Multidimensional Auctions of Contracts: An Empirical Analysis

Yunmi Kong[†] Isabelle Perrigne[‡] Quang Vuong[§]

October 21, 2021

Abstract

In this paper, we conduct a structural analysis of multi-attribute auctions of contracts with a general allocation rule when private information is multidimensional. Upon modeling bidders' contract value that accounts for their endogenous ex post actions, we nonparametrically identify bidders' private information from their bids and estimate their joint distribution. Analyzing cash-royalty auctions of Louisiana oil leases, we find government revenue worse and development rates no better than in a cash auction with a fixed royalty in view of adverse selection and moral hazard. Our findings revise conventional wisdom on the optimality of multi-attribute auctions.

Keywords: Auctions of Contracts, Adverse Selection, Moral Hazard, Multidimensional Private Information, Option Value, Oil Leases, Scoring Auctions

We thank the Editor and four referees for thoughtful and constructive comments. We are grateful for comments from the seminar/conference participants at University of Washington-St Louis, Texas A&M University, Lancaster Conference on Auctions, International Association of Applied Econometrics, Stanford Institute for Theoretical Economics, Banff Empirical Microeconomics workshop, Department of Justice, University of Toronto, Columbia University, Southern Economic Association Meeting, New York University and Colby College. We thank the Louisiana Department of Natural Resources for providing the data.

[†]Rice University, [‡]Rice University, [§]New York University.

Emails: yunmi.kong.01@gmail.com, iperrigne@gmail.com, qvuong@nyu.edu

1 Introduction

Contracts involving an upfront payment and a sharing rule based on the agent's output are commonly observed. Examples are the relationships between an author and a publisher, a licensee and a patent holder, a sharecropper and a landlord, a contracting firm and a government agency to name a few. The magnitude of economic activity governed by these contracts is large.[1] As a public finance issue, questions of how best to allocate and design these contracts are important because reducing spending and increasing revenue through better mechanisms means less distortionary taxation. In particular, auctions are often the mechanism employed by the principal to choose among competing agents and set contract terms. A common feature of these auctions is that agents bid on several attributes and bidders have multidimensional private information. Our paper contributes to the literature by proposing a structural analysis of auctions of contracts with multidimensional private information and a general allocation rule based on multivariate bids. Our empirical application studies cash-royalty auctions of oil leases where the contract takes the form of a real option.

Auctions of contracts were first studied by Laffont and Tirole (1987) and McAfee and McMillan (1987). This literature relies on one-dimensional private information and designs a menu of upfront cash and contingent payments that are negatively related.[2] In contrast, when private information is multidimensional, there is a paucity of theoretical results due to technical challenges arising from multidimensional screening as surveyed by Rochet and Stole (2003), who point out the 'uncomfortably restrictive' nature of one-dimensional types in empirical applications. Indeed, multidimensionality of bids is prevalent in practical contracting. The U.S. Government Accountability Office reports that the vast majority of large procurement contracts is selected based on more than one attribute. For instance, in design-build auctions used by departments of transportation in U.S. states and other countries, the allocation process considers design quality in addition to price. Similarly, both quality and price matter in private sector contracting and online freelancing auctions. Multidimensional private information provides a rich framework to account for the variability of observed multivariate bids.

In this paper, we propose a structural analysis of auctions of contracts with mul-

[1]The federal government alone procured \$586 billion worth of contracts in 2019. Among these, the Department of Defense awarded 18% of its procurement dollars to incentive contracts.

[2]See Hansen (1985) and DeMarzo, Kremer and Skrzypacz (2005) who show that royalties reduce bidders' information rents, Board (2007) and Cong (2020) who study optimal design when the contract takes the form of a real option, and Skrzypacz (2013) who surveys the literature.

tidimensional private information and a general allocation rule based on multivariate bids. Specifically, bidders propose both an upfront or 'cash' payment and a sharing rule or royalty in their bids. We make minimal assumptions about the allocation rule and allow for non-deterministic allocation, as may arise when no rule is announced. In view of the complexity of multidimensional screening in the presence of multidimensional private information, we adopt a best response approach. Our model allows for (i) adverse selection through the principal's payoff which depends not only on the bid components but also on the bidder's private information and (ii) moral hazard or incentive effects induced by the royalty paid as a share of production revenue. In our empirical application, the value of the contract takes the form of a real option because the winner is not obligated to produce. We then interpret the effect of revenue sharing on the agent's incentive to exercise the option as moral hazard.[3] We model bidders with bidimensional private information or types and a contract value that depends on each bidder's private information and royalty bid. The first type-component represents the bidder's productivity or expected production volume while the second type-component represents his economic cost of production.

We show that bidding a higher royalty rate is less costly for 'weak' types, i.e., agents with low productivity and/or high cost, than for 'strong' types. Intuitively, a given royalty percentage is less costly given lower expected production because royalty is a share of revenue. Also, it is less costly in expectation given a higher cost because a higher cost decreases the probability of production. As the principal does not observe bidders' private information, agents strategically choose to submit a cash-royalty bid that reduces their payments to the principal without compromising their winning probability. Upon characterizing the bidder's optimal cash-royalty bid as a function of his bidimensional type, we show that, under a known contract value function, the joint distribution of types is identified from bids upon exploiting the bidder's first-order conditions. We then develop an estimation method for the joint distribution of bidders' types allowing for affiliation of private information within and across bidders. Our methodology extends to a large class of multi-attribute auctions with multidimensional information including scoring auctions as well as other contract allocation procedures with general allocation rules that may be nonlinear, nondeterministic and/or unannounced.

We use a novel dataset of Louisiana auctions of oil leases in which bidders bid on both a cash payment and a royalty rate. Institutional and empirical evidence supports

[3]The traditional definition of moral hazard involves an action that is unobservable to the principal. Here we use the term more broadly to refer to incentive effects on agents' ex post actions.

a nondeterministic allocation rule as well as the multidimensional nature of bidders' private information. As firms are not obligated to drill, we model the contract value as an option value, borrowing insights from the option pricing literature. The option value takes into account that the winning bidder might not exercise his option, i.e., develop the tract. Our estimates of bidders' private information indicate correlations within and between bidders. We also find that for a large fraction of bidders, the cost of production is higher than expected revenue, explaining the low development rate observed on U.S. onshore tracts. Our estimated results on the predicted rate of development and expected production volumes are close to those observed in reality though we do not use any post-auction observations in estimation.

Given the limited theoretical guidance on how multiple versus single attribute bidding compare under multidimensional private information, our paper provides empirical responses to several questions through a rich set of counterfactuals. First, we compare cash-royalty auctions with fixed-royalty auctions, in which the principal fixes the royalty rate so that bidders bid on the cash component only. Cash-royalty auctions allow royalty flexibility by letting competitive forces determine the royalty. However, they are more susceptible to adverse selection because they give agents the freedom to select favorable contract terms which reduce their payments. Our empirical results show that cash-royalty auctions yield lower government revenue than fixed-royalty auctions while failing to improve the development probability or social surplus. Indeed, the potential benefits of royalty flexibility fail to dominate the adverse selection effects of cash-royalty bidding.[4] Second, in light of Che (1993) and Asker and Cantillon (2008), we simulate quasi-linear scoring auctions. Here again, reducing bidder-driven royalty variance through the score's curvature improves government revenue, and fixed-royalty auctions perform better than the scoring auctions, in contrast to existing results from the scoring auction literature. This is so because (i) the bidder's incentives to develop the lease are affected by his royalty bid, and (ii) the principal's payoff depends not only on the winner's bid but also on his private information. Beyond auction design, we also assess policy instruments such as increasing the lease duration and exploiting fluctuations of oil prices. Both have a positive impact on government revenue, though

[4] As a concrete example, consider a bidder who bids a 25% royalty and $820 of cash per acre in the Louisiana auction. From our counterfactual, this bidder would bid $940 per acre in a fixed-royalty auction with 23% fixed royalty, which is the empirical average royalty. Endogenizing the development probability, the government's ex ante expected royalty receipts are only $60 per acre higher in the cash-royalty auction while the cash bid is $120 less per acre, resulting in a net loss for the government.

increasing the lease duration decreases the development probability.

Related Literature

Theoretical guidance on auctions with multidimensional private information is sparse; important exceptions are Asker and Cantillon (2008, 2010) who study procurements where the principal's payoff is determined by the price and quality of a product. They show that quasi-linear scoring auctions strictly dominate price-only auctions with fixed quality standards. By characterizing an optimal mechanism in the 2-by-2 discrete type space, they also illustrate the difficulty of deriving optimal auctions given multidimensional types, finding that the optimal mechanism cannot be implemented by a simple auction format and differs significantly from its one-dimensional counterpart. This difficulty is linked to the challenges of multidimensional screening as surveyed by Rochet and Stole (2003), in which there is no longer an exogenous ordering of the type space and no a priori knowledge of which incentive compatibility constraints will be binding.

