below a mass point at w_f. There cannot be any non-degenerate interval where only a single firm posts.*

The reason there cannot be any gaps is that a firm posting just above the gap could always lower its wage without increasing turnover. However, if there is a mass point with multiple firms posting, then turnover discretely jumps for a firm at the mass point when reducing the wage as long as $\kappa < 1$. We have already established that the only mass point where multiple firms possibly post wages is w_f. To prove the second part of the lemma note that the lone firm on a non-degenerate interval could increase profits by only offering the lowest wage in the interval because its turnover cost would not rise.

We now want to prove that all firms offer wages on a connected support (except for below any masspoint at w_f) from highest admissible wage and that all firms of the same type offer jobs at the same rate and with the same distribution over wages. To do so, we proceed in a number of steps.

Lemma A.3. *Consider an interval (w_1, w_2) where firms offer all wages. Then, over this interval, the rate at which each firm offers jobs and the distribution of wages are identical.*

To see this, denote l the number of firms in the interval and $\hat{\psi}_j$ the rate of offer of jobs and $\hat{F}_j(w)$ the distribution of wages in the interval by firm j. For firm i in the interval, we require that

$$w - w_1 - sc \sum_{j \neq i} \hat{\psi}_j' \hat{F}_j(w) = 0 \tag{20}$$

since $\lim_{w \to w_1} \hat{F}_j(w) = 0$. This has to hold for all $w \in (w_1, w_2)$ and all of the l firms.

It follows that for any pair of firms i and z posting on this interval $sc\sum_{j\neq z}\hat{\psi}_j'\hat{F}_j(w) - sc\sum_{j\neq i}\hat{\psi}_j'\hat{F}_j(w) = sc\hat{\psi}_i\hat{F}_i(w) - sc\hat{\psi}_z\hat{F}_z(w) = 0$ for all $w \in (w_1, w_2)$. Given that $\lim_{w\to w_2}\hat{F}_j(w) = 1$ and $\lim_{w\to w_1}\hat{F}_j(w) = 0$, $sc\sum_{j\neq z}\hat{\psi}_j'\hat{F}_j(w_2) - sc\sum_{j\neq i}\hat{\psi}_j'\hat{F}_j(w_2) = sc\hat{\psi}_i - sc\hat{\psi}_z = 0$ and so all firms have the same contact rate. Again use (20) evaluated at w_2 and with symmetric contact rates to get that $\hat{\psi}_j' = \frac{w_2 - w_1}{sc(l-1)}$.

Plug in for the contact rate and use (20) once more to show that $\hat{F}_j(w) = \frac{w-w_1}{w_2-w_1}$. Therefore, all firms offer jobs at the same rate and with the same wage offer distribution over this interval.

Lemma A.4. *Suppose firm i posts a wage w_1 but does not post wages anywhere on an interval immediately above, (w_1, w_2). Then, no other firm j offers wages over this interval, unless i is in the cartel and $w_1 = w_f$.*

To prove this, assume the contrary. Firm i offers w_1 but not on (w_1, w_2) for some w_2. Denote by l the number of i's competitors posting on the first part of this interval, (w_1, w') with $w' \leq w_2$ (there might be intervals with a different numbers of firms posting). From (20), we have that $(l-1)sc\psi' = w' - w_1$, where ψ' now denotes the rate of offers in this interval by the l firms. This implies that the user cost is lower for firm i at w' compared with w_1 because its turnover cost over this interval falls by $lsc\psi' > w' - w_1$. This proves the Lemma.

Lemma A.5. *If there is a mass at w_f, no outside firm or cartel member that does not offer the most amount of jobs at w_f offers a wage below w_f.*

Assume first that at least one cartel firms offer a mass of jobs at w_f but that some other firm offers fewer jobs at w_f. Assume that that other firm also offers wages below w_f. Pick the firm i with the largest mass of jobs at w_f. If that firm was posting a wage just marginally below the w_f that job had additional turnover cost $sc(1-\kappa)\sum_{j\neq i}\psi_j(F_j(w_f^+) - F_j(w_f^-))$. It follows that it would not post any wage below w_f that is higher than w' which solves $w_f - w' = sc(1-\kappa)\sum_{j\neq i}\psi_j(F_j(w_f^+) - F_j(w_f^-))$. All other firms would post any wage on (w', w_f) either as their increase in turnover would be weakly larger. Thus, if $w' < w_r$ no firm posts any wage strictly below w_f.

If, in turn, $w' > w_r$ and the gap below w_f extended beyond w', then firm i would prefer not to post on the masspoint and instead post the highest wage below the gap. Thus, for firm i to post a mass of jobs at w_f, the gap has to span from w' to w_f.

Now consider cartel firms that post a strictly lower mass at w_f or outside firms that post right above w_f. These firms will have strictly higher user cost at w' and will hence not post there or immediately below. If they posted any wage below w_f it would be at a level strictly below w' but that is ruled out by Lemma A.4.

Lemma A.6. *All outside firms have the same user cost and hence the same level of employment.*

To prove this, note first that the user cost must be equated across all wages offered by any given firm since otherwise it could lower its total user cost. Assume now that there exist a firm i that has a lower user cost than another firm j of the same type. Denote by w' the highest wage offered by firm i. Take firm j (of the same type) that has a higher user cost. Firm j can offer w' and firm j must at this wage have weakly lower turnover and therefore weakly lower user cost of labor.[16] This contradicts that i has a lower user cost and all firms of the same type must consequently have the same user cost. A direct implication is that employment is the same at all firms of the same type when technology is the same.

Next, we establish the following.

Lemma A.7. *All cartel firms offer the same amount of jobs at w_f when κ is sufficiently low.*

Suppose, to the opposite, that i posted fewer jobs than j at w_f. By Lemma A.5, firm i posts no jobs below w_f. Firm j therefore has strictly higher employment than i since, by definition of w_f, neither posts above.

To prove that this cannot hold, abuse notation and denote by ψ_j firm j's offer rate at w_f (its total offer rate might be higher). Express firm i's offer rate at w_f as $\Delta\psi_j$, with $\Delta < 1$. We have that $N_i/N_j = a\Delta$ where $a \leq 1$ since firm j might post some jobs below w_f. Next, denote by γ the quit rate at w_f to firms other than i or j. Optimal employment implies that

$$\alpha x N_j^{\alpha-1} = w_f + (r+\delta+\gamma)c + s\kappa\psi_i c = w_f + (r+\delta+\gamma)c + s\kappa\psi_j c\Delta, \tag{21}$$

$$\alpha x N_i^{\alpha-1} = w_f + (r+\delta+\gamma)c + s\kappa\psi_j c. \tag{22}$$

[16] Firm i cannot offer a mass of jobs at its highest wage w'. If i did, j would only offer wages above w' (the logic analogous to lemma A.5). If j offered a nondegenerate distribution of wages above w', a contradiction would follow from lemma A.4; if it posted a mass, a contradiction would follow again from lemma A.5.

Taking the ratio yields

$$(a\Delta)^{1-\alpha} = 1 - \frac{s\kappa\psi_j c}{w_f + (r+\delta+\gamma)c + s\kappa\psi_j c} + \frac{s\kappa\psi_j c}{w_f + (r+\delta+\gamma)c + s\kappa\psi_j c}\Delta. \quad (23)$$

Note that the equation holds when $\Delta = 1$ and $a = 1$, that is in the limit where the two firms act identically. The Lemma immediately follows if this is the unique solution. We next show that this is indeed the case provided κ is low enough.

