An Aggregative Games Approach to Merger Analysis in Multiproduct-Firm Oligopoly*

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Abstract

Using an aggregative games approach, we analyze horizontal mergers in a model of multiproduct-firm price competition with nested CES or nested logit demands. We show that the Herfindahl index provides an adequate measure of the oligopoly distortions to consumer surplus and aggregate surplus, and that the induced change in the naively-computed Herfindahl index is a good approximation for the market power effect of a merger. We also provide conditions under which a merger raises consumer surplus, and conditions under which a myopic, consumer-surplus-based merger approval policy is dynamically optimal. Finally, we study the aggregate surplus and external effects of a merger.

Keywords: Multiproduct firms, aggregative game, oligopoly pricing, market power, horizontal merger, Herfindahl index.

1 Introduction

Using an aggregative games approach, we provide an analysis of horizontal mergers in a model of multiproduct-firm price competition with nested CES (NCES) or nested multinomial logit (NMNL) demand systems. The paper makes three contributions. First, we show that the

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Herfindahl index, which plays an important role in antitrust practice, provides an adequate measure of the oligopoly distortions to consumer surplus and aggregate surplus, and that the induced change in the (naively-computed) Herfindahl index is a good approximation for the market power effect of a merger. Second, we provide conditions under which a merger raises consumer surplus, and conditions under which a myopic, consumer-surplus-based merger approval policy is dynamically optimal. Third, we study the aggregate surplus and external effects of a merger.

Almost all mergers involve multiproduct firms selling differentiated products. This is reflected in the literature on merger simulation (e.g., Hausman, Leonard, and Zona, 1994; Nevo, 2000a; Peters, 2006; Miller and Weinberg, 2017) and in the literature on the upward-pricing pressure of mergers (e.g., Werden, 1996; Goppelsroeder, Schinkel, and Tuinstra, 2008; Farrell and Shapiro, 2010; Jaffe and Weyl, 2013), both of which have heavily influenced antitrust practice. Despite this, much of the theoretical literature on horizontal mergers and antitrust, including Farrell and Shapiro (1990), McAfee and Williams (1992), and Nocke and Whinston (2010, 2013), has focused on single-product firms in the homogeneous-goods Cournot setting. An open question is to what extent the insights derived in that earlier literature carry over to more realistic models of (price) competition with differentiated products and multiproduct firms.[1]

There are several desiderata for a flexible model of horizontal mergers and merger control: First, the underlying demand system should have sound micro-foundations and allow for flexible substitution patterns. Second, the model should allow for arbitrary firm and product heterogeneity (e.g., in terms of marginal costs, qualities, size of product portfolios). Third, the underlying oligopoly game should be tractable and give rise to a unique equilibrium. Fourth, the model should permit rich forms of merger-specific synergies (e.g., marginal cost reductions, quality improvements, new products). Finally, for the model to be useful for antitrust practitioners, its predictions should ideally relate to easily observable sufficient statistics such as firm-level market shares and concentration ratios.

The most important hurdle in developing such a model is that the flexibility and tractability desiderata are in conflict with each other. Multiproduct-firm pricing games are known to give rise to several technical difficulties: Among other issues, payoffs often fail to be quasi-concave and/or (log-)supermodular in own prices (Spady, 1984; Hanson and Martin, 1996; Whinston, 2007). To the best of our knowledge, existence and uniqueness of an equilibrium in a multiproduct-firm pricing game with an arbitrary marginal cost vector has only been established for classes of demand systems that satisfy some variants of the Independence of Irrelevant Alternatives (IIA) property (Spady, 1984; Konovalov and Sándor, 2010; Gallego

[1]For instance, Whinston (2007) notes: "[...] the Farrell and Shapiro analysis is based on the strong assumption that market competition takes a form that is described well by the Cournot model, both before and after the merger. [...] There has been no work that I am aware of extending the Farrell and Shapiro approach to other forms of market interaction. The papers that formally study the effect of horizontal mergers on price and welfare in other competitive settings [...] all assume that there are no efficiencies generated by the merger."

and Wang, 2014; Nocke and Schutz, 2018).[2,3] In short, to develop a model that gives rise to a unique equilibrium requires making compromises on substitution patterns.

In this paper, we develop a model that, despite its limitations, goes some way towards satisfying the above desiderata. The competitive setting underlying our merger analysis is a game of price competition with multiproduct firms and NCES/NMNL demands. This class of demand systems has discrete/continuous choice micro-foundations and, through its nest structure, allows products to be closer substitutes to some products than to others, thereby relaxing the strict IIA property. Indeed, variants of this class are ubiquitous in the empirical industrial organization literature (e.g., Berry, 1994; Berry, Levinsohn, and Pakes, 1995; Goldberg, 1995; Verboven, 1996; Goldberg and Verboven, 2001; Nevo, 2001; Björnerstedt and Verboven, 2016). We allow quality and marginal costs to differ arbitrarily across products.

In much of the analysis, we assume that firms own the property rights over arbitrary collections of nests of products, implying that competition between firms takes place across nests, and not within nests. This restriction can be motivated in several ways. For example, one can think of each firm as owning one or multiple brands, with consumers perceiving products as being closer substitutes within a brand than across brands. Alternatively, one can think of each firm as offering multiple products, with each product being available in different varieties. In Section 6, we relax this restriction by allowing for the coexistence of both "broad" firms that own entire nests of products and "narrow" firms that own only a subset of products within a single nest.

The NCES/NMNL demand specification, in conjunction with our restriction on the ownership structure of nests, gives rise to an aggregative game: Each firm's profit depends on rival firms' prices only through a single-dimensional aggregator. In equilibrium, each firm charges the same markup—the relative markup under NCES demand and the absolute markup under NMNL demand—for each of its products. Moreover, *type aggregation* obtains: All relevant information about a firm's product portfolio (the number of nests, the numbers of products within the various nests, as well as the qualities and marginal costs of the products) can be summarized in a single-dimensional sufficient statistic—the firm's "type."

Building on the aggregative games approach taken in Nocke and Schutz (2018), we show that there exists a unique pricing equilibrium, with intuitive comparative statics. The resulting levels of consumer surplus and aggregate surplus can be expressed as functions of firms' equilibrium market shares. The type aggregation property, the well-behaved comparative

[2]The linear demand system, which does not satisfy the IIA property, is well known to give rise to a unique equilibrium under the assumption of symmetric single-product firms. In recent work, Cumbul and Virág (2018) show that equilibrium uniqueness is no longer guaranteed when firms are allowed to be heterogeneous. They provide (generic) examples of single-product-firm pricing games with a continuum of equilibria that survive standard selection arguments.

[3]In Nocke and Schutz (2018), we develop an aggregative games approach to study multiproduct-firm pricing games based on IIA demand systems. We establish equilibrium existence and uniqueness, perform comparative statics, and extend our results to nested demand systems under the assumption that the ownership partition is a filtration of the nest partition. That article does not contain any results on mergers, although it briefly discusses some of the results derived in the present paper.

statics, and market shares being sufficient statistics for welfare are the reason why we have elected to use the NCES/NMNL demand specification rather than the more general class of discrete/continuous choice demand systems studied by Nocke and Schutz (2018).[4]

At the heart of the review of a horizontal merger by an antitrust authority is the Williamson (1968) trade-off between the merger's market power effect (which is due to the internalization of pricing externalities post merger) and its efficiency effect (which is due to potential merger-specific synergies). In our model, merger-induced synergies can take many forms: Some of the marginal costs of the merged firms' products may go down (while those of others may go up); some of the products' qualities may improve (while others may degrade); and the merged entity may offer new products (while possibly withdrawing others). The type aggregation property allows us to refrain from imposing any restrictions on the nature of the synergies as all relevant information can be summarized in the merged firm's post-merger type.

The Herfindahl index (HHI) is often used to quantify market power. Using the outcome under monopolistic competition as the appropriate competitive benchmark in our differentiated-products setting, we show that the Herfindahl index provides a measure of the distortions caused by oligopolistic behavior. Specifically, using Taylor approximations both around small market shares and around monopolistic competition "conduct," we show that the difference in the outcomes of our welfare measures (consumer surplus and aggregate surplus) under oligopoly and monopolistic competition is proportional to the Herfindahl index.

The Herfindahl index also plays an important role in merger control.[5] Defining the market power effect of a merger as its effect in the absence of synergies, we use Taylor approximations to show that the market power effect on consumer surplus and aggregate surplus is proportional to the naively-computed, merger-induced variation in the Herfindahl index. Our results thus provide some justification for the use of the Herfindahl index in antitrust practice.

We also provide an analysis of the consumer surplus effects of mergers that does not rely on approximations. We show that, for any merger, there exists a unique cutoff such that the merger increases consumer surplus if the post-merger type is above that cutoff, and decreases consumer surplus if it is below. As in the homogeneous-goods Cournot model (Farrell and Shapiro, 1990), for a merger to increase consumer surplus it must involve synergies. Moreover, the required synergies are larger the less competitive is the market pre-merger and the larger are the merging parties. This suggests that mergers inducing a larger increase in the naively-

[4]Those more general demand systems usually do not give rise to the type aggregation property. Moreover, comparative statics are much less well behaved than under NCES/NMNL demand (see Nocke and Schutz, 2018, Section 3.3). For instance, a reduction in the marginal cost of a product does not necessarily increase the equilibrium profit of the firm offering that product, nor does it necessarily raise equilibrium consumer surplus. Finally, firms' market shares in volume or in value are in general not sufficient statistics for consumer surplus and aggregate surplus.

[5]For instance, in the U.S. Horizontal Merger Guidelines, the pre-merger Herfindahl index and the "naively-computed" merger-induced change in the Herfindahl index are proposed as indicators of the "likely competitive effects of a merger."

computed Herfindahl index should indeed receive additional scrutiny.

Further, we embed the static pricing game in a dynamic model in which merger opportunities arise stochastically over time. In every period, firms involved in feasible but not-yet-approved mergers have to decide whether to propose their merger, and the antitrust authority has to decide which (if any) of the proposed mergers to approve. We show that, in this dynamic model, a completely myopic merger approval policy is dynamically optimal. This extends the main insight of Nocke and Whinston (2010), derived in a homogeneous-goods Cournot setting, to the case of differentiated-products price competition with NCES or NMNL demands.

Turning to the aggregate surplus effects of mergers, we show that there also exists a post-merger cutoff type above which a merger increases aggregate surplus, and below which it decreases aggregate surplus.[6] That cutoff type is lower than the one for a consumer surplus standard: For a merger to increase aggregate surplus requires fewer synergies than for it to increase consumer surplus, and may not require any synergies at all.

Building on Farrell and Shapiro (1990)'s analysis of the homogeneous-goods Cournot setting, we also study the external effect of a merger, defined as the sum of the effect on consumer surplus and the non-merging firms' profits. The aggregative properties of our oligopoly model allow us to decompose a merger into a sequence of *infinitesimal* mergers, where, along the sequence, the value of the aggregator changes continuously from its pre-merger to its post-merger equilibrium value. Using this insight, we show that a consumer-surplus-decreasing merger is more likely to have a positive external effect if the non-merging firms command larger pre-merger market shares and if these pre-merger market shares are more concentrated.[7] We also provide a simple and easily-implementable test to check whether a consumer-surplus-decreasing merger has a positive external effect. That test only requires knowledge of the *pre-merger* market shares and of a demand elasticity parameter.

Relaxing the assumption that competition takes place only across nests, we study the coexistence of broad firms (which own entire nests of products) and narrow firms (which own only a subset of the products within a single nest). We show that an aggregative games approach can still be applied in such a setting and that there exists a unique equilibrium, with intuitive comparative statics. This insight allows us to extend most of our results on the static and dynamic consumer surplus effects of mergers to both broad mergers (which involve only broad firms) and narrow mergers (which involve only narrow firms operating in the same nest). One notable exception is that a narrow merger giving rise to a larger naively-computed variation in the Herfindahl index may in fact require *fewer* synergies to benefit consumers.