Multi-attribute auctions, in which the winner is chosen based on more than one attribute, encompass a large set of auction mechanisms including the scoring auctions well known in the empirical literature. In the case of price-quality scoring auctions, a first set of studies takes the project quality as exogenous. Examples include Nakabayashi (2013), Yoganarasimhan (2016), Andreyanov (2018), Krasnokutskaya, Song and Tang (2020), and Laffont, Perrigne, Simioni and Vuong (2020). A second set of studies endogenizes the submitted qualities. Quasi-linear scoring rules are the most frequently studied there due to convenient theoretical properties. Examples include Lewis and Bajari (2011) where quality is replaced by project completion time, and Allen, Clark, Hickman and Richert (2019) on auctions of insolvent banks, where the weights in the scoring rule are unknown by bidders. See also Takahashi (2018) on price-per-quality procurement auctions of road construction projects, Sant'Anna (2018) on Brazilian oil leases and Hanazono, Hirose, Nakabayashi and Tsuruoka (2018) for an econometric method. We contribute to the multi-attribute auction literature by allowing for agents' endogenous ex post actions and the adverse selection and moral hazard that result, in contrast to work that treats ex post actions as fixed or exogenous. In addition, we provide a general methodology that extends to a broad set of allocation rules.

In using real options to study oil leases, our paper relates to Bhattacharya, Ordin and Roberts (2021) and Herrnstadt, Kellogg and Lewis (2020). The former paper investigates single-attribute auctions such as bidding on cash or royalty only under one-dimensional private information. Using New Mexico fixed-royalty auctions, the authors exploit lease development timing to estimate drilling costs. The latter paper abstracts from bidding and calibrates a model of firms to study the impact of royalty rates and lease duration on their drilling and production. More broadly, our paper relates to empirical studies of real options that are widely used to model decisions under uncertainty in various fields such as management, research and development, and resource economics. See e.g. Pakes (1986) on the analysis of patents.

Our paper contributes to the study of contract allocation policy by providing an empirical response on how to allocate contracts given multidimensional private information. We find that simpler can be better; single-attribute bidding can perform better than multi-attribute bidding in the presence of adverse selection. This contrasts with conventional wisdom from price-quality auctions, providing new insights on incentive contracts and the merits of multi-attribute auctions. Methodologically, our paper develops a flexible method to analyze endogenous, multivariate bids under multidimensional private information and a general allocation rule. We model contract values that allow for agents' endogenous ex post actions and account for the moral hazard and adverse selection that result.

The paper is organized as follows. Section 2 introduces the cash-royalty auctions of Louisiana oil leases and models the lease contract as a real option. Section 3 presents the auction model, establishes identification of the primitives and develops a semiparametric estimation method. Section 4 discusses empirical results, whereas Section 5 assesses the gain/loss of the Louisiana cash-royalty auctions relative to fixed-royalty auctions and scoring auctions as well as the efficacy of some policy instruments in counterfactual studies. Section 6 discusses robustness and an extension of our methodology to a general multi-attribute auction setting. Section 7 concludes.

2 Oil Lease Auctions and Option Value

This section introduces our empirical context: Auctions of contracts by the State of Louisiana for the allocation of oil leases. We present the data and some empirical evidence on the state's allocation rule. Given that the contract is an option to develop the leased tract for oil production, we model its value as an option value.

2.1 Institutional Background and Empirical Evidence

Auction Data

The Louisiana Department of Natural Resources (DNR) sells oil leases on lands owned by the State of Louisiana and its agencies. As is common in the United States, a lease grants the lessee the right, but not the obligation, to develop the tract for oil production. The lessee has a period of 3 years to develop the tract. If no development is performed, the lessee loses the lease. A significant proportion of leased tracts are not drilled. This is also the case in Hendricks and Porter (1988), Hendricks and Porter (1996), Haile, Hendricks and Porter (2010), Aradillas-Lopez, Haile, Hendricks and Porter (2018), and Bhattacharya, Ordin and Roberts (2021). We consider auctions of onshore leases between 1974 and 2003 which have at least forty acres and two or more bidders.[5] In their bid, bidders must specify both a positive cash payment and a royalty rate. We compute the cash component as the immediate payment plus the discounted present value of annual rental fees. Meanwhile, the royalty bid is a percentage. Contingent on oil production, the firm must pay this percentage of production volume times the price of oil. The State levies the royalty on revenue and not on profit as the latter would require a close monitoring of costs. This multi-attribute auction format differs from the standard one used in oil lease auctions studied by Hendricks and Porter (1988) and more recently by Kong (2020, 2021) and Bhattacharya, Ordin and Roberts (2021) in which the government fixes a common royalty rate and firms bid on the cash amount only. We refer to this latter format as a fixed-royalty auction.

The dataset contains 568 auctions. Figure 1 displays a scatterplot of the observed cash-royalty bids, while Table 1 provides summary statistics on the cash payment per acre, the royalty rate, number of bidders and acreage. All dollar amounts are expressed in 2009 dollars. The level of competition is low with 2 bidders in 80% of the sample. The median and mean cash payment are \$712 and \$1,015 per acre, respectively, suggesting skewness. The median and mean royalty are both around 23%. As a comparison, the Federal Bureau of Land Management uses a royalty rate of 12.5%, and Hendricks, Porter and Boudreau (1987) report a royalty of 16.67% for Outer Continental Shelf leases during 1954-1969 which was subsequently raised to 18.75% during 2008-2017, while the prevailing rate on private land is 25%.[6] Royalty rates display variability and are concentrated between 15% and 35%. Figure 1 suggests a

[5]We do not include data after 2003 because the boom in hydraulic fracturing caused a fundamental shift in the US oil industry. We also exclude auctions in which any bidder bid on only a portion of the tract offered for auction.

[6]www.americanprogress.org/issues/green/reports/2015/06/19/115580/federal-oil-and-gas-royalty-and-revenue-reform/

positive association between the logarithm of the cash payment per acre and the royalty rate with a correlation coefficient equal to 0.38.

State's Choice of Winner

In the absence of an announced allocation rule, we explore the data for patterns in the state's choice of winner among submitted bids.[7] Considering two-bidder auctions, we define a dominant bid as one which has both higher cash payment and royalty than its competing bid. We find that there is a dominant bid 64% of the time and that the State selects this bid as the winner 99% of the time. This is a consistent pattern that bidders can infer. When there is no dominant bid, the state selects the bid with higher cash payment 68% of the time and the bid with higher royalty 32% of the time. This suggests that the State's choice is not lexicographic. Figure 2 visualizes all the pairwise choices implied by the data. For instance, if bid A is chosen over bids B and C, we learn information about two pairwise choices: A over B and A over C. Figure 2 plots all the pairwise choices with a circle for the winner and a triangle for the loser. The x and y coordinates represent the royalty and cash components of the bid in relation to the competing bid in the pair. Points in the right (resp. upper) quadrants have higher royalty (resp. cash amounts) than the competing bid. Thus, points in the upper-right quadrant have both higher royalty and cash payment. The transition from triangles in the lower-left quadrant to circles in the upper-right quadrant visualizes the increased probability of winning as the bid moves in that direction. This figure indicates that the probability of winning is increasing in both cash and royalty. It also exhibits positive correlation between cash and royalty within auctions.[8]

This positive correlation contrasts with the main prediction in Laffont and Tirole (1987) and McAfee and McMillan (1987) who study optimal auctions of contracts. Their models lead to a decreasing mapping between cash payment and royalty. Intuitively, the principal selects the bidder with the lowest royalty and highest cash payment because the 'good' firm with high productivity or cost efficiency should be the residual claimant and benefit from informational rent. In contrast, a 'bad' firm should see its rent reduced to the minimum. This firm bids a low upfront payment and a large royalty. These models are based on a single dimension of private information. More generally, one-dimensional private information would result in bid pairs that ap-

[7]Our inquiry to the Louisiana DNR did not yield more details on the allocation procedure beyond a response that the "geological and engineering staff looks at each bid and make the determination".

[8]In the Online Appendix, we also check for bidders' asymmetry and stability of the allocation patterns.

proximate a curve in cash-royalty space. In our data, however, the within-auction bid distribution is widely scattered across two dimensions rather than exhibiting a curve, suggesting multi-dimensional rather than one-dimensional bidders' types.