To see this, note that the RHS is an increasing linear function in Δ with a positive intercept whereas the LHS is an increasing concave function function of Δ with intercept at 0. Thus, the slope of the LHS is lowest for $\Delta = 1$. Start with $a = 1$. If the derivative of the LHS is higher than the derivative of the RHS at $\Delta = 1$, the derivative of the LHS is higher for all $\Delta < 1$. It follows that the LHS is lower than the RHS for all $\Delta < 1$ since they reach the same value at $\Delta = 1$. Hence, no solution with $\Delta < 1$ exists. Of course, there is no solution for $a < 1$ since this implies a LHS that is even lower. The condition for the derivative of the LHS to be larger than the RHS is then

$$1 - \alpha > \frac{s\kappa\psi_j c}{w_f + (r+\delta+\gamma)c + s\kappa\psi_j c}. \quad (24)$$

This holds if firm j does not contribute too much to the user cost of firm i. For any set of parameters there exists a sufficiently small value of κ such that this holds.

Lemma A.8. *All firms of the same type act symmetrically.*

First consider outside firms. They post wages from the highest wage. Suppose that was not the case and instead the highest wage that some regular firm posts is w' with $w' < w_u$. Then, it would not post on any open interval above w', but then no other firm would post there either which would make w' the highest wage, from Lemma A.4.

It also follows from Lemma A.4 and Lemma A.2 that the outside firms post continuously distributed wages at least down to w_f. If there is mass at w_f then, by Lemma A.5, outside firms do not post any wages below and so behave identically.

If, in turn, there is no mass at w_f then, it follows jointly from Lemma A.2 and Lemma A.4 that each regular firm has a lowest wage above which it posts continuously up to the highest wage. Suppose two outside firms had differing lowest wages. Since they offer the same distribution at the same rate over any common interval by Lemma A.3, it follows that one is larger than the other which contradicts Lemma A.6.

Turning to the cartel, we know from Lemma A.7 that all cartel firms offer the same amount of jobs at w_f and have the same user costs and employment. If there is a mass of jobs offered at w_f, there is a gap below until w' where $w' = w_f - sc(1-\kappa)\sum_{j\neq i}\psi_j(F_j(w_f^+) - F_j(w_f^-))$ where i is a cartel firm. Note that all cartel firms offer the same amount of jobs at w_f by Lemma A.7. If there is no mass at w_f then $w' = w_f$. It also follows from Lemma A.4 and Lemma A.2 that firms post continuously distributed wages below w' down to w_r unless $w' \leq w_r$. The rest of the argument is symmetric to the outside firms.

A.2 Main Part of Proof of Proposition 1

Let ψ_c denote the offer rate of each of the cartel members and ψ_u the offer rate, strictly above w_f, of each of the outside firms. Define the employment given by the flows in the outside firms and cartel firms as $N_o^s(\psi_u, \psi_c)$ and $N_c^s(\psi_u, \psi_c)$, respectively. The implied employment demand by each type of firms is similarly defined by $N_o^d(\psi_u, \psi_c)$ and $N_c^d(\psi_u, \psi_c)$, respectively.

The proof proceeds as follows. We first show that the above Lemmas imply that, given fixed values of offer rates ψ_u and ψ_c, there is a unique allocation. We then show that these values of ψ_u and ψ_c are unique which then implies that the equilibrium is unique.

To begin with, we show that the equilibrium, given ψ_c and ψ_u such that the wage ceiling is binding, can be split into the following four cases, depending on how large $w_f - w_r$ is relative to ψ_c.

In all cases, the outside firms offer jobs uniformly over $(w_f, w_u]$ where $w_u = w_f + sc(M-k-1)\psi_u$ but in case III and IV, they also offer wages below w_f.

1. If $w_f - w_r \leq (k-1)s\psi_c(1-\kappa)c$, all cartel firms offer all their jobs at w_f.

   Given that there is a mass point at w_f, we know that no regular firm posts a lower wage. We further know that there are no gaps in the support and that the outside firms act symmetrically by Lemma A.8. It follows that they offer jobs uniformly over $(w_f, w_u]$ where $w_u = w_f + sc(M-k-1)\psi_u$.

   Thus, it remains to show that the cartel firms offer jobs only at w_f. Assume, to the contrary, that the cartel firms offer jobs at w_f at a rate $\tilde{\psi} < \psi_c$, implying that they also post jobs at wages below w_f. From Lemma A.5, it follows that there are no offers over the interval (w_1, w_f) where $w_1 = w_f - s(1-\kappa)(k-1)\tilde{\psi}c$. The remaining offers must therefore be over the interval $[w_r, w_1]$. From Lemma A.3, we know that in order for the firms to be indifferent over this interval, we must have that $w_1 = w_r + s(k-1)(\psi_c - \tilde{\psi})c$. These two conditions jointly imply

$$\tilde{\psi} = \frac{\psi_c}{\kappa} - \frac{w_f - w_r}{s(k-1)c\kappa}, \tag{25}$$

which implies $\tilde{\psi} \geq \psi_c$ for

$$\psi_c \geq \frac{w_f - w_r}{s(k-1)c(1-\kappa)}, \tag{26}$$

and therefore a contradiction. We note that, if $\tilde{\psi} = 0$, then outside firms might also make offers below w_f which reinforces the argument. It follows that the cartel only posts at w_f.

Given these posting decisions, employment at the respective firms satisfies

$$N_c^s(\psi_u, \psi_c) = \frac{\psi_c}{\delta + s(M-k)\psi_o} \frac{\delta}{\delta + k\psi_c + (M-k)\psi_u}, \tag{27}$$

$$N_o^s(\psi_u, \psi_c) = \psi_u \left(1 + \frac{sk\psi_c}{\delta + s(M-k)\psi_o}\right) \frac{\delta}{\delta + k\psi_c + (M-k)\psi_u}. \tag{28}$$

The user costs satisfies

$$\omega_c = w_f + sc\left(\kappa(k-1)\psi_c + (M-k)\psi_u\right), \tag{29}$$

$$\omega_o = w_f + sc(M-k-1)\psi_u, \tag{30}$$

which implies that the optimal employment demand satisfies

$$N_c^d(\psi_u, \psi_c) = \left(\frac{\alpha x}{w_f + sc\left(\kappa(k-1)\psi_c + (M-k)\psi_u\right)}\right)^{\frac{1}{1-\alpha}}, \tag{31}$$

$$N_o^d(\psi_u, \psi_c) = \left(\frac{\alpha x}{w_f + sc(M-k-1)\psi_u}\right)^{\frac{1}{1-\alpha}}. \tag{32}$$

2. If $w_f - w_r \in ((k-1)s\psi_c(1-\kappa)c, (k-1)s\psi_c c)$, the outside firms act as above whereas the cartel firms offer a mass of jobs at w_f at rate ψ_f and, additionally, uniformly distributed wages on $[w_r, w_1]$, at rate $\psi_c - \psi_f$.

w_1 and ψ_f are such that $sc\psi_f(1-\kappa)(k-1) = w_f - w_1$ and $w_1 - w_r = sc(\psi_c - \psi_f)(k-1)$. This implies that $\psi_f = \frac{\psi_c}{\kappa} - \frac{w_f - w_r}{sc(k-1)\kappa}$ and $w_1 = w_r + sc(k-1)\left(\frac{\psi_c(1-\kappa)}{\kappa} + \frac{w_f - w_r}{sc(k-1)\kappa}\right)$.
Employment is given by (27) and (28), as before. The user cost is given by

$$\omega_c = w_f + sc(M-k)\psi_u + sc(k-1)\kappa\psi_f = w_f + sc(M-k)\psi_u + sc(k-1)\left(\psi_c - \frac{w_f - w_r}{sc(k-1)}\right),$$
$$\omega_r = w_f + sc(M-k-1)\psi_u,$$

which implies that the employment demand satisfies

$$N_c^d(\psi_u, \psi_c) = \left(\frac{\alpha x}{w_f + sc(M-k)\psi_u + sc(k-1)\left(\psi_c - \frac{w_f - w_r}{sc(k-1)}\right)}\right)^{\frac{1}{1-\alpha}}, \tag{33}$$
$$N_o^d(\psi_u, \psi_c) = \left(\frac{\alpha x}{w_f + sc(M-k-1)\psi_u}\right)^{\frac{1}{1-\alpha}}. \tag{34}$$