We also compare the synergy levels required to make a broad merger and an "equivalent" narrow merger desirable for consumers. A broad and a narrow merger are deemed equivalent if both sets of merger partners command the same industry-level market shares before the merger, so that both sets of firms appear to have the same degree of market power. While

[6]An analogous result is unavailable in the homogeneous-goods Cournot model.

[7]The converse holds if the merger under consideration is consumer-surplus-increasing.

one might expect a narrow merger to raise more competitive concerns than an equivalent broad merger as the products of the merger partners are closer substitutes, we show that the narrow merger requires in fact fewer synergies if the nest in which the narrow firms are present has a sufficiently high market share.

Our paper is related to several strands of literature. In a diagrammatic analysis of a merger from perfect competition to monopoly, Williamson (1968) was the first to identify the welfare trade-off between the market power effect of a merger and its efficiency effect. Farrell and Shapiro (1990) provide a thorough analysis of this trade-off in a homogeneous-goods Cournot model. They give a necessary and sufficient condition for a merger to increase consumer surplus, and sufficient conditions for the external effect of a merger to be positive. In a dynamic setting with endogenous merger proposals (and approvals), Nocke and Whinston (2010) study the dynamic optimality of a myopic, consumer-surplus-based merger approval policy in a homogeneous-goods Cournot model. In Sections 4, 5.2, and 6, we extend Farrell and Shapiro (1990) and Nocke and Whinston (2010)'s analyses to the case of differentiated-goods price competition with multiproduct firms.[8,9,10]

The literature on upward pricing pressure and the related concept of compensating marginal cost reductions attempts to operationalize the Williamson (1968) trade-off using information local to the pre-merger equilibrium. Werden (1996) considers a merger between two single-product firms competing in prices and, using pre-merger markups, diversion ratios and prices as primitives, computes the critical level of synergies that makes the merger price-reducing. Goppelsroeder, Schinkel, and Tuinstra (2008) extend this approach to mergers among multiproduct firms under price or quantity competition. Farrell and Shapiro (2010) provide guidance on how to implement upward pricing pressure tests in practice. Using a Taylor approximation around zero upward pricing pressure in a multiproduct-firm setting, Jaffe and Weyl (2013) formalize Farrell and Shapiro (2010)'s intuition that local information on pass-through rates can be combined with upward pricing pressure indices to obtain the likely price effect of a merger. The approximation results we provide in Section 3.3 are of a different nature; those results are obtained around small market shares or around monopolistic competition conduct and relate explicitly the market power effect of a merger to easily-observable concentration ratios. We also derive exact conditions on the consumer

[8]A separate, less-related strand of literature studies the profitability of mergers in the absence of merger-specific synergies (Salant, Switzer, and Reynolds, 1983; Perry and Porter, 1985; Deneckere and Davidson, 1985). In recent work, Johnson and Rhodes (2018) study the profitability and consumer-surplus effects of mergers in a Cournot setting with pure vertical product differentiation, where each firm may provide one or two (exogenously-given) quality levels. Another literature, pioneered by Kamien and Zang (1990), studies the limits of monopolization through mergers in the absence of antitrust policy.

[9]A recent literature focuses on the effects of mergers and merger policy on investment and innovation (e.g., Gowrisankaran, 1999; Mermelstein, Nocke, Satterthwaite, and Whinston, forthcoming; Motta and Tarantino, 2017; Federico, Langus, and Valletti, 2018; Bourreau, Jullien, and Lefouili, 2018).

[10]Anderson, Erkal, and Piccinin (2013) use an aggregative games approach to study an oligopoly model with single-product firms under price or quantity competition. They show that a merger without synergies lowers consumer surplus in the short run. In the long-run free-entry equilibrium, ignoring integer constraints, such a merger does not affect consumer surplus.

surplus and aggregate surplus effects of mergers. Finally, in contrast to the literature on upward pricing pressure, we allow synergies to materialize not only through marginal cost reductions, but also through quality improvements and new products.

The Herfindahl index is a key sufficient statistic in our approximation results in Section 3. In previous work on the homogeneous-goods Cournot model, the Herfindahl index has been shown to provide a measure of an industry's average markup and profitability; see, for instance, Cowling and Waterson (1976), and Belleflamme and Peitz (2010) for a textbook treatment.[11] In recent work, Spiegel (2019) shows that, in that model, the ratio of consumer surplus to producer surplus is proportional to the Herfindahl index if demand is ρ-linear.[12] We are, however, aware of only few results linking the Herfindahl index to industry performance measures in models of differentiated-products industries. In a model with CES preferences and price or quantity competition, Grassi (2017) relates the industry average markup to the Herfindahl index. In Feenstra and Weinstein (2017)'s model with translog preferences, the representative consumer's indirect utility depends on the Herfindahl index both directly, due to translog preferences, and indirectly, due to endogenous markups. To the best of our knowledge, our paper is the first to link explicitly the oligopoly distortions to consumer surplus and aggregate surplus to the Herfindahl index, and to show that the market power effect of a merger is approximately proportional to the naively-computed, merger-induced variation in that index.

The remainder of the paper is organized as follows. In Section 2, we introduce the oligopoly model and solve it using aggregative games techniques. There, we also show that the type aggregation property permits a tractable analysis of mergers in multiproduct-firm oligopoly. Section 3 shows that the Herfindahl index provides an adequate approximation of the oligopoly distortions, and that the merger-induced, naively-computed variation in the Herfindahl index approximates the market power effect of a merger. Our results on the consumer surplus effects of mergers, in both static and dynamic settings, are derived in Section 4. Section 5 presents our results on the aggregate surplus and external effects of mergers. In Section 6, we extend the model by allowing for the coexistence of broad and narrow firms. Section 7 concludes. The proofs are gathered in an Appendix.

2 Mergers in Multiproduct-Firm Oligopoly

In this section, we present the oligopoly model that will serve as a workhorse throughout the paper. We describe the model in Section 2.1. Section 2.2 introduces the important benchmark of monopolistic competition. We solve the oligopoly model using aggregative-games techniques in Section 2.3. Section 2.4 uses the type aggregation property to simplify the treatment of mergers among multiproduct firms.

[11] Dansby and Willig (1979) show that, in the homogeneous-goods Cournot model, the industry performance gradient index, which measures the rate of potential improvement in aggregate surplus from a small variation in the output vector, is proportional to the square root of the Herfindahl index.

[12] Demand is ρ-linear if the inverse demand takes the form $P(Q) = a - bQ^{\delta}$.

2.1 The Oligopoly Model

Consider an industry with a set $\mathcal{N}$ of imperfectly substitutable products. Each product belongs to a nest of products; the set of nests is denoted $\mathcal{L}$, a partition of $\mathcal{N}$. Products within the same nest are viewed by consumers as closer substitutes with each other than products in different nests. Specifically, the representative consumer's quasilinear indirect utility function is given by

$$y + V(p) = y + V_0 \log \left[H^0 + \sum_{l \in \mathcal{L}} \left(\sum_{j \in l} h_j(p_j) \right)^{\beta} \right], \tag{1}$$

where $y > 0$ is the consumer's income, $V_0 > 0$ is a market size parameter, $0 < \beta \leq 1$ is a parameter measuring the substitutability of products across nests relative to that within nests,[13] $H^0 \geq 0$ is a baseline-utility parameter, and

$$h_j(p_j) = \begin{cases} \exp\left(\frac{a_j - p_j}{\lambda}\right) & \text{in the case of NMNL,} \\ a_j p_j^{1-\sigma} & \text{in the case of NCES.} \end{cases}$$

The parameter $a_j > 0$, $j \in \mathcal{N}$, summarizes vertical product characteristics, and will be referred to as the quality of product j; $\sigma > 1$ and $\lambda > 0$ measure the substitutability of products within nests.

Defining the nest- and industry-level aggregators

$$H_l(p_l) = \sum_{j \in l} h_j(p_j), \text{ where } p_l \equiv (p_j)_{j \in l} \ \forall l \in \mathcal{L},$$
$$\text{and } H(p) = H^0 + \sum_{l \in \mathcal{L}} (H_l(p_l))^{\beta}$$

allows us to rewrite the consumer's indirect utility as $V(p) = V_0 \log H(p)$.

Assuming that income y is sufficiently large, and applying Roy's identity, we obtain the demand for product i in nest l:

$$\begin{aligned} D_i(p) &= V_0 \beta \frac{-h_i'(p_i)}{h_i(p_i)} \frac{h_i(p_i)}{H_l(p_l)} \frac{H_l(p_l)^{\beta}}{H(p)} \\ &= V_0 \beta \frac{-h_i'(p_i) H_l(p_l)^{\beta - 1}}{H(p)}. \end{aligned} \tag{2}$$

As shown in Nocke and Schutz (2018), demand system (2) can alternatively be derived from discrete/continuous choice.[14] With such a micro-foundation, $V_0\beta$ is the total number of consumers, H_l^{β}/H is the probability that a given consumer chooses nest l, h_i/H_l is the

[13] If $\beta = 1$, the nest structure is irrelevant.

[14] Anderson, de Palma, and Thisse (1987) were the first to provide a micro-foundation for the non-nested CES demand system.

probability that a consumer picks product i conditional on having chosen nest l, and $-h_i'/h_i$ is the number of units of product i a consumer purchases conditional on having chosen product i.[15] Moreover, $(\log H^0)/\beta$ is the value of the outside option. In the remainder of the paper, we normalize V_0 to 1.

Each product $i \in \mathcal{N}$ has constant marginal cost of production $c_i > 0$. There is a set $\mathcal{F}$ of firms, which we assume to be a partition of $\mathcal{L}$. That is, each firm has property rights over the production of all products within one or more nests. (We relax this assumption in Section 6.) One way to interpret this restriction is that each firm owns one or several brands (nests), with products being closer substitutes within a brand than across brands. Another interpretation is that each firm owns multiple products, with each product being available in different varieties.[16]

The economic environment can thus be summarized by the tuple $(\mathcal{N}, \mathcal{L}, \mathcal{F}, (a_j)_{j\in\mathcal{N}}, (c_j)_{j\in\mathcal{N}})$ along with nest parameter β, and elasticity parameters σ under NCES demand and λ under NMNL demand. The profit of firm $f \in \mathcal{F}$ is given by

$$\Pi^f = \sum_{l\in f}\sum_{i\in l}(p_i - c_i)D_i(p).$$

Firms compete by simultaneously setting the prices of all of their products. We seek the Nash equilibrium of this multiproduct-firm pricing game. Aggregate surplus is the sum of consumer surplus, $\log H$, and industry-level profits, $\sum_{f\in\mathcal{F}}\Pi^f$.

Firms' market shares will play an important role in our analysis. We define the market share of firm f as

$$s^f = \sum_{l\in f}\frac{(H_l)^\beta}{H}.$$

In the discrete/continuous choice micro-foundation mentioned above, s^f corresponds to the probability that any given consumer chooses one of firm f's products. Moreover, s^f is equal to firm f's market share in volume under NMNL demand, and to firm f's market share in value under NCES demand. In both cases, the firms' market shares add up to $1 - H^0/H$, where H^0/H is the market share of the outside option.

In the presence of an outside option ($H^0 > 0$), computing market shares in practice is well known to be non-trivial as the potential market size may be hard to determine. This issue is ubiquitous in the literature on demand estimation in differentiated-products industries (see, e.g., Berry, 1994; Berry, Levinsohn, and Pakes, 1995; Nevo, 2001). Nevo (2000b) provides guidance on how to proceed. If the outside option represents consuming an imported good and importers have no market power, in that they form a perfectly or monopolistically competitive fringe, then computing the market share of the outside option is particularly

[15] Under NMNL demand, $-h_i'/h_i$, the conditional demand for product i, is constant and equal to $1/\lambda$; under NCES demand, it is equal to $(\sigma-1)/p_i$.

[16] In recent work, Hottman, Redding, and Weinstein (2016) structurally estimate a model of price competition with multiproduct firms, where each firm owns one nest of products.

simple: All that is required is knowledge of those importers' sales (in value under NCES demand, and in volume under NMNL demand).