2.2 Contract as an Option Value

Since the lessee has no obligation to develop the tract, we model the oil lease as an option value. Let $a \in [0, 1)$ be the royalty component of the bid, θ_1 the firm's expected production volume, and θ_2 the firm's economic cost of production. From a bidder's perspective, the contract is an option to obtain $(1-a)\theta_1$ expected units at a cost of θ_2, where $1 - a$ is the portion kept by the firm. The bidder's contract value internalizes the fact that the probability of exercise is less than one and depends on the future price of oil which is uncertain at the time of bidding. Letting p be the oil price, the firm exercises the option when $(1-a)p\theta_1 > \theta_2$, i.e., the profit is positive. In the option literature, one says that a stock option is exercised only if the stock price exceeds the strike price, which is the role played by θ_2. Modeling the bidder's value as an option is not new in the empirical literature on oil lease auctions. Bhattacharya, Ordin and Roberts (2021) and Herrnstadt, Kellogg and Lewis (2020) adopt option values in their analysis of fixed-royalty auctions and non-auctioned leases, respectively.

Borrowing insights from the option pricing literature, we model oil prices as following a geometric Brownian motion with known volatility that is constant for the duration of the option. As in Black and Scholes (1973), Merton (1973) and Black (1976), the geometric Brownian motion yields a closed-form expression of value for European options, which are exercised only at expiration. American options, which can be exercised at any time until expiration, do not have a closed form solution. We adopt European options for the following reasons. First, Hull (2017) explains that some properties of an American option are frequently deduced from those of its European counterpart. Second, we conduct a robustness analysis with American options and find that the difference in empirical results is small. See Section 6.1. Third, Bhattacharya, Ordin and Roberts (2021) and Herrnstadt, Kellogg and Lewis (2020) document substantial bunching of drilling times in the final months before lease expiration.

Let t be the duration in years until the lease expires, p the price of oil at the time of the auction, σ the price volatility, r the one-year interest rate and $\Phi(\cdot)$ the standard normal distribution. The oil price p follows a geometric Brownian motion with volatility σ and zero drift after adjusting for inflation. The variables t, p, σ and r are exogenous.

Upon production, the firm receives the price at the time of production. A bidder's value for the lease at the time of auction is

$$V(a;\theta_1,\theta_2) = e^{-rt}[\underbrace{(1-a)p\theta_1}_{\text{firm's share}}\Phi(x) - \theta_2\underbrace{\Phi(x-\sigma\sqrt{t})}_{\text{Pr(exercise)}}], \tag{1}$$

where

$$x \equiv \frac{\log((1-a)p\theta_1/\theta_2) + \sigma^2 t/2}{\sigma\sqrt{t}}. \tag{2}$$

The derivation of (1)-(2) is as follows. The firm exercises the option if $(1-a)p_t\theta_1 - \theta_2 > 0$ where p_t is the price at the time t of expiration. Thus, the present value of the option is $e^{-rt}\mathrm{E}[\max\{(1-a)p_t\theta_1 - \theta_2, 0\}]$. The geometric Brownian motion implies that p_t is lognormally distributed with $\mathrm{E}(p_t) = p$ and the standard deviation of $\log p_t$ is $\sigma\sqrt{t}$. From Hull (2017), $\mathrm{E}[\max\{(1-a)p_t\theta_1 - \theta_2, 0\}] = \mathrm{E}[(1-a)p_t\theta_1]\Phi(x) - \theta_2\Phi(x-\sigma\sqrt{t})$, where $x = [\log[\mathrm{E}((1-a)p_t\theta_1)/\theta_2] + \sigma^2 t/2]/(\sigma\sqrt{t})$.[9]

Intuitively, $\Phi(x-\sigma\sqrt{t})$ represents the ex ante probability of option exercise, i.e., the probability that the oil price will be high enough to make the firm's profit positive. If option exercise occurs exogenously with probability $\Phi(x-\sigma\sqrt{t})$, the value of the option would be instead $e^{-rt}[(1-a)p\theta_1 - \theta_2]\Phi(x-\sigma\sqrt{t})$, i.e., the discounted expected profit from exercise times the exercise probability. However, option exercise is in fact endogenous and occurs when the oil price is higher than some threshold. Thus, the expected price conditional on exercise is actually higher than the unconditional price. Therefore, $(1-a)p\theta_1$ is multiplied by $\Phi(x)$ to account for this.

In view of our empirical analysis, it is useful to state how the exogenous variables, the firm's types, and the bidder's royalty bid affect the option value and the exercise probability. Table 2 summarizes these effects. See also Hull (2017). Everything else constant, a higher productivity θ_1 has a positive effect on both the contract value and the exercise probability, while a higher cost θ_2 has a negative effect on both, in line with intuition. Also as expected, a higher royalty rate a has a negative effect on both option value and exercise probability while a higher price p has a positive effect on both. Meanwhile, the price volatility σ and the duration t have ambiguous effects on

[9]As explained above, the explicit expression of the option value in (1)-(2) depends on the log normality implied by geometric Brownian motion. Noting that $\mathrm{E}[\max\{(1-a)p_t\theta_1 - \theta_2, 0\}] = -\theta_2 + (1-a)\theta_1\mathrm{E}[\max\{p_t, \theta_2/[(1-a)\theta_1]\}]$, the latter expectation is $\int \max\{p_t, \theta_2/[(1-a)\theta_1]\}dH_t(p_t)$, where $H_t(\cdot)$ is the distribution of p_t. This distribution could be estimated using oil price data thereby providing more flexibility to the option value at the cost of not having a closed form expression.

the exercise probablity. We assess the effect of lease duration in a counterfactual study in Section 5.3. The negative effect of the royalty on the probability of option exercise highlights moral hazard, which we define as the effect of revenue sharing on the agent's incentive to exercise the option. The lack of obligation to develop the lease opens up the possibility of this type of moral hazard.[10] This reduced probability of option exercise is partially responsible for the negative effect the royalty rate has on contract value. Since royalty rates are determined endogenously through the allocation mechanism, moral hazard emphasizes that the allocation mechanism has consequences for agents' ex post incentives. Agents in turn take these effects on incentives and contract value into account as they choose their cash-royalty bids, as we discuss in Section 3.

3 Model, Identification and Estimation

We develop a model of auctions of contracts in which bidders bid an upfront payment and a royalty rate on future revenue. For general applicability, we consider a generic contract value $V(\cdot;\cdot,\cdot)$, the contract value in (1) being an application. We then establish identification of the model and develop a flexible estimation procedure.

3.1 A Model of Auctions of Contracts

Notations and Assumptions

A principal, who can be a buyer or a seller, organizes a sealed-bid auction or procurement auction in which each bidder submits a pair (a_i, b_i), where $a_i \in [0,1)$ is a share or royalty rate on future expected revenue and b_i is a cash payment. There are n bidders participating in the auction. The agents or bidders have bidimensional private information $(\theta_1, \theta_2) \in \Theta_1 \times \Theta_2 \equiv [\underline{\theta}_1, \overline{\theta}_1] \times [\underline{\theta}_2, \overline{\theta}_2]$. The term θ_1 represents expected production which incorporates productivity/efficiency as well as some degree of uncertainty. The term θ_2 represents the economic cost of production including opportunity costs. Upon winning the contract, the agent expects to produce θ_1 at a cost of θ_2. We allow θ_1 and θ_2 to be dependent as a higher production volume may entail a higher cost. Moreover, private information is affiliated among the n bidders as bidder i's production and cost may be large when other bidders have large production and cost values. Thus, the

[10]Incentive effects on drilling activity are referred to as moral hazard by e.g. Bhattacharya, Ordin and Roberts (2021). See Lewis and Bajari (2014) for moral hazard in procurement auctions.

vector $(\theta_{11}, \theta_{21}, \ldots, \theta_{1n}, \theta_{2n})$ is distributed as $F(\cdot, \ldots, \cdot|n)$ on $(\Theta_1 \times \Theta_2)^n$, which is exchangeable across bidders. Though the type distribution depends on n automatically through its dimension, the conditioning on n also allows for endogeneity of the number of bidders, which captures unobserved heterogeneity as in Campo, Perrigne and Vuong (2003). Bidders do not observe other bidders' private information, but they know the joint distribution $F(\cdot, \ldots, \cdot|n)$ and that other bidders' private information is affiliated with their own. Independence of types among the n bidders and among the two type components for each bidder is a special case. Our model is in the spirit of a private value paradigm where bidders know their expected production and cost.[11]

Bidder i's contract value is captured by the function $V(a; \theta_1, \theta_2) \geq 0$. We make the following natural assumptions on $V(\cdot; \cdot, \cdot)$: (i) a larger royalty reduces the bidder's contract value, (ii) larger expected production increases the contract value, and (iii) a larger cost decreases the contract value. Assumption A1-(i) summarizes these assumptions using a subscript to denote partial differentiaton.