3. If $w_f - w_r \geq (k-1)s\psi_c c$ and $w_f - w_r < (M-1)s\psi_c c$, cartel firms offer jobs uniformly over $[w_r, w_1]$ at rate ψ_1 and over $(w_1, w_f]$ at rate $\psi_2 = \psi_c - \psi_1$. outside firms offer jobs uniformly over $(w_1, w_f]$, at rate ψ_2, and over $(w_f, w_u]$, at rate ψ_u where $w_u = w_f + sc(M-k-1)\psi_u$.
Indifference implies both $w_1 = w_r + sc(k-1)\psi_1$ and $w_1 = w_f - sc(M-1)\psi_2$ which yields $\psi_1 = \frac{sc(M-1)\psi_c - (w_f - w_r)}{sc(M-k)}$ and $w_1 = w_r + sc(k-1)\frac{sc(M-1)\psi_c - (w_f - w_r)}{sc(M-k)}$. Employment satisfies

$$\begin{aligned}
N_c^s(\psi_u, \psi_c) &= \frac{\psi_1(\psi_u, \psi_c)}{\delta + sM\psi_2(\psi_u, \psi_c) + s(M-k)\psi_u} \frac{\delta}{\delta + k\psi_1(\psi_u, \psi_c) + M\psi_2(\psi_u, \psi_c) + (M-k)\psi_u} \\
&\quad + \frac{\psi_2(\psi_u, \psi_c)}{\delta + s(M-k)\psi_u}\left(1 + sk\frac{\psi_1(\psi_u, \psi_c)}{\delta + sM\psi_2(\psi_u, \psi_c) + s(M-k)\psi_u}\right)\frac{\delta}{\delta + k\psi_1(\psi_u, \psi_c) + M\psi_2(\psi_u, \psi_c) + (M-k)\psi_u}, \\
&= \frac{\frac{(M-1)\psi_c}{M-k} - \frac{w_f - w_r}{sc(M-k)}}{\delta + s\frac{M}{M-k}\frac{w_f - w_r}{sc} - s\frac{M(k-1)}{M-k}\psi_c + s(M-k)\psi_u} \frac{\delta}{\delta + \psi_c + \frac{w_f - w_r}{sc} + (M-k)\psi_u} \\
&\quad + \frac{\frac{w_f - w_r}{sc(M-k)} - \frac{k-1}{M-k}\psi_c}{\delta + s(M-k)\psi_u}\left(1 + sk\frac{\frac{(M-1)\psi_c}{M-k} - \frac{w_f - w_r}{sc(M-k)}}{\delta + s\frac{M}{M-k}\frac{w_f - w_r}{sc} - s\frac{M(k-1)}{M-k}\psi_c + s(M-k)\psi_u}\right)\frac{\delta}{\delta + \psi_c + \frac{w_f - w_r}{sc} + (M-k)\psi_u}, \\
N_o^s(\psi_u, \psi_c) &= \frac{1}{M-k}\left(\frac{\psi_c + \frac{w_f - w_r}{sc} + (M-k)\psi_u}{\delta + \psi_c + \frac{w_f - w_r}{sc} + (M-k)\psi_u} - kN_c\right).
\end{aligned}$$

The user cost is given by

$$\omega_c = w_f + sc(M-k)\psi_u, \tag{35}$$

and

$$\omega_o = w_f + sc(M-k-1)\psi_u. \tag{36}$$

which implies that the employment demand satisfies

$$\begin{aligned} N_c^d(\psi_u,\psi_c) &= \left(\frac{\alpha x}{w_f + sc(M-k)\psi_u}\right)^{\frac{1}{1-\alpha}}, && (37)\\ N_o^d(\psi_u,\psi_c) &= \left(\frac{\alpha x}{w_f + sc(M-k-1)\psi_u}\right)^{\frac{1}{1-\alpha}}. && (38)\end{aligned}$$

4. If $w_f - w_r \geq (M-1)s\psi_c c$, cartel firms offer jobs uniformly over $(w_1, w_f]$ at rate ψ_c. The outside firms offer jobs uniformly over $[w_r, w_1)$ at rate $\psi_1 = \frac{w_f - w_r - sc\psi_c(M-1)}{sc(M-k-1)}$, over $(w_1, w_f]$ at rate ψ_c, and over $(w_f, w_u]$ at rate ψ_u. The user costs are unchanged from the previous case,

$$\omega_c = w_f + sc(M-k)\psi_u \tag{39}$$

and

$$\omega_r = w_f + sc(M-k-1)\psi_u. \tag{40}$$

Employment satisfies (where the arguments (ψ_u, ψ_c) for ψ_1 and ψ_2 are dropped)

$$\begin{aligned} N_c^s(\psi_u,\psi_c) &= \frac{\psi_c}{\delta + s(M-k)\psi_u}\left(1 + s(M-k)\frac{\psi_1}{\delta + sM\psi_c + s(M-k)\psi_u}\right)\frac{\delta}{\delta + (M-k)\psi_1 + M\psi_c + (M-k)\psi_u},\\ &= \frac{\psi_c}{\delta + s(M-k)\psi_u}\,\frac{\delta}{\delta + sM\psi_2 + s(M-k)\psi_u}\,\frac{\delta + s(M-k)\psi_1 + sM\psi_c + s(M-k)\psi_u}{\delta + (M-k)\psi_1 + M\psi_c + (M-k)\psi_u},\\ &= \frac{\psi_c}{\delta + s(M-k)\psi_u}\,\frac{\delta}{\delta + sM\psi_c + s(M-k)\psi_u}\,\frac{\delta + \frac{M-k}{M-k-1}\frac{w_f - w_r}{c} - s\frac{k}{(M-k-1)}\psi_c + s(M-k)\psi_u}{\delta + \frac{1}{s}\frac{M-k}{M-k-1}\frac{w_f-w_r}{c} - \frac{k}{(M-k-1)}\psi_c + (M-k)\psi_u},\\ N_o^s(\psi_u,\psi_c) &= \frac{1}{M-k}\left(\frac{k\psi_1 + M\psi_2 + (M-k)\psi_u}{\delta + k\psi_1 + M\psi_2 + (M-k)\psi_u} - kN_c\right).\end{aligned}$$

There are some implication of the characterization that will be useful later on.

REMARK 1. The functions $N_c^s(\psi_u,\psi_c)$, $N_o^s(\psi_u,\psi_c)$, $N_c^d(\psi_u,\psi_c)$, and $N_o^d(\psi_u,\psi_c)$ are continuous in both argument.

To see this note that the expressions are clearly continuous within each case. We therefore just have to consider what happens when we cross between the cases.