2.2 The Monopolistic Competition Benchmark

Before analyzing the oligopoly model, it is instructive to consider first the monopolistic competition benchmark. Under monopolistic competition, firms do not internalize the impact of their behavior on the industry aggregator H, i.e., they behave as if $\partial H/\partial p_i = 0$ for every $i \in \mathcal{N}$.

Under this behavioral assumption, the first-order condition of profit maximization for product $i \in n \in f$ is given by

$$\frac{H_n^{\beta-1}}{H}\left(-h_i' - (p_i - c_i)h_i'' + (1-\beta)\frac{\partial H_n}{\partial p_i}\frac{\sum_{j\in n}(p_j - c_j)h_j'}{H_n}\right) = 0,$$

which can be rewritten as

$$\frac{p_i - c_i}{p_i}\frac{p_i h_i''}{-h_i'} = 1 + (1-\beta)\frac{\sum_{j\in n}(p_j - c_j)(-h_j')}{H_n}. \tag{3}$$

If $\beta = 1$ (i.e., in the absence of nests), we immediately obtain that firm f sets the Lerner index of product i equal to the reciprocal of the perceived price elasticity of demand. Under CES demand, that elasticity is equal to σ; under MNL demand, it is equal to p_i/λ. If $\beta < 1$, firm f internalizes self-cannibalization effects within its own nests, and it optimally sets a Lerner index that exceeds that in the absence of nests.

Following Nocke and Schutz (2018), we call the left-hand side of equation (3) the ι-markup on product i. As the right-hand side is the same for every $i \in n$, firm f charges the same ι-markup, $\tilde{\mu}_n > 1$, for each product i in nest n. Under NCES demand, this implies that the Lerner index of product i is equal to $\tilde{\mu}_n/\sigma$, whereas under NMNL demand, the absolute markup $p_i - c_i$ is equal to $\tilde{\mu}_n\lambda$.

Using the common ι-markup property within nest n, the sum on the right-hand side of equation (3) can be written as:

$$\sum_{j\in n}(p_j - c_j)(-h_j') = \sum_{j\in n}\frac{p_j - c_j}{p_j}\frac{h_j''}{-h_j'}\frac{(h_j')^2}{h_j''} = \tilde{\mu}_n\sum_{j\in n}\frac{(h_j')^2}{h_j''} = \tilde{\alpha}\tilde{\mu}_n\sum_{j\in n}h_j = \tilde{\alpha}\tilde{\mu}_n H_n, \tag{4}$$

where $\tilde{\alpha} = (\sigma-1)/\sigma < 1$ under NCES demand and $\tilde{\alpha} = 1$ under NMNL demand. Equation (3) therefore boils down to

$$\tilde{\mu}_n = \frac{1}{1-\tilde{\alpha}(1-\beta)} \equiv \mu^{\mathrm{mc}}. \tag{5}$$

As μ^{mc} does not depend on the identity of nest n nor on the identity of firm f, the monopolistically competitive ι-markup μ^{mc} is the same for each product $i \in \mathcal{N}$.

2.3 Equilibrium Analysis

We now turn to the equilibrium analysis of our multiproduct-firm pricing game. This requires adapting the aggregative-games approach taken in Nocke and Schutz (2018, Section 5), where each firm is restricted to own only a single nest.

The first-order condition for product i in nest n owned by firm f is given by

$$\frac{H_n^{\beta-1}}{H}\left(-h_i' - (p_i - c_i)h_i'' + (1-\beta)\frac{\partial H_n}{\partial p_i}\frac{\sum_{j\in n}(p_j - c_j)h_j'}{H_n} + \frac{H_n^{1-\beta}}{H}\frac{\partial H}{\partial p_i}\sum_{l\in f} H_l^{\beta-1}\sum_{j\in l}(p_j - c_j)h_j'\right) = 0.$$

The last term on the left-hand side, which is absent under monopolistic competition, captures the impact of the price change through the aggregator H. Simplifying, we obtain

$$\frac{p_i - c_i}{p_i}\frac{p_i h_i''}{-h_i'} = 1 + (1-\beta)\frac{\sum_{j\in n}(p_j - c_j)(-h_j')}{H_n} + \beta\frac{1}{H}\sum_{l\in f} H_l^{\beta-1}\sum_{j\in l}(p_j - c_j)(-h_j'). \tag{6}$$

Hence, despite the additional term on the right-hand side, firm f continues to charge the same ι-markup on every product i in nest n. That is, there exists $\tilde{\mu}_n > 1$ such that

$$\frac{p_i - c_i}{p_i}\frac{p_i h_i''}{-h_i'} = \tilde{\mu}_n$$

for every $i \in n$.

Using the common ι-markup property within each nest l as well as equation (4), equation (6) can be rewritten as

$$\tilde{\mu}_n\left(1 - \tilde{\alpha}(1-\beta)\right) = 1 + \tilde{\alpha}\beta\frac{1}{H}\sum_{l\in f}\tilde{\mu}^l H_l^{\beta}, \tag{7}$$

which immediately implies that $\tilde{\mu}_n = \tilde{\mu}_{n'} \equiv \tilde{\mu}^f$ for every $n, n' \in f$. Firm f therefore applies the same ι-markup $\tilde{\mu}^f$ to all the products in its portfolio. Using this common ι-markup property, both within and across nests, equation (7) simplifies to

$$\tilde{\mu}^f\left(1 - \tilde{\alpha}(1-\beta)\right) = 1 + \tilde{\alpha}\beta\tilde{\mu}^f\frac{\sum_{l\in f} H_l^{\beta}}{H} = 1 + \tilde{\alpha}\beta\tilde{\mu}^f s^f. \tag{8}$$

Define the elasticity measure $\alpha \equiv \tilde{\alpha}\beta/(1 - \tilde{\alpha}(1-\beta))$, and note that $\alpha < 1$ under NCES demand and $\alpha = 1$ under NMNL demand. Using equation (8), we can decompose firm f's

ι-markup as follows:

$$\tilde{\mu}^f = \underbrace{\frac{1}{1-\tilde{\alpha}(1-\beta)}}_{=\mu^{\text{mc}}} \underbrace{\frac{1}{1-\alpha s^f}}_{\equiv \mu^f}.$$

That is, under oligopoly, firm f's ι-markup $\tilde{\mu}^f$ is the product of the monopolistically competitive ι-markup μ^{mc} and a market power factor, the normalized markup $\mu^f > 1$. As μ^f is increasing in s^f, this decomposition reveals that firms with larger market shares have more market power, and therefore set higher ι-markups.

Equations (4) and (8) yield a simple formula for firm f's equilibrium profit:

$$\Pi^f = \tilde{\alpha}\beta\tilde{\mu}^f s^f = \mu^f - 1. \tag{9}$$

Next, we express firm f's market share as a function of the industry-level aggregator H and firm f's normalized markup μ^f. Under NCES demand,

$$\begin{aligned} s^f &= \frac{1}{H}\sum_{l\in f}\left(\sum_{j\in l} a_j \left(\frac{\sigma}{\sigma-\tilde{\mu}^f}c_j\right)^{1-\sigma}\right)^{\beta}, \\ &= \underbrace{\frac{1}{H}\sum_{l\in f}\left(\sum_{j\in l} a_j c_j^{1-\sigma}\right)^{\beta}}_{\equiv T^f} \left(1-(1-\tilde{\alpha})\tilde{\mu}^f\right)^{\frac{\tilde{\alpha}\beta}{1-\tilde{\alpha}}}, \\ &= \frac{T^f}{H}\left(1-(1-\alpha)\mu^f\right)^{\frac{\alpha}{1-\alpha}}. \end{aligned}$$

Under NMNL demand,

$$s^f = \frac{1}{H}\sum_{l\in f}\left(\sum_{j\in l}\exp\left(\frac{a_j-c_j}{\lambda}-\tilde{\mu}^f\right)\right)^{\beta} = \frac{1}{H}\underbrace{\sum_{l\in f}\left(\sum_{j\in l}\exp\left(\frac{a_j-c_j}{\lambda}\right)\right)^{\beta}}_{\equiv T^f}\exp(-\mu^f).$$

We call T^f firm f's type. As we shall see below, that uni-dimensional sufficient statistic aggregates all the relevant information about firm f's product portfolio—the *type aggregation property*.[17] If firm f were the only firm and priced all of its products at marginal cost, and if there were no outside option, then $\log T^f$ would be equal to consumer surplus.

The above analysis implies that, if H is an equilibrium aggregator level, then firm f's markup and market share μ^f and s^f jointly solve the following system of equations:

$$\mu^f = \frac{1}{1-\alpha s^f}, \tag{10}$$

[17] Nevo and Rossi (2008) were the first to obtain the type aggregation property in the case of non-nested MNL demand. They dubbed $\log T^f$ the adjusted inclusive value of firm f.

$$s^f = \begin{cases} \frac{T^f}{H}\left(1-(1-\alpha)\mu^f\right)^{\frac{\alpha}{1-\alpha}} & \text{under NCES demand,} \\ \frac{T^f}{H}\,\mathrm{e}^{-\mu^f} & \text{under NMNL demand.} \end{cases} \tag{11}$$

It is straightforward to show that this system has a unique solution $(m(T^f/H), S(T^f/H))$. We call $m(T^f/H)$ and $S(T^f/H)$ the firm's *markup fitting-in function* and *market-share fitting-in function*, respectively. Both fitting-in functions are increasing, $m' > 0$ and $S' > 0$, i.e., a firm that has a higher type and operates in a less competitive environment (lower H) sets a higher markup and commands a higher market share; moreover, the range of S is the entire interval $(0, 1)$. Using equation (9), we obtain the *profit fitting-in function* $\pi(T^f/H) = m(T^f/H) - 1$.

The equilibrium aggregator level is pinned down by the equilibrium condition

$$\frac{H^0}{H} + \sum_{f\in\mathcal{F}} S\left(\frac{T^f}{H}\right) = 1, \tag{12}$$

which says that market shares add up to unity. The continuity and monotonicity properties of S along with the fact that S has full range imply that equation (12) has a unique solution, establishing equilibrium existence and uniqueness.

We summarize these insights in the following proposition:

Proposition 1. *The multiproduct-firm pricing game has a unique equilibrium. The equilibrium aggregator level H^* is the unique solution of equation* (12)*. In equilibrium, firm $f \in \mathcal{F}$ sets a markup of $m(T^f/H^*)$, commands a market share of $S(T^f/H^*)$, and earns a profit of $\pi(T^f/H^*)$.*

Proof. The only thing left to prove is that first-order conditions are necessary and sufficient for global optimality. This is done in Appendix I. □

The following proposition, which follows immediately from Nocke and Schutz (2018), provides intuitive comparative statics:

Proposition 2 (Nocke and Schutz, 2018, Proposition 6)**.** *An increase in T^f raises firm f's equilibrium markup $m(T^f/H^*)$, market share $S(T^f/H^*)$, and profit $\pi(T^f/H^*)$, reduces firm $g \neq f$'s equilibrium markup $m(T^g/H^*)$, market share $S(T^g/H^*)$, and profit $\pi(T^g/H^*)$, and raises consumer surplus and aggregate surplus.*

The Monopolistic Competition Limit. In the monopolistic competition outcome studied in Section 2.2, each firm f sets a normalized markup μ^f of one. In the oligopoly model studied here, this outcome arises in the limit as firms' market shares tend to zero, i.e., when firms become atomless. Such a limiting outcome can be obtained by infinitely replicating the population of firms, or by making the value of the outside option, H^0, go to infinity.

Firm Conduct. Some of the approximation results derived in Section 3 will require bridging the gap between monopolistic competition conduct and fully-fledged "Bertrand-Nash" conduct. Specifically, let $\theta \in [0,1]$ be a conduct parameter, and assume that each firm believes that the impact of p_i, $i \in \mathcal{N}$, on the aggregator is $\theta \partial H/\partial p_i$ instead of $\partial H/\partial p_i$, i.e., firms internalize their impact on the aggregator only to a certain extent.[18]

The analysis proceeds along the same lines as above (see Appendix IV.1 for details). There exists a unique equilibrium aggregator level $H^*(\theta)$. It is easy to see that $H^*(\theta)$, $m(\cdot,\theta)$, $S(\cdot,\theta)$, and $\pi(\cdot,\theta)$ all tend to their value under monopolistic competition as θ tends to 0, and to their value under fully-fledged oligopoly as θ tends to 1.