Assumption A1: *The contract value* $V(\cdot; \cdot, \cdot)$ *satisfies*

(i) $V_a(a; \theta_1, \theta_2) < 0, V_{\theta_1}(a; \theta_1, \theta_2) > 0, V_{\theta_2}(a; \theta_1, \theta_2) < 0,$

(ii) $V_{a\theta_1}(a; \theta_1, \theta_2) \leq 0, V_{a\theta_2}(a; \theta_1, \theta_2) \geq 0.$

Assumption A1-(ii) on the cross-derivatives is intuitive. Since royalty is a share of revenue, paying a given royalty is costlier given larger expected production. Also, a larger cost decreases the probability of production, making a given royalty rate less costly in expectation for the bidder. A simple example of contract value is $V(a; \theta_1, \theta_2) = (1-a)p\theta_1 - \theta_2$, where p is the price of the product. The bidder keeps $(1-a)$ of revenue $p\theta_1$ from which he deducts cost θ_2. The context of the empirical application dictates the choice of the contract value function. In the Appendix, we prove that the specification of contract value in (1) satisfies assumption A1.

The principal is characterized by an allocation rule which can be deterministic or probabilistic. This rule incorporates various objectives, need not be announced and can be interdependent. We only require that the principal favors bidders submitting larger royalties and cash payments.[12] Bidders infer their winning probability given their bid from the history of past auctions. Because of affiliation, this probability is conditional on the bidder's own private information (θ_1, θ_2) as in the case of affiliated private values

[11] We discuss the common value paradigm in the Online Appendix.

[12] The purpose of our model is not to derive the optimal allocation rule which would imply defining a surplus function for the principal and his optimal behavior.

or more generally interdependent values. See Milgrom and Weber (1982) and Krishna (2009). Indeed, with affiliation, the bid distribution the bidder expects to compete against is conditional on his own type/value. This conditioning disappears if the types are independent. We denote the winning probability when submitting the pair (a, b) as $P(a, b|\theta_1, \theta_2, n)$. We make the following assumption on the winning probability.

Assumption A2: *At given n, the probability $P(a, b|\theta_1, \theta_2, n)$ satisfies $P_a(a, b|\theta_1, \theta_2, n) > 0$ and $P_b(a, b|\theta_1, \theta_2, n) > 0$.*

Assumption A2 reflects the principal's preference for larger cash and royalty payments. It assumes symmetric bidders as the winning probability does not depend on the bidder's identity. Bidders are symmetric when $F(\cdot, \ldots, \cdot|n)$ is exchangeable across bidders.

Cash-Royalty Bidding

We adopt a best response approach in the spirit of Guerre, Perrigne and Vuong (2000). Given his type $(\theta_{1i}, \theta_{2i})$, bidder i chooses his bid (a_i, b_i) to maximize his expected utility from the auction given his winning probability $P(\cdot, \cdot|\theta_{1i}, \theta_{2i}, n)$. His maximization problem is $\max_{a,b}[V(a; \theta_{1i}, \theta_{2i}) - b]P(a, b|\theta_{1i}, \theta_{2i}, n)$. We now omit the index i for sake of simplicity. Differentiating with respect to a and b, the first-order conditions give

$$V_a(a; \theta_1, \theta_2) = -\frac{P_a(a, b|\theta_1, \theta_2, n)}{P_b(a, b|\theta_1, \theta_2, n)} \tag{3}$$

$$V(a; \theta_1, \theta_2) = b + \frac{P(a, b|\theta_1, \theta_2, n)}{P_b(a, b|\theta_1, \theta_2, n)}. \tag{4}$$

Equation (4) resembles the first-order condition for first-price auctions in Guerre, Perrigne and Vuong (2000) for independent private values.[13]

To gain intuition on the bidder's trade-off between cash payment and royalty, we decompose his optimization problem as follows. For any given winning probability P, a choice of a automatically determines the b that satisfies $P(a, b|\theta_1, \theta_2, n) = P$ because $P(a, b|\theta_1, \theta_2, n)$ is increasing in both (a, b) by assumption A2. This defines a function $b(a, P|\theta_1, \theta_2, n)$ giving the iso-probability curve of cash-royalty combinations that achieve the winning probability P. Then the bidder's maximization problem is equivalent to $\max_{a,P}[V(a; \theta_1, \theta_2) - b(a, P|\theta_1, \theta_2, n)]P$. We break this problem into two steps. In the first step, we consider the choice of a given P, denoted $a(P; \theta_1, \theta_2, n)$. This step allows us to see how a (θ_1, θ_2)-bidder makes the trade-off between royalty

[13] See Li, Perrigne and Vuong (2002), Athey and Haile (2007) and Somaini (2020) for similar expressions for affiliated private and interdependent values.

and cash, holding fixed his winning probability. In the second step, we consider the bidder's optimal choice of P in light of $a(P;\theta_1,\theta_2,n)$ from the first step.

In the first step, the bidder's choice of a given P solves $a(P;\theta_1,\theta_2,n) \equiv \max_a V(a;\theta_1,\theta_2) - b(a,P|\theta_1,\theta_2,n)$, where the maximand is the payoff conditional on winning. The bidder chooses royalty a to maximize this payoff, accounting for its effect on the contract value and cash payment required to achieve P. This leads to the first-order condition

$$V_a(a;\theta_1,\theta_2) = b_a(a,P|\theta_1,\theta_2,n), \tag{5}$$

where $a = a(P;\theta_1,\theta_2,n)$. This choice of a implies $b = b[a(P;\theta_1,\theta_2,n),P|\theta_1,\theta_2,n]$. Equation (5) expresses the trade-off of the marginal cost of a higher royalty against its marginal benefit. Bidding a higher royalty reduces the contract value but allows a bidder to bid less cash by assumptions A1 and A2. This step has implications for adverse selection. Bidding a higher royalty rate is less costly for the undesirable 'weak' types (with low productivity θ_1 and/or high cost θ_2) than for the desirable 'strong' types (with high θ_1 and/or low θ_2). Given a, a strong type pays more in royalties than a weak type, making the cash component relatively more (less) attractive to the strong (weak) type. Because the principal does not observe (θ_1,θ_2), bidders exploit their private information to choose a favorable combination from the set $\{(a,b);P(a,b|\theta_1,\theta_2,n) = P\}$, resulting in adverse selection. In other words, bidders strategically choose a cash-royalty combination that reduces their total payment without compromising their winning probability. Since the royalty provides weak types a cheaper currency with which to bid, they win more often than they would in the absence of royalty bidding. This insight is similar in spirit to Che and Kim (2010) and Skrzypacz (2013).

In the second step, the bidder chooses the winning probability P. Plugging in the royalty $a(P;\theta_1,\theta_2,n)$ and cash amount $b[a(P;\theta_1,\theta_2,n),P|\theta_1,\theta_2,n]$ from the first step, the maximization problem reduces to $\max_P \pi(P;\theta_1,\theta_2,n)P$, where $\pi(P;\theta_1,\theta_2,n) \equiv V[a(P;\theta_1,\theta_2,n);\theta_1,\theta_2] - b[a(P;\theta_1,\theta_2,n),P|\theta_1,\theta_2,n]$ is the payoff conditional on winning. This leads to the first-order condition

$$\frac{1}{P} = -\frac{\pi_P(P;\theta_1,\theta_2,n)}{\pi(P;\theta_1,\theta_2,n)}, \tag{6}$$

giving the solution $P = P(\theta_1,\theta_2,n)$. Equation (6) expresses the trade-off between the bidder's desire to increase his winning probability through a more competitive bid and his payoff conditional on winning. This step implies that a strong bidder tends to bid a higher cash payment and/or royalty to increase his winning probability because his contract value is larger. Depending on the allocation rule, this can lead to the prevalence of dominant bids as documented in Section 2.1. The auction also induces moral hazard since higher royalty rates provide poor incentives for the winner to execute the contract.

Differentiating $P[a, b(a, P|\theta_1, \theta_2, n)|\theta_1, \theta_2, n] = P$ with respect to a and P gives $b_a(a, P|\theta_1, \theta_2, n) = -P_a(a, b|\theta_1, \theta_2, n) / P_b(a, b|\theta_1, \theta_2, n)$ and $b_P(a, P|\theta_1, \theta_2, n) = 1/P_b(a, b| \theta_1, \theta_2, n)$. Thus, using the definition of $\pi(\cdot; \cdot, \cdot, n)$ and $\pi_P(P; \theta_1, \theta_2, n) = -b_P[a(P; \theta_1, \theta_2, n), P|\theta_1, \theta_2, n]$ by the envelope theorem, the first-order conditions (5)-(6) give (3)-(4). As in the previous literature on the structural analysis of auction data, (3) and (4) constitute the basis for identification and estimation as discussed next.