Notice that when going from equilibrium type I to II the supply N_c^s and N_c^s are unchanged so the result is immediate. Similarly, at the left limit of this region where $w_f - w_r = (k-1)s\psi_c(1-\kappa)c$, the limit of the mass of offers at w_f is given by

$$\psi_f = \frac{\psi_c}{\kappa} - \frac{w_f - w_r}{sc(k-1)\kappa} = \psi_c. \tag{41}$$

Similarly, when going from case II to III. In the right limit of region II, the rate offer at w_f satisfies

$$\psi_f = \frac{\psi_c}{\kappa} - \frac{w_f - w_r}{sc(k-1)\kappa} = 0. \tag{42}$$

In the left limit of region III,

$$w_1 = w_r + sc(k-1)\frac{sc(M-1)\psi_c - (w_f - w_r)}{sc(M-k)} = w_f, \tag{43}$$

so there is no overlapping region which implies that that the supply and demand are continuous as we move from region II to III.

Lastly, considering case III to case IV. In the right limit of region III and on the left limit of region IV, the rate offer over $[w_r, w_f]$ are the same for the two types of firms.

remaining proof strategy The four cases above are ordered by w_f relative to an endogenous variable, ψ_c. We will therefore show that there exists a unique value ψ_c such that firms' optimal labor demand equals labor supply as implied by worker flows. To do so, we first map ψ_c to ψ_u by showing that, given ψ_u, there exists a unique ψ_c such that relative labor demand equals relative labor supply and that the mapping from ψ_u to ψ_c is continuous. Then, we show that there exists a unique value of ψ_u that equates labor demand and supply for the outside firms using the intermediate value theorem. As a preliminary step, we establish a bound for ψ_u in the next lemma.

Lemma A.9. *ψ_u is bounded above by $\overline{\psi}$ which is the unique value that satisfies*

$$\overline{\psi}\left(1-(1-s)k\left(\frac{\alpha x}{w_f+(r+\delta+(M-k)s\overline{\psi})c}\right)^{\frac{1}{1-\alpha}}-(M-k)\left(\frac{\alpha x}{w_f+(r+\delta+(M-k-1)s\overline{\psi})c}\right)^{\frac{1}{1-\alpha}}\right)$$
$$=\delta\left(\frac{\alpha x}{w_f+(r+\delta+(M-k-1)s\overline{\psi})c}\right)^{\frac{1}{1-\alpha}}. \quad (44)$$

Before proving the statement, it is instructive to briefly consider its content. In words, this is the average contact rate above w_f in an equilibrium with no mass at w_f where the cartel posts on $[w_r, w_f]$ and the outside firms post on $[w_f, w_u]$, that is the knife-edge case between Case II and Case III described in the main text.

Proof. We first note that since the right hand side is increasing in $\overline{\psi}$, the left hand side is decreasing, and the right hand side is smaller for $\overline{\psi} \to 0$ and larger for $\overline{\psi} \to \infty$), a unique positive value of $\overline{\psi}$ that solves (44) exists.

Assume then, to the contrary of the lemma, that an equilibrium exists with $\psi_u > \overline{\psi}$. The user cost of the cartel firms is therefore weakly higher than $w_f + (r + \delta + s(M-k)\psi_u)c$. (The cost is weakly higher because the cartel firms might post a mass of jobs at w_f.) Employment in the cartel firms is therefore bounded above by

$$N_c \leq \left(\frac{\alpha x}{w_f+(r+\delta+s(M-k)\psi_u)c}\right)^{\frac{1}{1-\alpha}} < \left(\frac{\alpha x}{w_f+(r+\delta+s(M-k)\overline{\psi})c}\right)^{\frac{1}{1-\alpha}}. \quad (45)$$

The lowest employment in the outside firms occurs if they do not post any additional job offers. Employment in the outside firms is therefore bounded below by $\tilde{N}_o$ which is if the only post above w_f which solves

$$\delta\tilde{N}_o \quad = \quad \psi_u\left(1-(M-k)N_o-(1-s)kN_c\right), \quad (46)$$

where N_o is the employment at the outside firms (since additional employment below w_f at the outside firms would raise their employment). From above it follows that the right hand side of this equation is bounded below (do to the upper bound for N_c) by

$$\psi_u\left(1-(M-k)N_o-(1-s)kN_c\right) > \overline{\psi}\left(1-(1-s)k\left(\frac{\alpha x}{w_f+(r+\delta+(M-k)s\overline{\psi})c}\right)^{\frac{1}{1-\alpha}}-(M-k)\left(\frac{\alpha x}{w_f+(r+\delta+(M-k-1)s\overline{\psi})c}\right)^{\frac{1}{1-\alpha}}\right) \quad (47)$$

We can similarly create an upper bound for the left hand side of (46). Note that since $\psi_u > \bar{\psi}$ it follows from optimal labor demand that

$$\delta \tilde{N}_o < \delta \left(\frac{\alpha x}{w_f + (r + \delta + (M-k-1)s\overline{\psi})c} \right)^{\frac{1}{1-\alpha}}. \tag{48}$$

However, equation (44) implies that the RHS of (47) and (48) to be the same, we get a contradiction. □

Lemma A.10. *For each $\psi_u \in [0, \overline{\psi}]$, there is a unique ψ_c such that $N_o^s(\psi_u, \psi_c)/N_c^s(\psi_u, \psi_c) = N_o^d(\psi_u, \psi_c)/N_c^d(\psi_u, \psi_c)$.*

Proof. We will prove this in two parts.

First, note that N_o^s/N_c^s exceeds N_o^d/N_c^d as $\psi_c \to 0$ since $N_c^s \to 0$ whereas $N_o^d/N_c^d \to \left(\frac{w_f + (r+\delta+s\psi_u(M-k)c}{w_f + (r+\delta+s\psi_u(M-k-1)c} \right)^{\frac{1}{1-\alpha}} > 0$ and $N_o^s > 0$. Second, as $\psi_c \to \infty$, $N_o^d/N_c^d \to \infty$ whereas N_o^s/N_c^s remain bounded. Since N_o^d/N_c^d and N_o^s/N_c^s are continuous functions in ψ_c, a solution $N_o^d/N_c^d = N_o^s/N_c^s$ exists.

We will now show that $N_o^d/N_c^d - N_o^s/N_c^s$ is strictly increasing in ψ_c which implies that there exist a unique solution.

First, from the characterization it follows that N_o^d/N_c^d is weakly increasing in ψ_c since N_c^d is weakly decreasing in ψ_c whereas N_o^d is independent of ψ_c.

We will now should that N_o^s/N_c^s is strictly decreasing in ψ_c.

We do it case by case and show that within each case, it holds. For case I and II, it follows directly from (27) and (28).

First, take a ψ_c that is strictly lower, the outside firms offer (weakly) more jobs below w_f. Denote w' the lowest wage paid by the outside firms. We get that from the characterization and ψ_c being lower, it follows that w' is weakly lower.

First start with case III. Denote by ψ' the rate of offers by the outside firms over the region $[w', w_f]$. In the new equilibrium ψ' has increased.

Total employment is given by

$$E = \frac{k(\psi_c - \psi') + M\psi' + (M-k)\psi_u}{\delta + k(\psi_c - \psi') + M\psi' + (M-k)\psi_u} = \frac{k\psi_c + (M-k)\psi' + (M-k)\psi_u}{\delta + k\psi_c + (M-k)\psi' + (M-k)\psi_u}, \tag{49}$$

using that $(M-k)\psi' = \frac{w_f - w_r}{sc} - (k-1)\psi_c$, this implies that a fall in ψ_c results in lower total employment. Total employment above w' solves

$$E' = \frac{M\psi' + (M-k)\psi_u}{\delta}(1 - E + s(E - E')) = \frac{M\psi' + (M-k)\psi_u}{\delta + sM\psi' + s(M-k)\psi_f}(1 - (1-s)E), \tag{50}$$

which is strictly higher when ψ_c is lower since ψ' has increased and employment E is lower. Thus, the outside firms employment increase unless the employment above w_f decrease sufficiently. Total employment strictly above w_f solves

$$\delta E_u = \frac{(M-k)\psi_u}{\delta}(1 - E + s(E - E_u)) \to E_u = \frac{(M-k)\psi_u}{\delta + s(M-k)\psi_u}(1 - (1-s)E) \tag{51}$$

which has strictly higher since E is lower. Total employment in the outside firms has therefore increased. Total employment by cartel firms have decreased since total employment is lower. Thus, N_o^s/N_c^s is decreasing in ψ_c.