2.4 Modeling Mergers

Consider a merger between the firms $\mathcal{M} \subsetneq \mathcal{F}$, and let $\mathcal{O} \equiv \mathcal{F} \setminus \mathcal{M}$ be the set of non-merging firms—the outsiders. The post-merger economic environment can be summarized by the tuple $(\overline{\mathcal{N}}, \overline{\mathcal{L}}, \overline{\mathcal{F}}, (\overline{a}_j)_{j\in\overline{\mathcal{N}}}, (\overline{c}_j)_{j\in\overline{\mathcal{N}}})$.

We assume that the merger does not directly affect the outsiders. Formally, this means that: For every $f \in \mathcal{O}$ and $l \in f$, the nest l belongs to $\overline{\mathcal{L}}$; for every $i \in l \in f \in \mathcal{O}$, we have $\overline{a}_i = a_i$ and $\overline{c}_i = c_i$. These assumptions imply that the post-merger type of each outsider $f \in \mathcal{O}$ is equal to its pre-merger type, T^f.

The merged firm M is defined as $M = \overline{\mathcal{L}} \setminus \bigcup_{f\in\mathcal{O}} \bigcup_{l\in f}\{l\}$. The post-merger set of firms is therefore $\overline{\mathcal{F}} = \{M\} \cup \mathcal{O}$. We allow for the possibility that the merger affects the merging firms' set of products by adding or dropping products as well as the marginal costs and qualities of their pre-existing products. Formally, this means that we do not impose any condition on the relationship between the merging firms' pre-merger products, $\bigcup_{f\in\mathcal{M}} \bigcup_{l\in f}\{l\}$, and the merged firm's post-merger products, M, implying no restriction on the relationship between the merged firm's type, T^M, and the merger partners' pre-merger types, $(T^f)_{f\in\mathcal{M}}$.

Our aggregative-games tools and the type aggregation property deliver important benefits in terms of tractability, as they allow us to view a merger as an event that simply turns the pre-merger type vector $(T^f)_{f\in\mathcal{F}}$ into $(T^M, (T^f)_{f\in\mathcal{O}})$. A special case of interest arises when the merger does not involve any synergies, so that $M = \bigcup_{f\in\mathcal{M}} \bigcup_{l\in f}\{l\}$, $\overline{a}_j = a_j$ and $\overline{c}_j = c_j$ for all $j \in l \in M$, implying that $T^M = \sum_{f\in\mathcal{M}} T^f$. We say that the merger involves synergies if $T^M > \sum_{f\in\mathcal{M}} T^f$.

3 The Herfindahl Index and Market Power

In antitrust practice, the Herfindahl index (HHI), defined as $\text{HHI}\left((s^f)_{f\in\mathcal{F}}\right) \equiv \sum_{f\in\mathcal{F}} (s^f)^2$, is often used to gauge the extent of market power as well as the potential market power effect of a merger (see, e.g., the 2010 U.S. Horizontal Merger Guidelines). One common

[18]Our treatment of firm conduct is closely related to the classical approach under quantity competition with homogeneous products surveyed by Bresnahan (1989).

presumption is that there is more market power in industries where the Herfindahl index is larger. Another common presumption is that the market power effect of a merger tends to be larger when (i) the pre-merger Herfindahl index is larger and (ii) the merger-induced increase in the Herfindahl index is larger.

In this section, we provide theoretical support for these presumptions. Specifically, we derive approximation results that show that this index is an adequate measure of the oligopoly distortions. Using similar approximation techniques, we also show that the naively-computed change in the Herfindahl index induced by a merger is an appropriate measure of the market power effect of the merger. To prove these results, we first relate measures of industry performance to the equilibrium market share vector. Such an analysis is useful for antitrust practice as information is easier to obtain on market shares than on firms' types.

3.1 Market Shares and Welfare

Let $(s^f)_{f\in\mathcal{F}}$ be the profile of equilibrium market shares. Assume that consumers have access to an outside option ($H^0 > 0$), so that $\sum_{f\in\mathcal{F}} s^f < 1$. Equation (12) implies that the equilibrium aggregator level H^* is equal to $H^0/(1-\sum_{f\in\mathcal{F}} s^f)$. As shown in Anderson and Nocke (2014), this implies that consumer surplus can be written as a function of market shares:[19]

$$\mathrm{CS}\left((s^f)_{f\in\mathcal{F}}\right) = \log H^0 - \log\left(1 - \sum_{f\in\mathcal{F}} s^f\right). \tag{13}$$

Note that consumer surplus depends only on the sum of the firms' market shares. The intuition is that, no matter in which nest, all the products are equally good substitutes for the outside option, as the elasticity of D_i with respect to H^0 is independent of $i \in \mathcal{N}$.

As firm f's equilibrium profit is $\mu^f - 1$ and $\mu^f = 1/(1-\alpha s^f)$, aggregate surplus can also be written as a function of market shares:

$$\mathrm{AS}\left((s^f)_{f\in\mathcal{F}}\right) = \log H^0 - \log\left(1 - \sum_{f\in\mathcal{F}} s^f\right) + \sum_{f\in\mathcal{F}} \frac{\alpha s^f}{1-\alpha s^f}.$$

Note that aggregate surplus is increasing in the vector of market shares. Moreover, by convexity of $s \mapsto s/(1-\alpha s)$, a mean-preserving spread of market shares raises industry profit and therefore aggregate surplus.[20]

3.2 The Herfindahl Index as a Measure of Oligopoly Distortions

We now argue that the Herfindahl index provides an adequate measure of the oligopoly distortions to consumer surplus and aggregate surplus. As a competitive benchmark, we use

[19] See Armstrong and Vickers (2018) on the related concept of consumer surplus as a function of quantities.

[20] This is akin to the homogeneous-goods Cournot model, where consumer surplus depends only on aggregate output, and aggregate profit is proportional to the Herfindahl index, for a fixed aggregate output.

the equilibrium outcome under monopolistic competition rather than perfect competition. If goods were homogeneous, then under both price and quantity competition, the equilibrium outcome would converge to perfect competition as the population of firms is infinitely replicated, so that each firm's limiting size is negligible relative to the size of the market. By contrast, in our differentiated-goods framework, such an infinite replication results in the monopolistic competition outcome, as shown in Section 2.3.

We provide two sets of approximation results: When firms have small market shares, and when industry conduct is close to monopolistic competition.

Approximation Results for Small Firms. For this set of approximations, we assume that consumers have access to an outside option. We proceed as follows. We first fix a vector of market shares $s = (s^f)_{f \in \mathcal{F}}$, and compute the welfare measures $\mathrm{CS}(s)$ and $\mathrm{AS}(s)$. Using s, we then back out the type vector $T(s) = (T^f(s))_{f \in \mathcal{F}}$ that gives rise to this profile of market shares under oligopoly. Next, using $T(s)$, we compute our welfare measures under monopolistic competition as functions of firms' market shares under oligopoly, $\mathrm{CS}^m(s)$ and $\mathrm{AS}^m(s)$. Finally, we apply Taylor's Theorem:

Proposition 3. *In the neighborhood of $s = 0$, the oligopoly distortions are:*[21,22]

$$\begin{aligned} CS(s) - CS^m(s) &= -\alpha HHI(s) + o\left(\|s\|^2\right), \\ \text{and } AS(s) - AS^m(s) &= -\alpha HHI(s) + o\left(\|s\|^2\right). \end{aligned}$$

Proof. See Appendix III.1. □

To see why the distortion to consumer surplus increases with the Herfindahl index, consider a mean-preserving spread of the market share vector s under oligopoly. This raises the Herfindahl index but leaves consumer surplus unchanged, as $\mathrm{CS}(s)$ depends only on the sum of market shares (see equation (13)). The concavity of the market-share fitting-in function $S(\cdot)$, which comes from the fact that a firm with a higher type tends to charge a higher markup, implies that the mean-preserving spread of the market share vector must have been caused by a sum-increasing change in the vector of firm types.[23] As consumer surplus under monopolistic competition depends only on the sum of those types (see equations (viii) and (ix) in Appendix III.1), this change increases $\mathrm{CS}^m(s)$.[24]

Recall that the approximation result provided in Proposition 3 require a positive outside option. Our Herfindahl index can easily be converted into the one commonly used by prac-

[21] $o(\cdot)$ is Landau's little-o notation: $f(x) = o(g(x))$ in the neighborhood of $x = x^0$ if $f(x)/g(x) \underset{x \to x^0}{\longrightarrow} 0$.

[22] While the Herfindahl index is unit-free, consumer surplus is usually measured in dollars. Here, consumer surplus is measured in units of the Hicksian composite commodity, the price of which was normalized to one in equation (1).

[23] The concavity of S is stated and proved in Lemma I in Appendix II.

[24] It may seem surprising that the distortion to consumer surplus is equal to that to aggregate surplus at the second order. In Appendix III.1, we show that those two distortions no longer coincide at the third order.

titioners ($\widehat{\text{HHI}}$, which does not account for an outside option): $\widehat{\text{HHI}} = \text{HHI}/(1-s^0)^2$, where s^0 denotes the market share of the outside option.

Approximation Results around Monopolistic Competition Conduct. We now provide an alternative approximation of the oligopoly distortions, namely one involving only small departures from monopolistic competition conduct, but without restricting the size of firms or imposing that there is a positive outside option.

Fix a conduct parameter $\theta \in [0,1]$ and a type vector $(T^f)_{f\in\mathcal{F}}$. The equilibrium aggregator level is denoted $H^*(\theta)$, and firm f's market share is $s^f(\theta) = S\left(T^f/H^*(\theta), \theta\right)$. Equilibrium consumer surplus and aggregate surplus are therefore given by

$$\text{CS}(\theta) = \log H^*(\theta) \quad \text{and} \quad \text{AS}(\theta) = \log H^*(\theta) + \sum_{f\in\mathcal{F}} \frac{\alpha s^f(\theta)}{1-\alpha\theta s^f(\theta)}.$$

The industry-level Herfindahl index is $\text{HHI}(\theta) = \sum_{f\in\mathcal{F}} s^f(\theta)^2$.

We can now provide a first-order Taylor approximation of the oligopoly distortions in the neighborhood of $\theta = 0$, i.e., close to monopolistic competition conduct:

Proposition 4. *In the neighborhood of $\theta = 0$, the oligopoly distortions are:*

$$CS(\theta) - CS(0) = -\alpha HHI(\theta)\theta + o(\theta),$$
$$\text{and} \;\; AS(\theta) - AS(0) = -\alpha HHI(\theta)\left(1 - \alpha\sum_{f\in\mathcal{F}} s^f(\theta)\right)\theta + o(\theta).$$

Proof. See Appendix IV.2. □

As in the approximation with small market shares, the oligopoly distortion to consumer surplus is proportional to the Herfindahl index. By contrast, the oligopoly distortion to aggregate surplus now contains a new term that depends on α and the aggregate market share. Holding fixed the aggregate market share, the distortion continues to be proportional to the Herfindahl index. Holding fixed the Herfindahl index, the distortion decreases with the aggregate market share. If the aggregate market share is small, the distortion is approximately the same as in Proposition 3.

3.3 The Herfindahl Index as a Measure of the Market Power Effect of a Merger

The market power effect of a merger is the impact that merger would have on consumer surplus or aggregate surplus if it involved no synergies. We now show that the naively-computed change in the Herfindahl index induced by the merger is an adequate measure of its market power effect. As in Section 3.2, we support this claim by providing approximation results around $s = 0$ and $\theta = 0$.