3.2 Identification of Model Primitives

Observables and Primitives

We consider L independent auctions. In each auction ℓ, we observe the bid vector $(a_{1\ell}, b_{1\ell}, \ldots, a_{n_\ell \ell}, b_{n_\ell \ell})$ of the n_ℓ bidders, which is distributed as $G(\cdot, \ldots, \cdot|n_\ell)$. For now, we omit exogenous variables characterizing the auctioned contracts. In each auction, we also observe the winning dummy $W_{i\ell}$ indicating that bidder i is selected by the principal. Because types are affiliated across bidders, the bid components are also affiliated across the n_ℓ bidders. Moreover, the bid components (a_i, b_i) for each bidder i are dependent within bidder. The joint bid distribution $G(\cdot, \ldots, \cdot|n)$ of $(a_1, b_1, \ldots, a_n, b_n)$ is exchangeable in the bidders' identities since they are symmetric. The model primitives are the contract value $V(\cdot; \cdot, \cdot)$, the joint distribution of types $F(\cdot, \ldots, \cdot|n)$ and the probability of winning $P(\cdot, \cdot|\cdot, \cdot, n)$.

About the Contract Value

We first discuss the identification of the value function $V(a; \theta_1, \theta_2)$. Suppose that the winning probability and its derivatives are known so that the right-hand sides in (3) and (4) denoted (say) Y_1 and Y_2, respectively, are observed. This gives a system of equations $V_a(a; \theta_1, \theta_2) = Y_1$ and $V(a; \theta_1, \theta_2) = Y_2$, where (θ_1, θ_2) are unobserved random terms. These resemble nonseparable models with multiple error terms. Matzkin (2003, Appendix A) addresses their identification under (i) monotonicity in (θ_1, θ_2), (ii) independence of (θ_1, θ_2) from a and (iii) some normalizations. While (i) is satisfied by assumption A1 and (iii) can be imposed, (ii) does not hold since $a = a(\theta_1, \theta_2; n)$ through bidders' best response.[14]

[14]This difficulty also arises in Luo, Perrigne and Vuong (2018) in nonlinear pricing with a one-

The value function $V(\cdot;\cdot,\cdot)$ is not identified without additional information and/or restrictions. Consider the following separable specification $V(a;\theta_1,\theta_2)=\theta_1 V_0(a)-\theta_2$, where $V_0(\cdot)$ is a positive decreasing function. The system (3)-(4) becomes $\theta_1 V_{0a}(a)=Y_1$ and $\theta_1 V_0(a)-\theta_2=Y_2$. Any choice of $V_0(\cdot)$ identifies $\theta_1=\theta_1(a,b,n)$ and $\theta_2=\theta_2(a,b,n)$ given (Y_1,Y_2). In other words, different specifications of $V_0(\cdot)$ lead to the same observables (Y_1,Y_2). Intuitively, any choice of $V_0(\cdot)$ is 'compensated' by the inverse best responses $\theta_1(\cdot,\cdot,n)$ and $\theta_2(\cdot,\cdot,n)$. This leads to the next assumption.

Assumption A3: *The contract value $V(a;\theta_1,\theta_2)$ is a known function.*

For instance, the agent's contract value could be $(1-a)p\theta_1-\theta_2$. Our empirical application takes $V(a;\theta_1,\theta_2)$ as an option value given by (1).

IDENTIFICATION OF $P(\cdot,\cdot|\cdot,\cdot,n)$ AND $F(\cdot,\dots,\cdot|n)$

We now turn to the identification of the winning probability $P(\cdot,\cdot|\cdot,\cdot,n)$ and its derivatives. If private information were independent across bidders, this probability would reduce to $P(a,b|n)$ which is identified as the conditional expectation of winning $\mathrm{E}[W|a,b,n]$, where W is observed. With affiliated private information, we note that bidder i's winning probability with a bid pair (a,b) is a composite of two objects: The conditional distribution $G_{a_-,b_-|\theta_1,\theta_2}(a_{-i},b_{-i}|\theta_{1i},\theta_{2i},n)$, which assesses the competition given bidder i's type $(\theta_{1i},\theta_{2i})$, and the choice probability $C[a,b,a_{-i},b_{-i}|n]$ that bidder i wins with (a,b) when his opponents submit the vector (a_{-i},b_{-i}) . Specifically,

$$P(a,b|\theta_{1i},\theta_{2i},n)=\int C[a,b,a_{-i},b_{-i}|n]\;dG_{a_-,b_-|\theta_1,\theta_2}(a_{-i},b_{-i}|\theta_{1i},\theta_{2i},n). \tag{7}$$

The choice probability $C[\cdot,\dots,\cdot|n]$ is identified from observed bids and the winner's identities as the conditional expectation $\mathrm{E}[W_i|a,b,a_{-i},b_{-i},n]$. Our approach concerning $C[\cdot,\dots,\cdot|n]$ is flexible and data-driven. We allow for uncertainty in the principal's allocation rule by not restricting the choice probability to be zero/one as in (say) scoring auctions. This uncertainty may be due to other factors considered beyond the bid components. Also, the auctioneer's choice may be the result of some information learned from the bid compositions (a,b) and (a_{-i},b_{-i}). Our approach does not exclude such a possibility as $C[\cdot,\dots,\cdot|n]$ is a nonparametric function of a,b,a_{-i},b_{-i}.

dimensional type. Identification is achieved by exploiting multiplicative separability of the consumer's utility and optimality of the observed nonlinear pricing. Here, the optimal mechanism remains an unresolved issue because of multidimensional screening. See Rochet and Stole (2003) for a survey, Asker and Cantillon (2010) and Carroll (2017). For this reason, we adopt a best response approach.

Next, we consider identification of $G_{a_-,b_-|\theta_1,\theta_2}(a_{-i}, b_{-i}|\theta_{1i}, \theta_{2i}, n)$ in (7). We note that the system (3)-(4) leads to the best responses $a_i = a(\theta_{1i}, \theta_{2i}, n)$ and $b_i = b(\theta_{1i}, \theta_{2i}, n)$. The next assumption says that this relationship is invertible.

Assumption A4: *The mapping* $(\theta_1, \theta_2) \to [a(\theta_1, \theta_2, n), b(\theta_1, \theta_2, n)]$ *is invertible.*

A necessary condition for assumption A4 is that the system $V_a(a; \theta_1, \theta_2) = Y_1$ and $V(a; \theta_1, \theta_2) = Y_2$ has at most a unique solution in (θ_1, θ_2) for any triplet (a, Y_1, Y_2). This condition becomes necessary and sufficient when bidders' private information are independent because the right-hand sides of (3)-(4) are independent of (θ_1, θ_2). It is satisfied for the example $V(a; \theta_1, \theta_2) = (1-a)p\theta_1 - \theta_2$ as shown below. In the Appendix we show that it is satisfied by the option value $V(a; \theta_1, \theta_2)$ given by (1). Assumption A4 implies $G_{a_-,b_-|\theta_1,\theta_2}(a_{-i}, b_{-i}|\theta_{1i}, \theta_{2i}, n) = G_{a_-,b_-|a,b}(a_{-i}, b_{-i}|a_i, b_i, n)$. Thus, the winning probability (7) can be written as $P(a, b|a_i, b_i, n)$.[15] The conditional bid distribution $G_{a_-,b_-|a,b}(\cdot, \dots, \cdot|\cdot, \cdot, n)$ is identified from the joint distribution $G(\cdot, \dots, \cdot|n)$ of the bid pairs $(a_1, b_1), \dots, (a_n, b_n)$. Thus $P(\cdot, \cdot|a, b, n)$ is identified from observables. Hence, its derivatives $P_a(\cdot, \cdot|a, b, n)$ and $P_b(\cdot, \cdot|a, b, n)$ are also identified.

With this information in hand, we rewrite the system (3)-(4) as

$$V_a(a; \theta_1, \theta_2) = -\frac{P_a(a, b|a, b, n)}{P_b(a, b|a, b, n)} \tag{8}$$

$$V(a; \theta_1, \theta_2) = b + \frac{P(a, b|a, b, n)}{P_b(a, b|a, b, n)}. \tag{9}$$

This system identifies the bidder's private information (θ_1, θ_2) from his observed bid (a, b). For instance, if the contract value is of the form $V(a; \theta_1, \theta_2) = (1-a)p\theta_1 - \theta_2$, then $\theta_1 = P_a(a, b|a, b, n)/[pP_b(a, b|a, b, n)]$ and $\theta_2 = (1-a)[P_a(a, b|a, b, n)/ P_b(a, b|a, b, n)] - b - [P(a, b|a, b, n)/P_b(a, b|a, b, n)]$ are identified. Once the pair (θ_1, θ_2) is identified for every bidder, the joint type distribution $F(\cdot, \dots, \cdot|n)$ is identified as stated next.