For case IV, consider again a lower value of ψ_c. A lower value of ψ_c implies that total employment has increased (see characterization above). Employment in the cartel firms satisfies

$$N_c^s = \frac{\delta}{\delta + s(M-k)\psi_u}\frac{\psi_c}{\delta + sM\psi_c + s(M-k)\psi_u}\frac{\delta + \frac{M-k}{M-k-1}\frac{w_f - w_r}{c} - s\frac{k}{(M-k-1)}\psi_c + s(M-k)\psi_u}{\delta + \frac{1}{s}\frac{M-k}{M-k-1}\frac{w_f - w_r}{c} - \frac{k}{(M-k-1)}\psi_c + (M-k)\psi_u}. \tag{52}$$

This expression is lower when ψ_c is lower. Thus, since total employment has increased and N_c^s is lower, we again get that N_o^s/N_c^s is decreasing in ψ_c.

□

Lemma A.11. *The function $\psi_c(\psi_u)$ such that $\frac{N_o^s(\psi_u,\psi_c(\psi_u))}{N_c^s(\psi_u,\psi_c(\psi_u))} = \frac{N_o^d(\psi_u,\psi_c(\psi_u))}{N_c^d(\psi_u,\psi_c(\psi_u))}$ is continuous in ψ_u for $\psi_u \in [0, \overline{\psi}_u]$.*

Proof. Note that the function $f(\psi_u, \psi_c) = N_o^d(\psi_u, \psi_c)/N_c^d(\psi_u, \psi_c) - N_o^s(\psi_u, \psi_c)/N_c^s(\psi_u, \psi_c)$ is continuous in both arguments (from the characterization) and, from the proof of Lemma A.10, strictly increasing in ψ_c. Assume that the function $\psi_c(\psi_u)$ is not continuous at ψ_u with $\lim_{\epsilon \to 0} \psi_c(\psi_u + \epsilon) = \psi_c(\psi_u) + \delta$. We therefore get a contradiction since $\lim_{\epsilon \to 0} f(\psi_u + \epsilon, \psi_c(\psi_u + \epsilon)) - f(\psi_u, \psi_c(\psi_u)) > 0$ since $f(\psi_u, \psi_c)$ is strictly increasing in ψ_c. □

Parameter Restriction II

ASSUMPTION 1. We assume, from here on, that

$$\min_{\psi_u\in[0,\overline{\psi}]}\frac{(\delta+s(M-k)\psi_u)^2/\delta\left(\frac{\alpha x}{w_f+(r+\delta+s(M-k)\psi_u)c}\right)^{\frac{1}{1-\alpha}}}{1-\left(\frac{\delta+s(M-k)\psi_u}{\delta}sM+(1-s)k\right)\left(\frac{\alpha x}{w_f+(r+\delta+s(M-k)\psi_u)c}\right)^{\frac{1}{1-\alpha}}-(1-s)(M-k)\left(\frac{\alpha x}{w_f+(r+\delta+s(M-k-1)\psi_u)c}\right)^{\frac{1}{1-\alpha}}}>\frac{w_f-w_r}{(M-1)sc}. \tag{53}$$

Lemma A.12. *No type IV equilibrium exists.*

Proof. Recall that a type IV equilibrium occurs when $\psi_c<\frac{w_f-w_r}{(M-1)sc}$. The following proof then just offers a lower bound on ψ_c in terms of primitives and assumption 1 guarantees that lower bound is sufficiently large. To see this note that under case IV, the offer rate of cartel firms is given by

$$\psi_c=\frac{(\delta+s(M-k)\psi_u)}{1-MN_c-sX-(1-s)(M-k)\left(N_o-N_c\right)}N_c \tag{54}$$

X denotes the employment strictly above w_f, which solves

$$\delta X = (M-k)\psi_u\left(1-(1-s)kN_c-(1-s)(M-k)N_o-sX\right) \tag{55}$$

$$X = \frac{(M-k)\psi_u}{\delta+s(M-k)\psi_u}(1-(1-s)kN_c-(1-s)(M-k)N_o) \tag{56}$$

We therefore have

$$\begin{aligned}\psi_c &= \frac{(\delta+s(M-k)\psi_u)N_c}{1-MN_c-s\frac{(M-k)\psi_u}{\delta+s(M-k)\psi_u}(1-(1-s)kN_c-(1-s)(M-k)N_o)-(1-s)(M-k)\left(N_o-N_c\right)}\\ &= \frac{(\delta+s(M-k)\psi_u)}{\frac{\delta}{\delta+s(M-k)\psi_u}-sMN_c-\frac{\delta}{\delta+s(M-k)\psi_u}(1-s)\left(kN_c+(M-k)N_o\right)}N_c\\ &= \frac{(\delta+s(M-k)\psi_u)^2/\delta}{1-\frac{\delta+s(M-k)\psi_u}{\delta}sMN_c-(1-s)\left(kN_c+(M-k)N_o\right)}N_c,\end{aligned} \tag{57}$$

We know $\psi_u\in[0,\overline{\psi}]$ and that, under case IV,

$$N_c = \left(\frac{\alpha x}{w_f+(r+\delta+s(M-k)\psi_u)c}\right)^{1/(1-\alpha)}, \tag{58}$$

$$N_o = \left(\frac{\alpha x}{w_f+(r+\delta+s(M-k-1)\psi_u)c}\right)^{1/(1-\alpha)}. \tag{59}$$

using these two equations, we get that

$$\begin{aligned}\psi_c &= \frac{(\delta+s(M-k)\psi_u)^2/\delta}{1-\left(\frac{s(M-k)\psi_u}{\delta}sM+s(M-k)+k\right)N_c-(1-s)(M-k)N_o}N_c, \\ &= \frac{(\delta+s(M-k)\psi_u)^2/\delta\left(\frac{\alpha x}{w_f+(r+\delta+s(M-k)\psi_u)c}\right)^{\frac{1}{1-\alpha}}}{1-\left(\frac{s(M-k)\psi_u}{\delta}sM+s(M-k)+k\right)\left(\frac{\alpha x}{w_f+(r+\delta+s(M-k)\psi_u)c}\right)^{\frac{1}{1-\alpha}}-(1-s)(M-k)\left(\frac{\alpha x}{w_f+(r+\delta+s(M-k-1)\psi_u)c}\right)^{\frac{1}{1-\alpha}}}.\end{aligned}$$

Assumption 1 then implies that $s(M-1)\psi_c c \geq w_f - w_r$ which immediately rules out case IV. □

Lemma A.13. *There exists a unique equilibrium value of ψ_u.*

Proof. First clearly as $\psi_u \to 0$, $N_o^s(\psi_u, \psi_c(\psi_u)) < N_o^d(\psi_u, \psi_c(\psi_u))$ since the fixed wage is binding. As $\psi_u \to \overline{\psi}$, the opposite is true by Lemma A.9. Note further that the functions $N_o^s(\psi_u, \psi_c(\psi_u)) - N_o^d(\psi_u, \psi_c(\psi_u))$ is continuous in both arguments. The function $\psi_c(\psi_u))$ is continuous from Lemma A.11. Therefore the function $N_o^s(\psi_u, \psi_c(\psi_u)) - N_o^d(\psi_u, \psi_c(\psi_u))$ is continuous and a solution exists.