Approximation Results for Small Firms. As in Section 3.2, we assume that consumers have access to an outside option. We proceed as follows. We fix the pre-merger vector of market shares $s = (s^f)_{f\in\mathcal{F}}$, and use this vector to recover the pre-merger type vector $(T^f(s))_{f\in\mathcal{F}}$ and compute the pre-merger market performance measures $\mathrm{CS}(s)$ and $\mathrm{AS}(s)$. Assuming no synergies, the merged firm's type is $T^M(s) = \sum_{f\in\mathcal{M}} T^f(s)$. We then use the post-merger type vector $(T^f(s))_{f\in\overline{\mathcal{F}}}$ to obtain the post-merger vector of market shares $\bar{s}(s) = (\bar{s}(s))_{f\in\overline{\mathcal{F}}}$. The post-merger welfare measures are $\mathrm{CS}(\bar{s}(s))$ and $\mathrm{AS}(\bar{s}(s))$. Hence, the market power effect of the merger is $\mathrm{CS}(\bar{s}(s)) - \mathrm{CS}(s)$ or $\mathrm{AS}(\bar{s}(s)) - \mathrm{AS}(s)$.

The merged-induced, naively-computed variation in the Herfindahl index is:

$$\Delta^M \mathrm{HHI}(s) = \left(\left(\sum_{f\in\mathcal{M}} s^f\right)^2 + \sum_{f\in\mathcal{O}} (s^f)^2\right) - \sum_{f\in\mathcal{F}} (s^f)^2 = \left(\sum_{f\in\mathcal{M}} s^f\right)^2 - \sum_{f\in\mathcal{M}} (s^f)^2.$$

Applying Taylor's theorem, we obtain the following second-order approximation results:

Proposition 5. *In the neighborhood of $s = 0$, the market power effect of the merger is:*

$$CS(\bar{s}(s)) - CS(s) = -\alpha\Delta^M \, HHI(s) + o(\|s\|^2),$$
$$AS(\bar{s}(s)) - AS(s) = -\alpha\Delta^M \, HHI(s) + o(\|s\|^2).$$

Proof. See Appendix III.1. □

Hence, the market power effect of a merger is proportional to the naively-computed variation in the Herfindahl index, where the proportionality coefficient is the elasticity measure α. As was the case in Proposition 3, this holds regardless of whether the market power effect is measured in terms of consumer surplus or aggregate surplus.

Approximation Results around Monopolistic Competition Conduct. Let θ be a conduct parameter and $(T^f)_{f\in\mathcal{F}}$ be the pre-merger type vector. The merged firm's type is $T^M = \sum_{f\in\mathcal{M}} T^f$, assuming no synergies. Let $\overline{\mathrm{CS}}(\theta)$ and $\overline{\mathrm{AS}}(\theta)$ denote post-merger consumer surplus and aggregate surplus, respectively. The naively-computed, merger-induced change in the Herfindahl index is denoted $\Delta^M \mathrm{HHI}(\theta)$. We provide a linear approximation of the market power effect of the merger around monopolistic competition conduct:

Proposition 6. *In the neighborhood of $\theta = 0$, the market power effect of the merger is:*

$$\overline{CS}(\theta) - CS(\theta) = -\alpha\Delta^M \, HHI(\theta)\theta + o(\theta),$$
$$\overline{AS}(\theta) - AS(\theta) = -\alpha\Delta^M \, HHI(\theta) \left(1 - \alpha \sum_{f\in\mathcal{F}} s^f(\theta)\right) \theta + o(\theta).$$

Proof. See Appendix IV.3. □

As in the approximation of the oligopoly distortions, the merger's market power effect on aggregate surplus when approximated around monopolistic competition conduct differs slightly from that when approximated around small market shares. That difference vanishes as market shares become small.

4 Consumer Surplus Effects of Mergers

We now turn to the consumer surplus effects of mergers without approximations. We study a static setting in Section 4.1 and a dynamic one with endogenous mergers in Section 4.2.

4.1 Static Analysis

Consider a merger M between the firms in $\mathcal{M}$. Let H^* (resp., $\overline{H}^*$) denote the equilibrium value of the aggregator before (resp., after) the merger. As consumer surplus is increasing in the value of that aggregator, we say that the merger is *CS-increasing* (resp., *CS-decreasing*) if $\overline{H}^* > H^*$ (resp., $\overline{H}^* < H^*$); it is *CS-neutral* if $\overline{H}^* = H^*$.

Suppose the merger is CS-neutral. This implies that the market share of each outsider $g \in \mathcal{O}$, $S(T^g/H^*)$, and the market share of the outside option, H^0/H^*, is unaffected by the merger. Since the market shares of the firms and the outside option have to add up to one (equation (12)), this means that the post-merger market share of the merged firm is equal to the sum of the pre-merger market shares of the merger partners:

$$S\left(\frac{T^M}{H^*}\right) = \sum_{f \in \mathcal{M}} S\left(\frac{T^f}{H^*}\right),$$

where we have used the fact that $\overline{H}^* = H^*$.

As S is strictly increasing and has full range, it follows that there exists a unique cutoff type $\hat{T}^M$ such that the merger is CS-neutral if and only if $T^M = \hat{T}^M$. By Proposition 2, $\overline{H}^*$ is strictly increasing in T^M, implying that the merger is CS-increasing if $T^M > \hat{T}^M$, and CS-decreasing if the inequality is reversed.

As the market-share fitting-in function S is strictly concave (see Lemma I in Appendix II) and satisfies $S(0) = 0$, that function is sub-additive. This implies that the cutoff type satisfies $\hat{T}^M > \sum_{f \in \mathcal{M}} T^f$. That is, for the merger to be CS-nondecreasing it has to involve synergies.[25]

We summarize these insights in the following proposition:

Proposition 7. *For a merger among the firms in $\mathcal{M}$, there exists a unique $\hat{T}^M > \sum_{f \in \mathcal{M}} T^f$ such that the merger is CS-neutral if the post-merger type satisfies $T^M = \hat{T}^M$, CS-decreasing if $T^M < \hat{T}^M$, and CS-increasing if $T^M > \hat{T}^M$.*

[25]Farrell and Shapiro (1990) obtain the same conclusion in the homogeneous-goods Cournot model. By contrast, Johnson and Rhodes (2018) show that a merger without synergies may benefit consumers in a Cournot setting with pure vertical product differentiation.

We now turn to the comparative statics of the post-merger cutoff-type $\hat{T}^M$. First, we consider the thought experiment of changing the pre-merger aggregator level H^* while holding fixed the characteristics of the merger. Second, we compare two alternative mergers in a given industry, thus holding fixed the pre-merger aggregator level H^*.

The first comparative statics result shows that the synergies required for a merger to be CS-nondecreasing are smaller the more competitive is the market before the merger:

Proposition 8. *For a merger among the firms in $\mathcal{M}$, the post-merger cutoff type $\hat{T}^M$ is strictly decreasing in the pre-merger level of the aggregator, H^*.*

Proof. See Appendix V.1. □

To see the intuition, consider a merger between two symmetric single-product firms, producing products i and j at pre-merger marginal cost c, and charging the pre-merger price p^*. Suppose the merger-induced synergies materialize only through a symmetric marginal cost reduction. As shown by Werden (1996), for the merger to be CS-neutral, the common post-merger marginal cost $\hat{c}$ must be such that

$$\frac{c-\hat{c}}{c} = \frac{d(H^*)}{1-d(H^*)}\frac{(p^*-c)}{c}, \tag{14}$$

where $d(H^*) \equiv -(\partial D_j/\partial p_i)/(\partial D_i/\partial p_i)$ is the diversion ratio between goods i and j.

The left-hand side of equation (14) gives the required percentage change in marginal cost whereas the right-hand side represents the increase in market power due to the post-merger internalization of competitive externalities. An increase in the pre-merger aggregator level H^* does not affect the left-hand side but reduces the right-hand side through two channels: It reduces both the pre-merger equilibrium price p^* and the diversion ratio $d(H^*)$.[26] Proposition 8 generalizes this intuition to mergers between arbitrary sets of firms, involving arbitrary forms of synergies.

We now turn to our second comparative statics result. It shows that the synergies required for a merger to be CS-nondecreasing are larger for mergers involving larger firms, holding fixed the pre-merger aggregator level H^*.

Proposition 9. *Consider a merger between the firms in $\mathcal{M} = \{f, g\}$, resp., $\mathcal{M}' = \{f', g'\}$, where $T^f \geq T^{f'}$ and $T^g > T^{g'}$. Then, the "larger" merger $\mathcal{M}$ requires larger synergies than $\mathcal{M}'$, in the sense of a larger fractional increase in type:*[27]

$$\frac{\hat{T}^M}{T^f+T^g} > \frac{\hat{T}^{M'}}{T^{f'}+T^{g'}}.$$

[26] In our model, the diversion ratio between two symmetric single-product firms can be shown to be equal to $\alpha s^*/(1-\alpha s^*)$, which is increasing in the equilibrium market share s^*, and thus decreasing in H^*.

[27] If the merger partners were the only firms and were pricing all of their products at marginal cost both pre- and post-merger, and if there were no outside option, then the logarithm of this fractional increase would give the merger-induced increase in consumer surplus. The proposition implies that the larger merger also requires a larger absolute increase in type $\hat{T}^M - (T^f + T^g) > \hat{T}^{M'} - (T^{f'} + T^{g'})$.

Proof. See Appendix V.2. □

To see the intuition, suppose each of the two mergers involves symmetric single-product firms, and that merger-induced synergies materialize only through a symmetric reduction in the common marginal cost. The right-hand side of equation (14) is larger for merger $\mathcal{M}$ than $\mathcal{M}'$ as each merger partner in $\mathcal{M}$ has a higher pre-merger market share, implying that both its pre-merger diversion ratio $d(H^*)$ and its markup $(p^* - c)/c$ are larger. Hence, the percentage cost reduction necessary for the merger to be CS-neutral is larger for the larger merger.

Propositions 8 and 9 provide theoretical support for the use of the merger-induced, naively-computed variation in the Herfindahl index to screen mergers. For merger $\mathcal{M} = \{f, g\}$, the naively-computed increase in the Herfindahl index is $\Delta^M \text{HHI} = 2s^f s^g$, and is thus larger for mergers involving larger firms.

Proposition 8 shows that, holding fixed the types of the merger partners, a decrease in the pre-merger equilibrium aggregator level H^*, resulting in a higher Δ^M HHI, raises the required level of synergies for the merger to be CS-increasing. Proposition 9 shows that, holding fixed the pre-merger equilibrium aggregator level, a merger involving firms with higher types, and thus resulting in a higher Δ^M HHI, also raises that required level of synergies.

4.2 Dynamic Analysis

In industries in which merger opportunities are not isolated events, a static analysis of the consumer surplus effect of a given proposed merger may be inappropriate: The approval decision on a merger may affect both the consumer surplus effects of future mergers, and therefore the set of mergers that will be approved in the future, as well as the profitability of future mergers, and therefore the set of mergers that will be proposed in the future.

In the following, we show that a completely myopic merger approval policy, whereby, in every period, the antitrust authority approves only those mergers that raise consumer surplus given current market conditions, is dynamically optimal. This extends the main insight of Nocke and Whinston (2010), derived in the context of a homogeneous-goods Cournot model, to the case of differentiated-goods price competition with NMNL or NCES demands.

Framework. Following Nocke and Whinston (2010), we assume that there is a collection of potential mergers, $M_1, \ldots, M_K$, corresponding to sets of merger partners $\mathcal{M}_1, \ldots, \mathcal{M}_K$, and that all of these mergers are disjoint, i.e., $\mathcal{M}_k \cap \mathcal{M}_l = \emptyset$ for $k \neq l$. Disjointness means that each firm has a distinct set of natural merger partners that have the potential to create sizable synergies by merging.

There are $\tau < \infty$ periods in which mergers may become feasible, and be proposed to the antitrust authority for approval. Any merger M_k may become feasible at the beginning of period $1 \leq t \leq \tau$ with probability $p_t^{M_k}$, where $\sum_t p_t^{M_k} \leq 1$. Once merger M_k has become feasible, the merger partners learn the realization of their post-merger type T^{M_k}, drawn from a continuous probability distribution $G_t^{M_k}$.