Proposition 1: *Under assumptions A1–A4, the bidder type* (θ_1, θ_2) *associated with bid* (a, b) *is identified. Therefore, the joint type distribution* $F(\cdot, \dots, \cdot|n)$ *is identified.*

The separate identification of (θ_1, θ_2) implies that bidders' types cannot be reduced to scalar representations. To provide a contrasting example, suppose that royalties were levied on profit instead of revenue. The value function would be $V(a; \theta_1, \theta_2) = (1-a)(p\theta_1 - \theta_2)$ and $V_a(a; \theta_1, \theta_2) = -(p\theta_1 - \theta_2)$. Thus, (θ_1, θ_2) can replaced by the

[15] This probability is similar to $G_{B|b}(b|b_i)$ in the case of first-price sealed-bid auctions where b denotes an arbitrary bid and B is the maximum of the competitors' bids. See, e.g., Laffont and Vuong (1996) and Li, Perrigne and Vuong (2002).

'aggregate' type $t = p\theta_1 - \theta_2$ in (8)-(9). This implies that all (θ_1, θ_2) pairs such that $p\theta_1 - \theta_2 = t$ are observationally equivalent. This case is ruled out by assumption A4, which requires that θ_1 and θ_2 play distinctive roles in determining the bid components.[16]

3.3 An Estimation Method

The estimation method follows the identification argument. There are several possible estimation methods ranging from nonparametric to parametric ones. We adopt a semiparametric procedure. In auction models, the equilibrium strategies depend on the whole distribution of bidders' private information. Thus, estimating a few moments of this distribution is not sufficiently informative. Because one has little information on this distribution, and parametric families can lead to an inadequate fit to the observed bid distributions, we favor data driven methods to uncover the bidders' private information distribution. This is important as this latent distribution drives the economics and counterfactuals. We present an estimation method taking into account (i) functional form flexibility, (ii) the curse of dimensionality associated with nonparametric estimators and (iii) interpretability of results. First, we estimate the conditional density of bid pairs semiparametrically by $\hat{g}_{a_-,b_-|a,b}(\cdot, \ldots, \cdot|\cdot, \cdot, n)$ using a Gaussian copula upon estimating the marginal bid densities nonparametrically. Second, we estimate the choice probability that a bidder wins with bid components (a, b) when his opponents bid (a_-, b_-) by $\hat{C}[a, b, a_-, b_-|n]$ via sieve approximation of the conditional expectation $\mathrm{E}[W = 1|a, b, a_-, b_-, n]$. Third, by (7) we estimate the winning probability by $\hat{P}(\cdot, \cdot|\cdot, \cdot, n)$ as a composite function of $\hat{G}(\cdot, \ldots, \cdot|n)$ and $\hat{C}[a, b, a_-, b_-|n]$. Finally, using $\hat{P}(\cdot, \cdot|\cdot, \cdot, n)$, we solve the system (8)-(9) at each observed bid $(a_{i\ell}, b_{i\ell})$ to estimate the bidder type $(\theta_{1i\ell}, \theta_{2i\ell})$. Using the latter, we estimate the joint density of $(\theta_{11}, \theta_{21}, \ldots, \theta_{1n}, \theta_{2n})$ semiparametrically using a Gaussian copula. Readers who wish to skip the technical details may proceed to Section 4 for the estimation results.

We omit the exogenous variables Z_ℓ characterizing contract ℓ and incorporate them in Section 3.4. We impose exchangeability of distributions/densities throughout since bidders are symmetric. We first estimate the conditional bid density $g_{a_-,b_-|a,b}(\cdot|a_i, b_i, n)$. This density is the ratio of the $2n$-variate joint density $g(\cdot, \ldots, \cdot|n)$ over the bivariate density $g_{a,b}(\cdot, \cdot|n)$. To alleviate the curse of dimensionality, we use the semiparametric estimator of Genest, Ghoudi and Rivest (1995) for the $2n$-multivariate joint den-

[16] Aggregating types is traditionally used in mechanism design with multidimensional screening. See Armstrong (1996) and Rochet and Stole (2003). Our best response approach avoids this technique.

sity $g(\cdot,\dots,\cdot|n)$ of bid pairs. This requires estimating the two marginal distributions $G_a(\cdot|n)$ and $G_b(\cdot|n)$, which are obtained by integrating the kernel density estimators $\hat{g}_a(\cdot|n)$ and $\hat{g}_b(\cdot|n)$. For interpretability of empirical results, we choose the Gaussian copula density $c_{2n}(\cdot,\dots,\cdot;R)$, where the subscript $2n$ indicates its dimension and R is the correlation matrix. Exchangeability imposes equalities on the correlation coefficients, thereby reducing R to the four correlations $\rho_{a_1b_1}$, $\rho_{a_1b_2}$, $\rho_{a_1a_2}$ and $\rho_{b_1b_2}$ that are estimated by maximizing with respect to R the likelihood $\prod_{\ell=1}^{L_n} c_{2n}[\hat{G}_a(a_{1\ell}|n),\hat{G}_b(b_{1\ell}|n),\dots,\hat{G}_a(a_{n\ell}|n),\hat{G}_b(b_{n\ell}|n);R]$, where L_n is the number of auctions with n bidders. This gives the estimated joint density $c_{2n}[\hat{G}_a(a_1|n),\hat{G}_b(b_1|n),\dots,\hat{G}_a(a_n|n),\hat{G}_b(b_n|n);\hat{R}]\prod_{j=1}^{n}\hat{g}_a(a_j|n)\hat{g}_b(b_j|n)$ as well as the bivariate marginal density $c_2[\hat{G}_a(a_i|n),\hat{G}_b(b_i|n);\hat{\rho}_{a_1b_1}]\hat{g}_a(a_i|n)\hat{g}_b(b_i|n)$. The ratio of these two gives $\hat{g}_{a_-,b_-|a,b}(a_-,b_-|a_i,b_i,n)$.

Second, we estimate the choice probability $C[a,b,a_-,b_-|n]=\mathrm{E}[W=1|a,b,a_-,b_-,n]$, where W indicates that the bidder wins. We approximate this expectation with sieves. To alleviate the curse of dimensionality, we reduce the number of arguments by considering differences in bid components, namely $C(a-a_-,b-b_-|n)$. When $n=2$, the log-likelihood is $\sum_{\ell=1}^{L_2}\{W_{1\ell}\log C(a_{1\ell}-a_{2\ell},b_{1\ell}-b_{2\ell})+W_{2\ell}\log[1-C(a_{1\ell}-a_{2\ell},b_{1\ell}-b_{2\ell})]\}$, where L_2 is the number of auctions with 2 bidders and $W_{i\ell}=1$ $(=0)$ if bidder i is the winner (loser). The function $C(\cdot,\cdot)$ is approximated with Bernstein polynomials while imposing it to be increasing in both arguments in view of assumption A2.[17]

Third, given $\hat{g}_{a_-,b_-|a,b}(\cdot,\dots,\cdot|\cdot,\cdot,n)$ and $\hat{C}(a-a_-,b-b_-|n)$, we estimate $P(\cdot,\cdot|\cdot,\cdot,n)$ as

$$\hat{P}(\cdot,\cdot|a,b,n)=\int\hat{C}(\cdot-a_-,\cdot-b_-|n)\hat{g}_{a_-,b_-|a,b}(a_-,b_-|a,b,n)\;da_-db_-. \tag{10}$$

This integral can be computed using Monte Carlo integration. Its partial derivatives with respect to the first two arguments are estimated by differentiating inside the integral. In the last step, we solve the system (8)-(9) with the estimated winning probabilities and derivatives evaluated at each observation $(a_{i\ell},b_{i\ell})$. This gives $(\hat{\theta}_{1i\ell},\hat{\theta}_{2i\ell})$, $i=1,\dots,n_\ell$, $\ell=1,\dots,L_n$. We can also estimate the marginal density of the cost per unit $\hat{\theta}_{2\ell}/\hat{\theta}_{1\ell}$, which provides interesting economic content. This step is performed

[17] When $n=3$, the likelihood becomes $\sum_{\ell=1}^{L_3}\{W_{1\ell}\log C(a_{1\ell}-a_{2\ell},b_{1\ell}-b_{2\ell},a_{1\ell}-a_{3\ell},b_{1\ell}-b_{3\ell})+W_{2\ell}\log C(a_{2\ell}-a_{1\ell},b_{2\ell}-b_{1\ell},a_{2\ell}-a_{3\ell},b_{2\ell}-b_{3\ell})+W_{3\ell}\log C(a_{3\ell}-a_{1\ell},b_{3\ell}-b_{1\ell},a_{3\ell}-a_{2\ell},b_{3\ell}-b_{2\ell})\}$, where L_3 is the number of auctions with 3 bidders. We impose that (i) the three choice probabilities sum up to one, (ii) the first and second arguments are exchangeable with the third and fourth arguments in the choice probabilities, and (iii) the choice probability is increasing in all its arguments. These restrictions are imposed on the coefficients of the Bernstein polynomial expansion which are estimated by maximum likelihood after normalization of each argument to the interval $[0,1]$ through its quantile.

using a kernel density estimator subject to some trimming to correct for boundary effects following Guerre, Perrigne and Vuong (2000). To assess the degree of type dependence within and between bidders, we use a Gaussian copula estimator as above while imposing exchangeability of the joint type distribution.