Second, for a higher value of ψ_u, $N_o^d(\psi_u, \psi_c(\psi_u))$ is lower. Consider now ψ_c that is weakly lower. This implies that $N_o^s(\psi_u, \psi_c(\psi_u))$ is higher (see proof of Lemma A.10) which results in the contradiction. Consider a case in which ψ_c has increased. Under Case I, II, and III this implies that the job offer rate over $[w_r, w_f]$ weakly increase. Employment must therefore go up. This results in the contradiction since $N_o^d(\psi_u, \psi_c(\psi_u)), N_c^d(\psi_u, \psi_c(\psi_u))$ have strictly decreased.

□

B Proof of Proposition 2

Part A

Proof. For any equilibrium with a binding w_f, the characterization of the equilibrium obtained in proving Proposition 1 directly implies that the user cost for the cartel members is higher. Since employment and profits are strictly declining in the user cost of labor, these firms also have lower profits and employment.

To show that the impact on the average wage is ambiguous consider the following case in which all firms are in the cartel. Let w_u and ψ denote the highest wage and offer arrival rate in the absence of collusion. Take the wage ceiling $w_f = w_u - s(M-1)\psi\kappa c$. We can now guess and verify that employment remains unchanged. Given an unchanged employment, the equilibrium is in case I with an unchanged user cost since the lower highest wage is offset by the turnover at the mass point. Clearly the average wage will increase provided that κ is sufficiently small relative to δ. In contrast, if $k = M$ and $w_f = w_r$, clearly wages are lower. □

Part B

Cartel Firms

Proof. We first cover the cartel firms where we proceed by example. Equation (11) in the main body gives a closed form expression for the cartel's user cost in the special case where $s = 1$, $k < M$, $\kappa \to 0$, and we are in an equilibrium of type I. There, the cartel's user cost is declining in the user cost of the outside firms (and therefore in w_f) for sufficiently low w_f. In contrast, the user cost of the cartel members is increasing in w_f when $k = M$. This establishes that reducing the wage ceiling may or may not reduce the cartel's user cost.

Additionally, in the case covered by (11), the cartel's user cost goes to infinity for sufficiently low w_f which gives a case where the cartel members' profits are lower than without a binding wage ceiling. The opposite is the case for any binding wage ceiling above w_r when $k = M$. The impact of wage fixing on the cartel's profit is therefore ambiguous.

□

Outside Firms

Proof. We now turn to the outside firms. We prove that the highest wage w_u is strictly increasing in w_f. Since the user costs of the outside firms are given by $w_u + (r+\delta)c$, this is sufficient to establish that their user cost is strictly increasing in w_f, the part of the proposition that remains to be proven.

We proceed by contradiction. Suppose, to the contrary, that there exist two (binding) values for the wage ceiling with $w'_f < w''_f$ such that w_u is higher under w'_f than w''_f. This has two implications that we will use: first, employment at outside firms is higher under

w''_f since the user cost are lower; second, the amount of offers above w_f, ψ_u, is strictly lower under w''_f. The latter follows from $w_u = w_f + s(M-k-1)\psi_u c$, since w_f is higher but w_u is weakly lower.

The rest of the proof establishes contradictions to these implications in a case-by-case fashion. We break the proof into two exhaustive cases, a) the case where the cartel members' user cost is weakly lower under w''_f compared with w'_f, and b) the case where the cartel members' user cost is strictly higher under w''_f compared with w'_f.

Part a) lower cartel user cost under higher ceiling First, consider the case where the cartel members' user cost is weakly lower under w''_f compared with w'_f. This implies that cartel employment N_c is weakly higher.

Part a-I) Case I and II equilibrium under w''_f We first show that it cannot be that under w''_f, there is an equilibrium of Case I or II. Equilibrium employment in outside firms under an equilibrium of these types is given by:

$$N_o = \frac{\psi_u}{\delta + (M-k)\psi_u}\left(1-(1-s)kN_c\right). \tag{60}$$

Employment under w'_f is

$$N_o = \frac{\psi_u}{\delta + (M-k)\psi_u}\left(1-(1-s)kN_c-(1-s)(M-k)\underline{N}_o\right) + \underline{N}_o, \tag{61}$$

where $\underline{N}_o$ is the level of employment below w'_f in an outside firm. Since ψ_u is higher, N_c is lower, and $\underline{N}_o \geq 0$, employment is higher at the outside firms at the lower w'_f. This contradicts the earlier observation that employment in the outside firms is higher at w''_f, ruling out an equilibrium of Case I or II under w''_f.

Part a-II) Case III under w''_f What remains to be shown is then that there is no equilibrium of Case III under w''_f. Denote the lowest wage posted by the outside employers associated with w''_f as w''_1. We will first consider the case in which we are in case I or II under w'_f and then cover the case III.

Part a-II-i) Equilibrium under w'_f is either in Case I or II

Step 1: it must be that $w''_1 < w'_f$.

Assume, to the contrary, that $w''_1 \geq w'_f$. Let $\tilde{\psi}$ denote the total rate of offers above w''_1. From the characterization of Case III with w''_f, we know that $\tilde{\psi} = \frac{M-k}{M-k-1}\frac{w''_u - w''_f}{sc} + \frac{M}{M-1}\frac{w''_f - w''_1}{sc}$. In turn, under w'_f we have that $\tilde{\psi} = \frac{M-k}{M-k-1}\frac{w'_u - w''_1}{sc}$, which is larger than under w''_f because $w'_u > w''_u$.

But recall the implication from above that employment at outside firms must be higher under w''_f. The total rate of job offers, denoted $\bar{\psi}$, must therefore be higher under w''_f.

Total employment above w''_1 solves

$$N_1 = \frac{\tilde{\psi}}{\delta + s\tilde{\psi}}\left(1 - (1-s)\frac{\bar{\psi}}{\delta + \bar{\psi}}\right). \tag{62}$$

This expression is increasing in $\tilde{\psi}$ but declining in $\bar{\psi}$. N_1 is therefore lower under w''_f since $\tilde{\psi}$ is lower whereas $\bar{\psi}$ is higher compared to w'_f.

Under w'_f, the total employment by the outside firms $(M-k)N_o$ is weakly higher than N_1, since only the outside firms post above w_1 and $w''_1 \geq w'_f$. Employment at the outside firms is lower than employment N_1 under w''_f since the cartel firms make offers over $[w''_1, w''_f]$. This implies that employment in the outside firms is lower under w''_f than under w'_f (since N_1 is lower under the former). This yields a contradiction and so it must be the case that $w''_1 \leq w'_f$.

Step 2: Case I or II equilibrium under w'_f

Next, we show that there are fewer offers strictly above w'_f under w''_f compared with w'_f. Step 1 proved that $w''_1 < w'_f$. This implies that all M firms offer wages over the region $[w'_f, w''_f]$ under w''_f whereas only the $M-k$ outside firms make offer over this region under w'_f. The total amount of offers above w'_f is thus $\frac{M}{M-1}\frac{w''_f - w'_f}{sc} + \frac{M-k}{M-k-1}\frac{w''_u w''_f}{sc}$ under w''_f. Under w'_f, it is $\frac{M-k}{M-k-1}\frac{w'_u w'_f}{sc}$ which is strictly larger since, under our conjecture, $w'_u \geq w''_u$.