If merger M_k has become feasible in period t, or became feasible earlier but has not yet been approved, the merger partners decide whether to propose it to the antitrust authority. We assume that the merger is proposed if and only if it is in the merger partners' joint interest to do so. When doing so, they observe the type not only of their own merger but also that of any other feasible but not yet approved merger (as well as the type of every firm).

If a feasible merger is proposed, the antitrust authority observes its efficiency (i.e., the post-merger type); the authority also observes the types of all firms. Market structure (as summarized by the vector of firm types) changes according to the authority's approval decisions. Importantly, while a blocked merger cannot be consummated, it can be proposed again in the future.

At the end of period t, firms compete in prices under complete information, as described in Section 2.1. Payoffs in each period therefore depend only on the market structure at the end of that period. Firms as well as the authority discount future payoffs with factor $\delta \leq 1$.

Results. The main result of this subsection is that the myopically CS-maximizing merger policy is dynamically optimal in that it maximizes the discounted sum of consumer surplus. The myopically CS-maximizing merger policy is the merger approval rule that, in each period t, maximizes consumer surplus in that period, given current market structure and the set of proposed mergers.[28]

Our result on the dynamic optimality of a CS-maximizing merger policy comes in two parts. First, we ignore the incentive constraints for proposing mergers and show that the myopically CS-maximizing merger policy maximizes discounted consumer surplus *if all feasible but not yet approved mergers are proposed in each period*. Second, we show that there exists a subgame-perfect equilibrium in which all feasible but not yet approved mergers are indeed proposed in each period. Moreover, any subgame-perfect equilibrium induces the same optimal sequence of period-by-period consumer surpluses.

To show the first part, we begin by establishing a sign-preserving complementarity in the consumer surplus effects of mergers. Consider two disjoint mergers M_k and M_l, and suppose first that each is CS-nondecreasing given current market structure, i.e., $T^{M_k} \geq \hat{T}^{M_k}$ and $T^{M_l} \geq \hat{T}^{M_l}$. If merger M_k is implemented first, then H^* weakly increases as the merger is CS-nondecreasing. By Proposition 8, this implies that $\hat{T}^{M_l}$ weakly decreases so that the condition for merger M_l to be CS-nondecreasing, $T^{M_l} \geq \hat{T}^{M_l}$, continues to hold. By the same argument, if both mergers are CS-decreasing given current market structure, then

[28] There may be more than one set of merger approvals that maximizes consumer surplus in a given period but, if so, these sets differ only by mergers that are CS-neutral given the other mergers in those sets. Proposition 2 and the fact that post-merger types are drawn from continuous distributions imply that any given merger is CS-neutral with probability zero. We thus assume that the myopically CS-maximizing set of merger approvals is unique.

implementing merger M_k increases the cutoff type for the other merger M_l, implying that M_l remains CS-decreasing. This insight is summarized in the following proposition:

Proposition 10. *If merger M_l is CS-nondecreasing in isolation, it remains CS-nondecreasing if another merger M_k, $k \neq l$, that is CS-nondecreasing in isolation takes place. If merger M_l is CS-decreasing in isolation, it remains CS-decreasing if another merger M_k, $k \neq l$, that is CS-decreasing in isolation takes place.*

Proposition 8 implies that a CS-increasing merger M_k can induce an otherwise CS-decreasing merger M_l to become CS-nondecreasing. In this case, we have:

Proposition 11. *Suppose that merger M_k is CS-nondecreasing in isolation whereas merger M_l is CS-decreasing in isolation but CS-nondecreasing once merger M_k has taken place. Then, merger M_k is CS-increasing conditional on merger M_l taking place.*

Proof. This follows immediately from the assumption that consumer surplus is higher after both mergers are implemented, but lower after only M_l is implemented. □

Propositions 10 and 11 imply that if the antitrust authority approves only mergers that are CS-nondecreasing at the time of approval, then it will not have ex post regret about previously approved mergers (as these remain CS-nondecreasing) nor about previously rejected mergers (as these remain feasible and therefore can be implemented once they become CS-nondecreasing). This intuitively explains the following result:

Corollary 1. *Suppose that all feasible but not yet approved mergers are proposed in each period. Then, the myopically CS-maximizing merger policy maximizes discounted consumer surplus, no matter what the realization of feasible mergers is.*

Proof. See Appendix VI.1. □

We now turn to the second part by showing that there exists a subgame-perfect equilibrium in which, in each period, every feasible but not yet approved merger is proposed.

The first step in showing this is that a CS-nondecreasing merger is privately profitable in the sense that it raises the joint profit of the merger partners, holding fixed the market structure in the rest of the industry. We first argue that a merger that does not involve synergies is profitable, as is usually the case in models of price competition with differentiated products (see, e.g., Deneckere and Davidson, 1985). Intuitively, such a merger lowers the equilibrium aggregator level, and therefore reduces the outsiders' contribution to the aggregator. It follows that the merging parties face less competition, and therefore make strictly higher profits after the merger. By Proposition 7, a CS-nondecreasing merger must involve synergies. Hence, by Proposition 2, a merger involving synergies must be more profitable than one that does not. This explains the following result:

Proposition 12. *A CS-nondecreasing merger M_k is privately profitable, holding fixed the market structure among outsiders.*

Proof. See Appendix VI.2. □

The second step consists in showing that a CS-nondecreasing merger is still privately profitable even if it induces (directly or indirectly) other mergers to become CS-nondecreasing, resulting in their approval:

Proposition 13. *Suppose that merger M_k is CS-nondecreasing given current market structure whereas merger M_l is CS-decreasing but becomes CS-nondecreasing once M_k has been implemented. Then, the joint profit of the firms in M_k is strictly higher if both mergers take place than if none does.*

Proof. Think of implementing merger M_l at step one. As that merger is CS-decreasing by assumption, the equilibrium level of the aggregator, H^*, must decrease, which strictly raises the profit of each firm in $\mathcal{M}_k$. Next, implement merger M_k at step two: As that merger remains, by Proposition 11, CS-nondecreasing after M_l has taken place, it is profitable by Proposition 12. Thus, the joint profit of the firms in $\mathcal{M}_k$ strictly increases at each step. □

Propositions 12 and 13 imply that if the antitrust authority adopts a myopically CS-maximizing merger policy, then—in the last period, τ—there exists an equilibrium in which all feasible but not yet approved mergers are proposed. Consider now period $\tau - 1$. As the set of mergers that the antitrust authority would want to approve can only increase over time, the set of approved mergers in period τ is independent of firms' proposal decisions in period $\tau - 1$. By the same argument as for the last period, there therefore exists an equilibrium in which all feasible but not yet approved mergers are proposed in period $\tau - 1$. Folding backward, the same holds for each of the previous periods.

The following proposition states the main result of this subsection:

Proposition 14. *Suppose that the antitrust authority adopts the myopically CS-maximizing merger policy. Then, all feasible mergers being proposed in each period after any history is a subgame-perfect equilibrium. The resulting outcome maximizes discounted consumer surplus, no matter what the realized sequence of feasible mergers. Moreover, every subgame-perfect equilibrium results in the same optimal level of consumer surplus in each period.*

Proof. See Appendix VI.3. □

5 Aggregate Surplus and External Effects of Mergers

Although most antitrust authorities have adopted a consumer surplus standard, or something close to it, it is also important to understand the impact of mergers on aggregate surplus, which we undertake next.

5.1 Aggregate Surplus Effects

Consider a merger M among the firms in $\mathcal{M}$, and let T^M be the merged firm's type. If $T^M = \hat{T}^M$, where $\hat{T}^M$ is the cutoff type defined in Proposition 7, then the merger is CS-neutral. Moreover, as the merger does not affect the equilibrium value of the aggregator, it has no impact on the outsiders' equilibrium profits. Since the merger is profitable by Proposition 12, it is therefore aggregate-surplus-increasing (AS-increasing). Conversely, it is straightforward to show that the merger is AS-decreasing if T^M is small.[29] The continuity of aggregate surplus in types implies the existence of a cutoff type $\widetilde{T}^M$ that makes the merger AS-neutral. By monotonicity of aggregate surplus (Proposition 2), that cutoff type is unique, and the merger is AS-increasing if $T^M > \widetilde{T}^M$, and AS-decreasing if $T^M < \widetilde{T}^M$.

We summarize these insights in the following proposition:

Proposition 15. *For a merger among the firms in $\mathcal{M}$, there exists a unique $\widetilde{T}^M < \hat{T}^M$ such that the merger is AS-neutral if the post-merger type satisfies $T^M = \widetilde{T}^M$, AS-decreasing if $T^M < \widetilde{T}^M$, and AS-increasing if $T^M > \widetilde{T}^M$.*

Note that there is no counterpart to Proposition 15 in Farrell and Shapiro (1990)'s classical analysis. The reason is that, in the homogeneous-goods Cournot model, equilibrium aggregate surplus is not a monotonic function of firms' marginal costs (Lahiri and Ono, 1988; Zhao, 2001). By contrast, we are able to leverage the monotonicity of aggregate surplus in firms' types to obtain Proposition 15.

That $\widetilde{T}^M < \hat{T}^M$ follows immediately from the fact that a CS-neutral merger is AS-increasing. Whether an AS-neutral merger must involve synergies (i.e., $\tilde{T}^M > \sum_{f \in \mathcal{I}} T^f$) is unclear. On the one hand, a merger without synergies lowers the equilibrium aggregator. On the other, it reallocates market shares toward the outsiders, which can raise aggregate surplus if those firms are initially producing too little relative to the merger partners.

An example where a merger without synergies is AS-increasing can easily be constructed in the case of NMNL demand without an outside option ($H^0 = 0$). Let there be three firms, 1, 2, and 3, with pre-merger types $T^1 = 1$ and $T^2 = T^3 = 1/2$. In the aggregate-surplus-maximizing pre-merger allocation, which can be obtained by setting all markups equal to zero, firm 1 commands a market share of 1/2, whereas firms 2 and 3 each receive a market share of 1/4. The equilibrium allocation is efficient if and only if it replicates that allocation, which arises if and only if all firms charge the same markup. As firm 1's type is higher than its rivals', that firm sets an equilibrium markup that strictly exceeds that of its rivals, resulting in an inefficient equilibrium allocation. Consider now a merger M between firms 2 and 3, and, assuming no synergies, let $T^M = 1$. As firm 1 and the merged firm have the same type, they charge the same equilibrium markups, implying that the post-merger equilibrium

[29] As T^M tends to zero, post-merger aggregate surplus converges to equilibrium aggregate surplus when firm M does not exist. This limiting value is also equal to equilibrium aggregate surplus pre-merger if the firms in $\mathcal{M}$ do not exist. As aggregate surplus is strictly increasing in types, that value is strictly lower than actual pre-merger aggregate surplus.

allocation is efficient. The merger is therefore AS-increasing.[30]

5.2 External Effects

We now extend Farrell and Shapiro (1990)'s analysis of the external effects of a merger, defined as the sum of its impact on consumer surplus and outsiders' profits. To the extent that a merger is proposed by the merger partners only if it is in their joint interest to do so, a positive external effect is a sufficient ("safe harbor") condition for the merger to raise aggregate surplus. The idea behind focusing on the external effect is that the profitability of a merger depends on the magnitude of internal cost savings, and that these are hard to assess for an antitrust authority. As we shall see below, the sign of the external effects of a merger can be related to pre-merger market shares.

Consider a merger M among the firms in $\mathcal{M}$, and let $\mathcal{O}$ be the set of outsiders. Let H^* and $\overline{H}^*$ denote the pre- and post-merger equilibrium values of the aggregator, respectively. The external effect of the merger is defined as

$$\mathcal{E}^M = \log \overline{H}^* - \log H^* + \sum_{f \in \mathcal{O}} \left(m\left(\frac{T^f}{\overline{H}^*}\right) - m\left(\frac{T^f}{H^*}\right)\right) = -\int_{H^*}^{\overline{H}^*} \frac{\eta(H)}{H} dH,$$

where

$$\eta(H) \equiv -1 + \sum_{f \in \mathcal{O}} \frac{T^f}{H} m'\left(\frac{T^f}{H}\right).$$

Hence, as in Farrell and Shapiro (1990), the merger can be thought of as a sequence of infinitesimal mergers dH, where, along the sequence, the value of the aggregator changes progressively from H^* to $\overline{H}^*$. The sign of the external effect of an infinitesimal CS-decreasing (resp. CS-increasing) merger is thus given by $\eta(H)$ (resp. $-\eta(H)$).