3.4 Incorporating Auction Covariates

We now discuss how to introduce auction covariates in the estimation method. We consider a vector Z_ℓ of covariates in the estimation of the $2n$-variate joint bid distribution $G(\cdot,\ldots,\cdot|z,n)$ and the choice probability $C(a,b,a_-,b_-|z,n)$. This leads to the winning probability $P(a,b|a,b,z,n)$ in (10). To alleviate the curse of dimensionality, we propose a demeaning approach using an index to account for auction heterogeneity in the estimation procedure, as in Haile, Hong and Shum (2005).

Given n, we estimate the means of $a_{i\ell}$ and $\log b_{i\ell}$ conditional on Z_ℓ using a 'leave-one-out' regression of $a_{i\ell}$ and $\log b_{i\ell}$ on Z_ℓ, respectively. Leave-one-out refers to not using data from auction ℓ when predicting the means for auction ℓ. To estimate the joint bid distribution, we convert each observed $a_{i\ell}$ and $\log b_{i\ell}$ to deviations from the leave-one-out conditional mean royalty $\hat{\mathrm{E}}(a_{-\ell}|z,n)$ and logarithm of cash bid $\hat{\mathrm{E}}(\log b_{-\ell}|z,n)$, respectively. This normalizes the bids and allows pooling of bids across heterogeneous auctions. Let $\tilde{a}_{i\ell} = a_{i\ell} - \hat{\mathrm{E}}(a_{-\ell}|z,n)$ and $\tilde{b}_{i\ell} = \log b_{i\ell} - \hat{\mathrm{E}}(\log b_{-\ell}|z,n)$ denote these normalized bids distributed as $\tilde{G}(\cdot,\ldots,\cdot|n)$. Using the estimation method in Section 3.3, the estimated conditional density $\hat{g}(\cdot,\ldots,\cdot|z,n)$ is obtained from $\hat{g}(a_1,b_1,\ldots,a_n,b_n|z,n) \equiv c_{2n}(\tilde{G}_a(\tilde{a}_1|n),\ldots,\tilde{G}_b(\tilde{b}_n|n);\hat{R})\Pi_{j=1}^n \tilde{g}_a(\tilde{a}_j|n)\tilde{g}_b(\tilde{b}_j|n)$.

To estimate the choice probability $C(a-a_-,\log b-\log b_-|z)$ for $n=2$, we reduce the dimensionality of z through the leave-one-out conditional expectation $\mathrm{E}(\log b_{-\ell}|z,n)$ defined above by adding $\hat{\mathrm{E}}(\log b_{-\ell}|z,n)$ as an argument to the choice probability, i.e., $C(a-a_-,\log b-\log b_-,\hat{\mathrm{E}}(\log b_{-\ell}|z,n))$. We then follow the estimation procedure of Section 3.3 to obtain $\hat{C}(\cdot,\cdot,\cdot)$. With these estimates in hand, following (10) we obtain $\hat{P}(\cdot,\cdot|a,b,z,n) = \int \hat{C}(\cdot - a_-, \cdot - \log b_-, \hat{\mathrm{E}}(\log b_-|z,n))\hat{g}_{a_-,b_-|a,b,z}(a_-,b_-|a,b,z,n)da_-db_-$, and its derivatives $\hat{P}_a(\cdot,\cdot|a,b,z,n)$ and $\hat{P}_b(\cdot,\cdot|a,b,z,n)$ by numerical differentiation.

Lastly, the type distribution depends on the covariates Z_ℓ, which are aggregated in the single index $\hat{\mathrm{E}}(\log b_{-\ell}|z,n)$ that we normalize through its quantile $q(z,n)$. Upon estimating $(\hat{\theta}_{1i\ell},\hat{\theta}_{2i\ell})$ for $i=1,2$ and $\ell=1,\ldots,L_2$ by solving (8)-(9), we assess the degree of affiliation among $(\theta_{11},\theta_{21},\theta_{21},\theta_{22},q(z,n))$ using a Gaussian copula with smooth kernel estimates of the marginal distributions as explained in Section 3.3.

4 Estimation Results

This section presents the estimation results for the Louisiana oil lease auction data preceded by a description of the covariates that characterize auction heterogeneity.

4.1 Characterizing Auction Heterogeneity

We present additional data sources needed in the computation of the option value. For the one-year risk-free interest rate r, we convert the nominal one-year treasury rates from the Federal Reserve Economic Data (FRED) to real rates via application of the Fisher equation, using percentage changes in the GDP implicit price deflator as a measure of inflation. We use the latter to adjust all values subject to inflation. For the oil price p, we use the West Texas Intermediate price provided by FRED. Including p as a covariate allows the distribution of θ_1, θ_2 to be conditional on p. For the volatility σ, we use the expected volatility implied by contemporary crude oil option prices.[18] Since crude oil options have been traded since November 1986, we use bids from 1987 onwards in the last step which estimates the joint type distribution by solving (8)-(9). For more details on the derivation of implied volatility, see the Online Appendix.

In addition to the oil price, volatility and interest rate which enter in the option value, bids depend on other characteristics that we now define. To account for geographical and geological heterogeneity, we use historical data from Drillinginfo to compute a production index based on production in the lease's township during the five years preceding its auction. To account for unobserved geographical and geological heterogeneity, following Kong (2020), we exploit the spatial continuity of land-based heterogeneity to construct smooth and location-based 'heatmap' indices as follows. To first eliminate time effects from the indices, we regress observed bid components on year fixed effects and obtain the residuals. We then define heatmap royalty and cash indices for each tract as the residuals predicted by geographic location. Hereafter we consider the logarithm of the cash component because of its skewness from Table 1. The predictions are obtained from leave-one-out local quadratic regressions of the residuals on geographic coordinates of the lease township. The prediction for auction ℓ uses only those auctions occurring up to the auction year to avoid using information from the 'future'. Lastly, for some auctions, the royalty recipient is a state agency other than the Department of Natural Resources (DNR), which is captured by a dummy.

[18]Access to these market data was purchased from the Chicago Mercantile Exchange Inc.

Table 3 gives the results of the regressions of the bid components on the auction covariates using the full sample of 1,291 bids. As year fixed effects are included, the monthly oil price and interest rate do not always have statistically significant coefficients. The logarithm of the cash bid per acre is affected by acreage and its square. When the royalty recipient is not the DNR, there is a negative effect on both bid components suggesting that these leases are less desirable. Lastly, leases with larger heatmap indices attract higher bid components, suggesting that these indices successfully capture unobserved geographical and geological heterogeneity.

4.2 Estimation Results

Given that 80% of the dataset contains two bidders per auction, we present the results for $n = 2$. Data on implied volatility are available after 1987. To exploit the maximum number of observations, we estimate the winning probability using the 904 (452 auctions times 2) bids including observations before 1987.[19] Figure 3 plots level contours of the estimated pairwise choice probability at the median value of the quality index $q(z)$ along with the observations. The contours, ranging from 0.1, 0.2,..., 0.9, show the transition from low to high probability of being chosen as bid components increase.

Because the option value is a function of implied volatility, we estimate bidders' types using the 258 (129 times 2) bids after 1987. The estimated values for θ_1 (expected production volume) are mostly between 100 and 3,000 barrels per acre. Though we have not used any ex post information on production, our estimates are comparable to general statistics on onshore oil production. Based on data from Drillinginfo, the median township in our data produces about 270,000 barrel-of-oil-equivalents (BOE) per well. Conditional on the median quality index, our estimate of expected production is 700 barrels per acre or about 230,000 barrels for the average-sized lease, which is close to the actual production statistic. We also examine the relationship between expected production volume $\hat{\theta}_1$ and realized production from wells spudded in the lease's township during the three years following its auction.[20] As there are no such wells for many of the auctioned leases, we estimate a type I Tobit model of log realized production on log $\hat{\theta}_1$ treating the latent volume of production as censored from below

[19]Because the nonparametric estimation of the choice probability is data demanding, we pool the data across n and consider the $n-1$ pairs $(a - a_{-}, \log b - \log b_{-})$ when estimating $C(\cdot, \cdot, \cdot)$.

[20]Since we do not have production data at the lease level, as a substitute, we use knowledge of each lease's township and auction date to compute production from wells spudded in the lease's township during the three years following the auction.

at the minimum observed non-zero value in the sample. The estimated Tobit coefficient is 0.71 with standard error 0.24, meaning a one percent higher value of $\hat{\theta}_1$ is associated with a 0.71 percent higher value of latent production at the township level.