Next, consider the total amount of offers weakly below w'_f. This is strictly higher under w'_f (if we are in case I or II under w'_f) since it is bounded below by $\frac{k}{k-1}\frac{w'_f - w_r}{sc}$ whereas under w''_f, it is given by $\frac{M}{M-1}\frac{w'_f - w''_1}{sc} + \frac{k}{k-1}\frac{w''_1 - w_r}{sc}$, which is less since $w''_1 < w'_f$ (step 1). Jointly with the previous paragraph this implies that, under w'_f, employment would be strictly higher compared with w''_f, a contradiction.

This rules out Case I or II under w'_f and therefore completes subcase **a-II-i)**.

Part a-II-ii) Case III equilibrium under w_f' This leaves case III under w_f'.

Total employment by outside firms under either wage ceiling can conveniently be broken into two pieces: $\frac{M-k}{M}$ times total employment above w_1 plus $\frac{k}{M}$ times total employment above the wage ceiling w_f.

We will next show that under w_f' each of these is larger compared with w_f''. This implies that employment at the outside firms is larger, resulting in a contradiction.

Start with employment above w_f. The total rate of offers above w_f solves $\tilde{\psi} = \frac{M-k}{M-k-1}\frac{w_u - w_f}{sc}$. This is higher under w_f' compared to w_f'' since w_u is higher while w_f is lower. The overall rate of offers $\bar{\psi}$ in turn is lower under w_f' since total employment is lower under the conjecture. The level of employment above w_f is given by $\frac{\tilde{\psi}}{\delta+s\tilde{\psi}}\left(1-(1-s)\frac{\bar{\psi}}{\delta+\bar{\psi}}\right)$ which is thus larger under w_f' compared with w_f''.

Next consider the second piece, the total level of employment above w_1. First, the total number of offers $\bar{\psi}$ is $\frac{k}{k-1}\frac{w_1-w_r}{sc} + \frac{M}{M-1}\frac{w_f-w_1}{sc} + \frac{M-k}{M-k-1}\frac{w_u-w_f}{sc}$. Since total employment is lower and w_u is higher under w_f' compared to w_f'' (under the conjecture), w_1 must therefore be lower under w_f' since otherwise the rate of offers would be higher. The total rate of offers above w_1 solves $\tilde{\psi} = \frac{M}{M-1}\frac{w_f-w_1}{sc} + \frac{M-k}{M-k-1}\frac{w_u-w_f}{sc}$ which is higher under w_f' compared to w_f'' since w_u is higher while w_f and w_1 are lower. Employment above w_1 can then again be written as $\frac{\tilde{\psi}}{\delta+s\tilde{\psi}}\left(1-(1-s)\frac{\bar{\psi}}{\delta+\bar{\psi}}\right)$, which is thus higher under w_f' compared to w_f''.

Jointly, it follows from the previous two paragraphs that employment at the outside firms is larger under w_f', a contradiction. That rules out subcase **a-II-ii)** and thus completes **part a)**.

Part b) higher cartel user cost under higher ceiling Consider now the case where the user cost for the cartel members is strictly higher under w_f'' compared with w_f'. We again split the proof into subcases.

Before turning to the subcases, we will derive an inequality which we will use in each subcase. Define ψ_u as the rate of offers strictly above w_f. As per the initial conjecture, the user cost of the outside firms $w_f + s(M-k-1)\psi_u c$ is lower under w_f''. For this to be the case, it must be that $s(M-k-1)c(\psi_u'' - \psi_u') \leq -(w_f'' - w_f')$. We therefore have that

$$w_f'' - w_f' + s(M-k)c(\psi_u'' - \psi_u') \leq w_f'' - w_f' - \frac{M-k}{M-k-1}(w_f'' - w_f') = -\frac{w_f'' - w_f'}{M-k-1} < 0. \tag{63}$$

Part b-I) Case III equilibrium under w_f''. Equation (63) implies that the user cost of the cartel under case III, $w_f + s(M-k)\psi_u c + (r+\delta)c$, is strictly lower under w_f'' which contradicts the conjecture that the cartel's user cost is lower under w_f''.

Part b-II-i) Case I or II equilibrium under w_f'' and ψ_c is weakly lower under w_f''. We will derive a contradiction by showing that the user cost of the cartel firms is higher under w_f''. To do so, we will first show that we are in case I or II also under w_f'.

The condition for being in case I or II reads $\frac{w_f - w_r}{\psi} < (k-1)sc$. This condition is met under w_f'' (because this is the case currently under consideration). Since the left hand side is smaller when w_f is lower and ψ_c is higher, it is also met under w_f', which means that we must be in case I or II under w_f' as well.

The offer rate of jobs paying w_f under case I or II is $\psi_f = \min\left\{\psi_c, \frac{\psi_c}{\kappa} - \frac{w_f - w_r}{sc(k-1)\kappa}\right\}$. Since w_f is higher and ψ_c is lower under w_f'' this is strictly lower.

This implies that then user cost for the cartel firms would be lower under w_f'' since $w_f + s\psi_u(M-k)c$ is lower via (63) and so is $s\psi_f(k-1)c$ which contradicts the initial conjecture that the cartels user cost is higher under w_f''. Thus, ψ_c must be strictly higher under w_f''.

Part b-II-ii) Case I or II equilibrium under w_f'' and ψ_c is strictly higher under w_f''. Denote by $\hat{\psi}$ the total rate of offers weakly below the wage ceiling w_f. Employment below w_f satisfies

$$\frac{\hat{\psi}}{\delta + s(M-k)\psi_u} \frac{\delta}{\delta + \hat{\psi} + (M-k)\psi_u}. \tag{64}$$

In a case I or II equilibrium this is equal to total cartel employment. In a case III equilibrium this is strictly larger than total cartel employment.

In order to rank cartel employment between the two wage ceilings we will thus rank $\hat{\psi}$ and ψ_u.

$\hat{\psi}$ is lower under w_f'' compared with w_f'. This follows directly from $\psi_c' < \psi_c''$ if we are in case I or II under w_f'; if we are in case III under w_f', it follows from the fact that $\hat{\psi} = \frac{M}{M-1}\frac{w_f' - w_1}{sc} + \frac{k}{k-1}\frac{w_1 - w_r}{sc} < \frac{k}{k-1}\frac{w_f'' - w_r}{sc} \geq \psi_c''$ where the last inequality comes from the fact that we are in a case I or II equilibrium under w_f''.

Turning to ψ_u (the rate of offers by a regular firm above the ceiling), we have that $\psi_u = \frac{M-k}{M-k-1}\frac{w_u - w_f}{sc}$. Since w_u is higher and w_f' is strictly lower this is strictly larger under w_f'. Finally, note that the above expression is an upper bound for cartel employment under a case III equilibrium. Jointly, it follows that cartel employment is higher under w_f'', a contradiction. This completes the proof.