In the following, we focus on CS-decreasing mergers to fix ideas. Such a merger necessarily has a positive impact on outsiders' profits. We now derive conditions under which this positive effect on outsiders outweighs the negative effect on consumers.

An infinitesimal CS-decreasing merger $dH < 0$ reduces consumer surplus by $d \log H = dH/H$, which corresponds to the first term in the definition of η. It also raises the profit of every outsider $f \in \mathcal{O}$ by dH/H times $(T^f/H)m'(T^f/H)$. In Appendix VII.1, we show that $\eta(H)$ can be rewritten as

$$\eta(H) = -1 + \sum_{f \in \mathcal{O}} \frac{\alpha s^f (1 - s^f)}{(1 - \alpha s^f)(1 - s^f + \alpha (s^f)^2)}, \tag{15}$$

where $s^f = S(T^f/H)$. The results stated in this section are derived by exploiting the properties of the right-hand side of equation (15).

[30] By the same token, with NMNL demand, no outside option, and three firms 1, 2, and 3 such that $T^1 = T^2 = T^3$, a merger between firms 2 and 3 is AS-decreasing if it does not give rise to synergies.

We first show that a CS-decreasing merger has a negative external effect when products are poor substitutes:

Proposition 16. *Let* $\bar{\alpha} = \frac{3}{2}\left(\sqrt{57} - 7\right) \simeq 0.82$. *If* $\alpha \leq \bar{\alpha}$, *then any CS-decreasing merger has a negative external effect. If instead* $\alpha > \bar{\alpha}$, *then there exist CS-decreasing mergers that have a positive external effect, and CS-decreasing mergers that have a negative external effect.*

Proof. See Appendix VII.2. □

In the non-nested CES case, the condition $\alpha \leq \bar{\alpha}$ translates into $\sigma \leq \bar{\sigma} \simeq 5.7$. More generally, in the case of NCES demand, it translates into a low value of β and/or σ. The intuition for the result is the following. After a CS-decreasing merger, the aggregate market share of the insiders falls, meaning that consumers substitute away from the insiders' products into the outsiders' products. If the insiders' and outsiders' products are poor substitutes, which is the case if σ is small and/or β is small, then such substitution gives rise to a large fall in consumer surplus, which the increase in the outsiders' profits cannot offset, implying a negative external effect.

In the following, we assume that $\alpha > \bar{\alpha}$, and derive conditions under which a CS-decreasing merger is more likely to have a positive external effect. Note that the positive impact on outsiders' profits can be decomposed into two effects. First, holding fixed outsiders' markups, the infinitesimal merger increases the profit of each outsider f by $\Pi^f \times |dH/H|$.[31] Hence, the "direct" effect on outsiders' joint profit is proportional to their joint profit. Second, outsiders respond by increasing their markups. As the outsiders' aggregate profit is increasing and convex in outsiders' market shares (see Section 3.1), the first, direct effect is larger when those market shares are higher and/or more concentrated. We would therefore expect the external effect of a merger to be more likely to be positive in such cases. The following propositions show that this intuition is indeed correct under the appropriate formalization of the notions of high and concentrated market shares, respectively.

We formalize both notions by defining partial order relations over the set of pre-merger industry structures among outsiders. A pre-merger industry structure among outsiders is a vector $(s^f)_{f\in\mathcal{O}}$ of arbitrary length, where $\mathcal{O}$ is a finite set, $s^f \in (0,1)$ for every $f \in \mathcal{O}$, and $\sum_{f\in\mathcal{O}} s^f < 1$. Let $s = (s^f)_{f\in\mathcal{O}}$ and $s' = (s'^f)_{f\in\mathcal{O}'}$ be two pre-merger industry structures.

Outsiders have *higher market shares* under s than under s', i.e., $s \geq_1 s'$, if there exists an injection $\iota : \mathcal{O}' \longrightarrow \mathcal{O}$ such that $s^{\iota(f)} \geq s'^f$ for every $f \in \mathcal{O}'$.

To every outsider industry structure s, we associate a discrete probability measure $P_s(\cdot)$:

$$P_s(x) = \frac{1}{|\mathcal{O}|}\left|\left\{f \in \mathcal{O} : \quad s^f = x\right\}\right|, \quad \forall x \in \mathbb{R}.$$

[31]This holds, as

$$\Pi^f = \alpha\mu^f s^f = \begin{cases} \alpha\frac{T^f}{H}\mu^f\left(1-(1-\alpha)\mu^f\right)^{\frac{\alpha}{1-\alpha}} & \text{under NCES,} \\ \frac{T^f}{H}\mu^f\,\mathrm{e}^{-\mu^f} & \text{under NMNL.} \end{cases}$$

Outsiders' market shares are *more concentrated* under outsider industry structure s than under s', i.e., $s \geq_2 s'$, if s and s' have the same length and the same mean, and $P_{s'}$ second-order stochastically dominates P_s. Note that s and s' having the same length and the same mean implies that the aggregate market share of the outsiders is the same under s and s'.

Using these partial order relations, we obtain the following proposition:

Proposition 17. *Let $\alpha > \bar{\alpha}$ and consider two infinitesimal CS-decreasing mergers, M and M', with pre-merger outsider industry structures $s = (s^f)_{f \in \mathcal{O}}$ and $s' = (s'^f)_{f \in \mathcal{O}'}$. Suppose one (or both) of the following holds:*

(i) $s \geq_1 s'$ and $s^f \leq s^* \simeq 0.68$ for every $f \in \mathcal{O}$.

(ii) $s \geq_2 s'$, $s^f \leq \hat{s} \simeq 0.29$ for every $f \in \mathcal{O}$, and $s'^f \leq \hat{s}$ for every $f \in \mathcal{O}'$.

If merger M' has a positive external effect, then so does merger M.

Proof. See Appendix VII.3. □

Thus, as long as outsiders' market shares are not too high, the external effect of an infinitesimal CS-decreasing merger is more likely to be positive when the outsiders have higher and/or more concentrated market shares, in line with the intuition outlined above. The reason why this intuition may fail if some of the outsiders are too large (i.e., if $s^f > s^*$ or $s^f > \hat{s}$ for some firm f) is the result of the second, indirect effect, namely, the fact that outsiders respond to the reduction in H by increasing their markups. Holding H fixed, the merger-induced increase in an outsider's markup decreases its profit. This holds since oligopolistic markups are always above those of monopolistically competitive firms (that perceive H as fixed), so any further increase must reduce profit for a fixed H. This second effect becomes quantitatively important when outsiders become too large.

Condition (ii) in Proposition 17 suggests that relying on the level of the pre-merger Herfindahl index to evaluate a merger can be misguided. To see this, consider two industries and suppose that the vector of insiders' market shares is the same in both industries. Suppose also that outsiders' market shares are more concentrated in the first industry than in the second. Then, the first industry's Herfindahl index is higher than the second's, despite the first merger being more likely to have a positive external effect than the second one.

We close this section by discussing the external effect of a non-infinitesimal CS-decreasing merger. From Proposition 16, such a merger always has a negative external effect if $\alpha \leq \bar{\alpha}$. Suppose now that $\alpha > \bar{\alpha}$. By continuity, the comparative statics in Proposition 17 continue to obtain as long as the mergers under consideration do not have too much of an impact on the equilibrium aggregator level.

Moreover, regardless of the magnitude of the merger-induced decrease in H, a sufficient condition for the merger to have a positive external effect is that $\eta(H^*) > 0$ (i.e., at the pre-merger aggregator level, an infinitesimal CS-decreasing merger has a positive external effect). The reason is the following. The external effect of the merger is the integral of the external effects of the infinitesimal mergers along the path from H^* to $\overline{H}^* < H^*$. As the merger is CS-decreasing by assumption, outsiders' market shares increase along that sequence. Hence, if $\eta(H^*) > 0$, then, by Proposition 17–(i), $\eta(H)$ remains positive along the sequence (provided no outsider reaches a market share larger than s^*), and so the external effect of the merger is positive. Note that checking whether $\eta(H^*) > 0$ involves using only the outsiders' *pre*-merger market shares (see equation (15)).

6 Merger Analysis with Broad and Narrow Firms

In our analysis so far, we have assumed that each firm owns all of the products in one or more nests, implying that competition takes place only across nests but not within nests. To relax this restriction, we partition the set of firms $\mathcal{F}$ into two subsets: The set of "broad" firms $\mathcal{F}^b$ in which each firm $f \in \mathcal{F}^b$ owns all the products in one or several nests (as assumed so far); and the set of "narrow" firms $\mathcal{F}^n$ in which each firm $f \in \mathcal{F}^n$ owns only a strict subset of products in a single nest. This partitioning induces a partitioning of the set of nests $\mathcal{L}$ into two subsets, $\mathcal{L}^b$ and $\mathcal{L}^n$, where $\mathcal{L}^b$ is a filtration of $\mathcal{F}^b$ and $\mathcal{L}^n$ is a coarsening of $\mathcal{F}^n$.

In this section, we provide an informal overview of a merger analysis with broad and narrow firms, referring the reader to Appendix VIII for a formal treatment. We show that an aggregative games approach can still be used to study oligopoly games with broad and narrow firms. Building on this, we extend the analysis of the static and dynamic consumer surplus effects of mergers to this more general setting. Finally, we show that, to be CS-nondecreasing, a merger between narrow firms in the same nest may require fewer or more synergies than an "equivalent" merger between broad firms.

Oligopoly with broad and narrow firms. The fitting-in functions of broad firm f are unaffected by the presence of narrow firms and are therefore as characterized in Section 2. Consider now narrow firm f in nest l. Its industry-level market share s^f can be decomposed into its nest-level market share $\widetilde{s}^f$ and the market share of its nest s_l: $s^f \equiv \widetilde{s}^f s_l$, where

$$\widetilde{s}^f = \frac{\sum_{j \in f} h_j(p_j)}{H_l} \quad \text{and} \quad s_l = \frac{H_l^\beta}{H}.$$

From the first-order condition, we find that narrow firm f charges the same ι-markup

$$\widetilde{\mu}^f = \frac{1}{1 - \tilde{\alpha}\widetilde{s}^f(1 - \beta + \beta s_l)}. \tag{16}$$

for all its products. Intuitively, the firm sets a high markup if it has a high market share in its nest or if its nest commands a high market share at the industry level. Using the definition of market shares and the common ι-markup property, we obtain

$$\widetilde{s}^f = \frac{\left(T^f\right)^{\frac{1}{\beta}}}{H_l} \times \begin{cases} \left(1-(1-\tilde{\alpha})\widetilde{\mu}^f\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} & \text{under NCES demand,} \\ \mathrm{e}^{-\widetilde{\mu}^f} & \text{under NMNL demand,} \end{cases} \tag{17}$$

where the firm's type T^f is given by $T^f = \left(\sum_{j\in f} h_j(c_j)\right)^{\beta}$. Note that $\log T^f$ corresponds again to the consumer surplus firm f would deliver if it priced all of its products at marginal cost and no other products were offered.