$\square$

C Derivations of harm

We will calculate a simple expression for the average wage above w_f under the case when w_f is sufficiently low such that we are in case I. We first calculate the average wage above some w_f assuming that wages are offered at a rate λ above and are uniformly distributed over $[w_f, w_u]$. The average wage in the outside firms satisfies

$$
\begin{aligned}
\mathrm{E}[w_o] &= \int_{w_f}^{w_u} w g_o(w) dw \\
&= w_f + \int_{w_f}^{w_u} (1 - G_o(w)) dw \\
&= w_f + \int_{w_f}^{w_u} \left(1 - \frac{1 - \frac{w_u - w}{w_u - w_f}}{1 + \frac{s\lambda}{\delta} \frac{w_u - w}{w_u - w_f}} \right) dw \\
&= w_u - \int_{w_f}^{w_u} \left(\frac{\delta}{s\lambda} \frac{\frac{s\lambda}{\delta} + 1 - \left(1 + \frac{s\lambda}{\delta} \frac{w_u - w}{w_u - w_f}\right)}{1 + \frac{s\lambda}{\delta} \frac{w_u - w}{w_u - w_f}} \right) dw \\
&= w_u - \int_{w_f}^{w_u} \left(\frac{\delta}{s\lambda} \frac{\frac{s\lambda}{\delta} + 1}{1 + \frac{s\lambda}{\delta} \frac{w_u - w}{w_u - w_f}} - \frac{\delta}{s\lambda} \right) dw \\
&= w_u - \frac{\delta(w_u - w_f)}{s\lambda} \left(1 + \frac{\delta}{s\lambda}\right) \log\left(1 + \frac{s\lambda}{\delta}\right) + \frac{\delta}{s\lambda}(w_u - w_f) \\
&= w_f + (w_u - w_f)\left(1 + \frac{\delta}{s\lambda}\right) - \frac{\delta(w_u - w_f)}{s\lambda} \left(1 + \frac{\delta}{s\lambda}\right) \log\left(1 + \frac{s\lambda}{\delta}\right)
\end{aligned}
$$

To calculate the average wage without wage fixing, we use that $\lambda = M\psi$, $w_f = w_r$, $w_u - w_f = s(M-1)\psi c$ and $M\psi = \frac{\delta \bar{N}}{1 - \bar{N}}$, where $\bar{N}$ is the total employment.

$$
\begin{aligned}
\mathrm{E}[w]_{k=0} &= w_r + s\frac{M-1}{M}\frac{\delta\bar{N}}{1-\bar{N}} c \left(1 + \frac{\delta}{s\frac{\delta\bar{N}}{1-\bar{N}}}\right) - \frac{\delta(M-1)c}{M} \left(1 + \frac{\delta}{s\frac{\delta\bar{N}}{1-\bar{N}}}\right) \log\left(1 + \frac{s\frac{\delta\bar{N}}{1-\bar{N}}}{\delta}\right) \\
&= w_r + \frac{M-1}{M}\delta c \left(\left(\frac{1-(1-s)\bar{N}}{1-\bar{N}} \right) - \left(\frac{1-(1-s)\bar{N}}{s\bar{N}} \right) \log\left(\frac{1-(1-s)\bar{N}}{1-\bar{N}} \right) \right)
\end{aligned}
$$

To calculate the average wage strictly above w_f under wage fixing, we use that $w_u - w_f = \frac{M-k-1}{M-k} s\lambda c$ where $\lambda = (M-k)\psi_o$,

$$\mathrm{E}[w_o] = w_f + \frac{M-k-1}{M-k}\delta c\left(\frac{s\lambda}{\delta}+1\right) - \frac{M-k-1}{M-k}\delta c\left(1+\frac{\delta}{s\lambda}\right)\log\left(1+\frac{s\lambda}{\delta}\right)$$

next use that $\psi_o = \frac{\delta N_o}{1-(M-k)N_o-(1-s)kN_c}$ and therefore $s\lambda/\delta = s(M-k)\psi_o/\delta = \frac{s(M-k)N_o}{1-(M-k)N_o-(1-s)kN_c}$ to get that

$$\begin{aligned}\mathrm{E}[w_o] &= w_f + \frac{M-k-1}{M-k}\delta c\Bigg(\frac{1-(1-s)(M-k)N_o-(1-s)kN_c}{1-(M-k)N_o-(1-s)kN_c} \\ &\quad - \frac{1-(1-s)(M-k)N_o-(1-s)kN_c}{s(M-k)N_o}\log\left(\frac{1-(1-s)(M-k)N_o-(1-s)kN_c}{1-(M-k)N_o-(1-s)kN_c}\right)\Bigg).\end{aligned}$$

To get mean wages, we weight this expression by the relative employment at the outside firms

$$\begin{aligned}\mathrm{E}[w] &= w_f + \frac{\bar{N}-kN_c}{\bar{N}}\frac{M-k-1}{M-k}\left(\frac{1-(1-s)\bar{N}}{1-\bar{N}+skN_c} - \frac{1-(1-s)\bar{N}}{s(\bar{N}-kN_c)}\log\left(\frac{1-(1-s)\bar{N}}{1-\bar{N}+skN_c}\right)\right)\delta c \\ &= w_f + \frac{\bar{N}-\bar{N}_c}{\bar{N}}\frac{M-k-1}{M-k}\left(\frac{1-(1-s)\bar{N}}{1-\bar{N}+s\bar{N}_c} - \frac{1-(1-s)\bar{N}}{s(\bar{N}-\bar{N}_c)}\log\left(\frac{1-(1-s)\bar{N}}{1-\bar{N}+s\bar{N}_c}\right)\right)\delta c.\end{aligned}$$

C.1 Backing out s from observables

In this section, we show how to back out s in an equilibrium with collusion (Case I and using that $\kappa \approx 0$). Denote by β_o the average offer acceptance rate of workers employed at outside employers. The rate at which workers switch employers ξ is given by the weighted average of the transition rates at the two different types of employers,

$$\xi = \frac{kN_c}{kN_c+(M-k)N_o}s(M-k)\psi_o + \frac{(M-k)N_o}{kN_c+(M-k)N_o}s(M-k-1)\psi_o\beta_o.$$

This uses that workers at outside firms receive job offers from other outside firms at rate $s(M-k-1)\psi_o$, while cartel workers receive offers from outside employers at rate $s(M-k)\psi_o$.

We will next show how to express ψ_o and β_o just in terms of observables. Denote by $G_o(x)$ the fraction of workers in an outside firm with a wage that is below rank x of the wage offer distribution of outside firms. This satisfies $G_o(x) = \frac{x}{1+s(M-k)\psi_o/\delta(1-x)}$. Next note that the acceptance rate of a job offer of rank x to a worker employed at a outside firm is simply $G_o(x)$. To get the average acceptance rate β_o we can just integrate,

$$\begin{aligned}
\beta_o &\equiv \int_0^1 G_o(x)dx = \int_0^1 \frac{x}{1+\frac{s(M-k)\psi_o}{\delta}(1-x)}dx \\
&= \int_0^1 \frac{\delta}{s(M-k)\psi_o} \frac{1+\frac{s(M-k)\psi_o}{\delta} - 1 - \frac{s(M-k)\psi_o}{\delta}(1-x)}{1+\frac{s(M-k)\psi_o}{\delta}(1-x)}dx \\
&= \frac{\delta}{s(M-k)\psi_o}\frac{\delta}{s(M-k)\psi_o}\left(1+\frac{s(M-k)\psi_o}{\delta}\right)\log\left(1+\frac{s(M-k)\psi_o}{\delta}\right) - \frac{\delta}{s(M-k)\psi_o} \\
&= \frac{\delta}{s(M-k)\psi_o}\left(\left(\frac{\delta}{s(M-k)\psi_o}+1\right)\log\left(1+\frac{s(M-k)\psi_o}{\delta}\right)-1\right).
\end{aligned}$$

Finally, use that

$$\psi_o = \frac{\delta N_o}{(1-(1-s)kN_c-(M-k)N_o)}, \tag{65}$$

to write ξ fully in terms of employment and the number of each type of firm as well as δ. This allows to construct s just from information on worker flows ξ and employment inside and outside the cartel.