There exists a unique pair of markup and nest-level market share,

$$\widetilde{m}\left(\frac{\left(T^f\right)^{\frac{1}{\beta}}}{H_l}, s_l\right) \quad \text{and} \quad \widetilde{S}\left(\frac{\left(T^f\right)^{\frac{1}{\beta}}}{H_l}, s_l\right),$$

that solves equations (16)–(17). Given industry-level aggregator H, the nest-level aggregator $H_l(H)$ is pinned down by nest-level market shares having to add up to one:[32]

$$\sum_{f\in l} \widetilde{S}\left(\frac{\left(T^f\right)^{\frac{1}{\beta}}}{H_l}, \frac{H_l^{\beta}}{H}\right) = 1.$$

The equilibrium industry-level aggregator H^* is the unique solution to industry-level market shares adding up to unity:

$$\frac{H^0}{H} + \sum_{f\in\mathcal{F}^b} S\left(\frac{T^f}{H}\right) + \sum_{l\in\mathcal{L}^n} \frac{H_l(H)^{\beta}}{H} = 1.$$

We show in Appendix VIII that there exists a unique equilibrium with intuitive comparative statics, extending the results in Nocke and Schutz (2018, Section 5).

Consumer surplus effects of mergers. We now revisit the results of Section 4 when broad and narrow firms coexist, making again use of the type aggregation property, which applies not only to broad but also to narrow firms. First, note that all those results extend immediately when confining attention to "broad mergers," i.e., to mergers between broad firms. In the following, we consider the consumer surplus effects of a merger between narrow firms that operate in the same nest, assuming throughout that all of the products of the merged firm are still within that same nest. We refer to those mergers as "narrow mergers." The reason for the restriction is that after the merger, it must still be possible to partition the set of firms into broad and narrow firms for our aggregative games approach to apply.

Consider a narrow merger M between the firms in $\mathcal{M}$ operating in the same nest l. The

[32] Anderson, Erkal, and Piccinin (2016) were the first to use the concept of nest-level aggregator, which they refer to as "sub-aggregator."

merger is CS-neutral if the post-merger nest-level market share of the merged firm is equal to the combined pre-merger market shares of the merger partners:

$$\sum_{f\in\mathcal{M}} \widetilde{S}\left(\frac{(T^f)^{\frac{1}{\beta}}}{H_l^*}, \frac{(H_l^*)^{\beta}}{H^*}\right) = \widetilde{S}\left(\frac{(T^M)^{\frac{1}{\beta}}}{H_l^*}, \frac{(H_l^*)^{\beta}}{H^*}\right),$$

where H_l^* and H^* are the pre-merger nest-level and industry-level aggregators and T^M is the post-merger type. This equation uniquely pins down a cutoff type $\widehat{T}^M(H_l^*, H^*)$ below which a merger is CS-decreasing and above which it is CS-increasing. As $\widetilde{S}$ is strictly sub-additive in its first argument, a CS-nondecreasing narrow merger must involve synergies:

$$\widehat{T}^M(H_l^*, H^*) > \left(\sum_{f\in\mathcal{M}} (T^f)^{\frac{1}{\beta}}\right)^{\beta}.$$

Note that the right-hand side of the inequality does indeed give the post-merger type in the absence of synergies: The logarithm of that term corresponds to the consumer surplus delivered by the firms in $\mathcal{M}$ under marginal cost pricing when no other products are offered.

Turning our attention to the comparative statics of the cutoff type, we find that $\widehat{T}^M(H_l^*, H^*)$ is decreasing in both of its arguments, implying that a merger requires fewer synergies to be CS-nondecreasing if the merging firms face more competition within their nest (higher H_l^*) or from other nests (higher H^*)—the counterpart of Proposition 8. That earlier proposition implied that, holding fixed the types of the merger partners, changes in market structure give rise to a positive correlation between the naively-computed, merged-induced change in the Herfindahl index, and the synergy level required for the merger to be CS-nondecreasing. This is no longer true for a narrow merger: Holding fixed pre-merger types and H^*, an increase in H_l^* may result in larger pre-merger industry-level market shares for the merger partners, despite decreasing the required synergies.[33] (On the other hand, a change in H^*, holding fixed pre-merger types and H_l^*, results in a positive correlation between pre-merger market shares and the required synergy level.)[34]

Interactions between mergers and dynamic merger policy. The sign-preserving complementarity in the consumer-surplus effect of disjoint broad mergers (Proposition 10) carries over to any admissible pair of mergers M_1 and M_2, where each M_i can be a narrow or a broad merger. In case both M_1 and M_2 are narrow mergers, we do not impose any restriction on whether they are in the same nest or not. The result follows as $\widehat{T}^{M_i}(H_l^*, H^*)$, the cutoff type for a narrow merger M_i in nest l, is decreasing in both of its arguments and any CS-increasing merger raises the industry aggregator and all nest-level aggregators. Moreover,

[33] In the NMNL case, this arises when β and s_l are high enough and the merger partners have sufficiently high nest-level market shares.

[34] We also establish the counterpart of Proposition 9. That is, we show that holding fixed H_l^* and H^*, a narrow merger in nest l requires larger synergies to be CS-nondecreasing if pre-merger types are larger.

it is still the case that a CS-increasing merger remains CS-increasing even if it induces an otherwise CS-decreasing merger to become CS-increasing—the counterpart of Proposition 11.

A CS-neutral merger, whether narrow or broad, is privately profitable as it must involve synergies and affects none of the aggregators. Since comparative statics are well-behaved, this implies that a CS-nondecreasing merger is profitable as well. Moreover, firms involved in a CS-nondecreasing merger are better off even if their merger induces otherwise CS-decreasing mergers to become CS-nondecreasing, resulting in their approval.

In the dynamic framework sketched in Section 4.2, the results outlined above imply the dynamic optimality of a myopic CS-based merger policy—the counterpart of Proposition 14.

Broad vs. narrow mergers. We close this section by comparing the synergy levels required for a broad merger M_b and an "equivalent" narrow merger M_n to be CS-nondecreasing. Intuitively, one would expect the required synergy level to be higher for the narrow merger as the merger partners compete more fiercely within the same nest than across nests. As we show below, this intuition is incomplete: To be CS-nondecreasing, a narrow merger may in fact require fewer synergies than an equivalent broad merger.

Formally, a broad merger is equivalent to a narrow one if the pre-merger vector of industry-level market shares of the merger partners is $(s^f)_{f\in\mathcal{M}}$ for both mergers, so that both sets of merger partners provide the same contribution to the industry-level aggregator before the merger. Pre-merger types for the broad and narrow mergers can be recovered (up to a multiplicative constant) by solving

$$s^f = S\left(\frac{T_b^f}{H^*}\right) \quad \text{and} \quad \frac{s^f}{s_l} = \widetilde{S}\left(\frac{(T_n^f)^{\frac{1}{\beta}}}{H_l^*}, s_l\right).$$

The cutoff types $\widehat{T}_b^M$ and $\widehat{T}_n^M$, which make the broad and the narrow merger CS-neutral, can be backed out by solving similar equations, recalling that a merger is CS-neutral if and only if the industry-level market share of the merged firm is equal to the combined pre-merger market shares of the merger partners. Our goal is to compare

$$E_b = \frac{\widehat{T}_b^M}{\sum_{f\in\mathcal{M}} T_b^f} \quad \text{and} \quad E_n = \frac{\widehat{T}_n^M}{\left(\sum_{f\in\mathcal{M}}(T_n^f)^{\frac{1}{\beta}}\right)^{\beta}},$$

the required synergy levels for the broad and the narrow merger.

For simplicity, we confine attention to mergers between symmetric firms.[35] We find:

Proposition 18. *Consider two equivalent broad and narrow mergers between N symmetric firms. Let $\overline{s}$ be the combined pre-merger industry-level market shares of the merger partners and s_l the pre-merger market share of the narrow merger's nest. There exists a threshold*

[35] In the case of NMNL demand, Proposition 18 extends to mergers involving asymmetric firms provided the nest-level market share of each narrow merger partner does not exceed 3/4. (See Appendix VIII.5.1.)

$\widehat{s}_l \in (\overline{s}, 1)$ *such that the broad merger requires fewer synergies than the narrow one,* $E_b < E_n$*, if* $s_l < \widehat{s}_l$*, whereas the opposite is true if* $s_l > \widehat{s}_l$*.*

Proof. See Appendix VIII.5. □

Intuitively, two opposing effects are at work. On the one hand, narrow firms face more intense competition, and therefore charge lower markups, compared to equivalent broad firms. A narrow merger eliminating that competition therefore requires stronger synergies than an equivalent broad merger. On the other hand, if there are non-merging rivals within the same nest, the merged narrow firm still faces more intense competition than an equivalent merged broad firm, implying that the narrow merger requires fewer synergies. The magnitude of the latter effect is increasing in the market share of the narrow merger's nest, holding fixed the industry-level market shares of the merger partners.

7 Conclusion

We provide a merger analysis in a multiproduct-firm oligopoly model with NCES or NMNL demands. That model goes some way towards satisfying a number of desiderata: The underlying demand system has discrete/continuous choice micro-foundations and allows for substitution patterns that go beyond those implied by the IIA property (notwithstanding the restrictions on the relationship between the ownership and nest structures). The model allows for arbitrary product heterogeneity in terms of marginal costs and qualities, and allows firms to differ in their product portfolios. The demand system gives rise to an aggregative pricing game; the equilibrium is unique and has intuitive comparative statics. Moreover, the type aggregation property permits rich forms of merger-specific synergies through marginal cost reductions, quality improvements, or new products. Finally, consumer surplus and aggregate surplus can be expressed as functions of firm-level equilibrium market shares.

We derive three sets of results. First, we relate the Herfindahl index to market performance measures using approximation techniques. The Herfindahl index provides a measure of the oligopoly distortions, relative to the monopolistic competition benchmark. Moreover, the naively-computed, merger-induced variation in the Herfindahl index approximates the merger's market power effect on consumer surplus and aggregate surplus.

Second, we study the consumer surplus effects of mergers in both static and dynamic settings. For a merger to be CS-increasing requires that the merger generates efficiencies. These efficiencies need to be larger when the industry is less competitive before the merger, or when the merger partners are larger—thus providing additional justification for the use of the naively-computed change in the Herfindahl index.[36] In a dynamic context, in which merger opportunities arise stochastically over time and merger proposals are endogenous, a completely myopic consumer-surplus-based merger approval policy is dynamically optimal. Most of these results on consumer surplus hold not only in our baseline model, where firms

[36] Recall, however, from Section 6 that these results must be qualified for narrow mergers.

own entire nests, but also in more general settings in which broad and narrow firms coexist. Strikingly, we show that a merger between firms operating in the same nest (and thus producing close substitutes to each other) may in fact require fewer synergies to be beneficial for consumers than an "equivalent" merger between firms owning different sets of nests.

Third, we study the aggregate surplus and external effects of mergers. For a merger to be AS-increasing requires fewer efficiencies than for it to be CS-increasing and may, in fact, not require any efficiencies at all. The external effect of a CS-decreasing merger is always negative when products are poor substitutes. When instead products are good substitutes, the external effect is positive if the outsiders' pre-merger market shares are sufficiently large or sufficiently concentrated.

The arguably biggest limitation of our analysis relates to the assumption that each firm either owns entire nests of products or only a subset of products within a single nest. If we were to relax this assumption, the type aggregation property would no longer hold, the game would no longer be aggregative, and no known results of equilibrium existence and uniqueness would apply.

Another limitation implied by the NCES/NMNL demand specification is that a given firm sets the same absolute markup (resp., Lerner index) for all its products under NMNL (resp., NCES) demand. This could be relaxed by using the more general class of demand systems studied in Nocke and Schutz (2018). However, under this more general class, the type aggregation property would, in general, no longer hold. Moreover, comparative statics would be much less well behaved, and market shares would no longer be sufficient statistics for welfare. (See footnote 4.)

We believe that our analysis has implications well beyond industrial organization and antitrust. The model of monopolistic competition with CES preferences is a major building block in the macroeconomics and international trade literatures. Yet, many industries are highly concentrated, with firms wielding market power. Such market power within an industry introduces several forms of misallocation, as it shifts output towards the outside good (representing other industries), the within-industry outside option, and smaller, less efficient firms that charge lower markups.[37] We show that the welfare loss associated with those misallocations is well approximated by the industry-level Herfindahl index—a measure that is often readily available in industry-level data.

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[37] There is a large recent literature that attempts to measure empirically the extent of misallocation arising from market imperfections. See Restuccia and Rogerson (2017) for a survey.

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