Figure 4 plots the density of the per unit cost $\hat{\theta}_2/\hat{\theta}_1$. This ratio represents the cost per barrel of production, which is a more intuitive measure of cost than $\hat{\theta}_2$. Our median estimate of cost per barrel is $21 in 2009 dollars, which is comparable to figures reported by the Wall Street Journal.[21] Figure 4 includes a vertical dotted line representing the sample average firm's revenue per barrel $(1-a)p$. Assuming a fixed revenue per barrel, this gives a rough idea of which firms would exercise their options, i.e., develop their tracts. Firms to the left of this value would exercise their option, while firms to the right would not. As there is a substantial mass to the right, Figure 4 provides a rationale as to why so many leases remain undeveloped in the U.S.[22] Next, we discuss the estimated correlation matrix of the joint density of types $(\theta_{11}, \theta_{21}, \theta_{12}, \theta_{22})$ and the index $q(z)$. The correlation coefficient between θ_{1i} and θ_{2i} is 0.86 indicating correlation between production and cost. The correlations between bidders (0.81 for production and 0.90 for cost) show affiliation of private information among bidders. If production and cost are expected to be high for one bidder, they are likely to be high for the other bidder as well. Lastly, the correlation between the types and the quality index $q(z)$ is 0.68 for both production and cost, confirming the explanatory power of tract heterogeneity through our index $q(z)$. Part of the correlation among the components $(\theta_{11}, \theta_{21}, \theta_{12}, \theta_{22})$ is due to their collective correlation with $q(z)$.

In light of the government's concern about undeveloped tracts, we assess the model's predicted development rate. We compare the ex ante predicted exercise probability $\Phi(x - \sigma\sqrt{t})$ to the ex post observed probability of development. We consider a tract developed if the Louisiana DNR has a record of receipts of royalties from the lease or of a well attached to the lease. Our average predicted ex ante exercise probability is equal to 0.44. For the same set of leases, we observe an exercise probability of 0.42. The closeness of these two values assesses the good fit of our model. Indeed, our model and our estimates of production and cost are consistent with the observed low rate of development, though we did not use any post-auction information in estimation. Relating to our discussion of Figure 4 above, our model explains this phenomenon by moral hazard induced by the combination of bidders' production costs, royalty bidding

[21] See graphics.wsj.com/oil-barrel-breakdown/.

[22] The Department of the Interior reports that by the end of 2011, about 56% of total acres of public land under lease in the lower 48 states are not undergoing either production or exploration activities.

and the optional nature of production. The option model also explains why firms are willing to pay for leases they may not develop. Even if the oil price is below the 'strike price' at the time of auction, the leases have option value.

5 Counterfactual Analysis

Fixed-royalty auctions are commonly used for public mineral leasing in the U.S. In that auction format, the principal fixes the royalty rate and bidders bid on the cash payment only, leading to one-dimensional bidding. Since little is known about optimal auction design with multidimensional private information, we compare our cash-royalty auctions to fixed-royalty auctions. In addition, since scoring auctions are the most studied multi-attribute auctions, we also investigate their performance. Lastly, in view of recent policy debates on the poor performance of oil lease auctions in the U.S., we assess the effects of increasing the lease duration and exploiting auction timing.

Regarding the pros and cons of cash-royalty versus fixed-royalty auctions, allowing competitive forces to determine royalties can potentially reap a higher share of revenue for the principal by driving up royalties on strong tracts while increasing the development probability by driving down royalties on weak tracts. Royalty rates, however, could be too high, exacerbating moral hazard, or too low, ceding information rents to the winner. Also, cash-royalty bidding gives firms more room to exploit information asymmetry. This adverse selection can depress the principal's revenue and distort lease allocation. In our counterfactual, we investigate whether the pros outweigh the cons.

Asker and Cantillon (2008) show that the principal is better off using a price-quality scoring auction than imposing a fixed quality and selecting the winner on the basis of price only. Cash-royalty auctions differ, however, from price-quality auctions in two key aspects. First, the winning bidder chooses whether to develop the lease, while his incentives are affected by the endogenous royalty. Second, since royalty revenue depends on the exercise probability and production volume, the principal's payoff $\mathrm{e}^{-rt}ap\theta_1\Phi(x) + b$, where x is given in (2), depends directly on bidders' private information. This is not the case in a price-quality scoring auction, where the principal's payoff is entirely determined by the observable bid components (price, quality). The bidder can exploit this extra gap created by asymmetric information in cash-royalty auctions by trading off cash and royalty to the detriment of the principal.[23]

[23]Thus the proof of Asker and Cantillon (2008, Theorem 6) no longer applies, and the superior performance of scoring auctions is no longer guaranteed.

5.1 Cash-Royalty vs Fixed-Royalty Auctions

We compare the observed cash-royalty auctions to simulated fixed-royalty auctions. Using our estimation results, we simulate auctions with a fixed royalty ranging from 0% to 50%. For a fixed royalty A, we compute for every bidder i in auction ℓ the lease value $V_{i\ell} = V(A; \hat{\theta}_{1i\ell}, \hat{\theta}_{2i\ell})$. Using random draws $(\theta^s_{11\ell}, \theta^s_{21\ell}, \theta^s_{12\ell}, \theta^s_{22\ell})$ from $\hat{F}(\cdot, \cdot, \cdot, \cdot | q(Z_\ell), n)$, we simulate the joint value distribution $F_{V_1,V_2|q(Z)}(\cdot, \cdot|\cdot, n)$ for $n = 2$. Second, from Milgrom and Weber (1982)'s equilibrium in a first-price sealed-bid auction with affiliated private values, we compute the cash bid that each bidder would have submitted in a fixed-royalty auction. We use notation A for a fixed royalty and a for a royalty bid.

In Figures 5–7, the solid curve represents the outcome of the fixed-royalty auction with A on the x-axis. For comparison, the horizontal dashed line represents the outcome of the Louisiana cash-royalty auction. We examine royalty revenue, cash revenue and their sum which gives total government revenue. Figure 5 displays the ex ante expected royalty revenue given by $e^{-rt} pA\theta_1\Phi(x)$ from (1)-(2). The solid curve exhibits a Laffer curve, in which royalty revenue initially rises but eventually falls after $A = 43\%$ as moral hazard overwhelms the gains from taking a higher share of revenue because a higher royalty induces a lower exercise probability. The average royalty from the Louisiana cash-royalty auction is 23% (vertical dotted line). We see that royalty revenue would have been similar had the royalty been fixed at 23%. Thus, allowing flexibility in royalties would not achieve any benefit compared to fixing it at the average. Meanwhile, Figure 6 displays cash revenue. Higher fixed royalties decrease the cash bid because bidders' lease values decline as A increases. We find that the Louisiana cash-royalty auction gives \$112 per acre or 11% less in cash than the 23% fixed-royalty auction.

Figure 7 displays total government revenue. The Louisiana auctions outperform fixed-royalty auctions when the fixed royalty is either higher than 48% or lower than 18%: This includes the federal fixed rate of 12.5%. For middle royalties ranging from 19% to 47%, the fixed-royalty auction outperforms the Louisiana auction. This includes the 25% rate common on private lands. In this middle range, the adverse selection effects of cash-royalty bidding dominate the benefits of royalty flexibility. As a result, a 23% fixed-royalty auction would generate an average gain of 4% or \$103 per acre relative to the cash-royalty auction. Meanwhile, the optimal royalty for fixed-royalty auctions is 34% allowing a gain of 8% or \$213 per acre compared to the cash-royalty auction. This optimal royalty rate is higher than most rates seen in Table 1. This confirms that Louisiana's underperformance is not caused by high royalties per se but by adverse selection in the cash-royalty combinations bidders choose.[24]

To get a sense of the firms' surplus, we consider their information rents defined as the firm's ex ante value for the contract $V(a; \theta_1, \theta_2)$ evaluated at the relevant royalty rate minus the cash payment. According to our simulations, bidders in Louisiana enjoy 4% or \$52-per-acre larger information rents than under a 23% fixed royalty, consistent with greater adverse selection in the cash-royalty auction. Meanwhile, one of the main concerns of the government is the low development rate. We find that royalty bidding in Louisiana does not yield special benefits for option exercise compared to fixing the royalty at 23%. Lastly, we consider social surplus defined as the firm's lease value plus the government's expected royalty revenue, accounting for the endogenous probability of development. We find that royalty bidding in Louisiana does not yield special benefits for social surplus compared to fixing the royalty at 23%. Graphs associated with these simulations are provided in the Online Appendix.

5.2 Scoring vs Fixed-Royalty Auctions

Scoring auctions, especially with quasi-linear scoring rules, are the most studied class of multi-attribute auctions. In light of Asker and Cantillon (2008), one may wonder whether the underperformance of cash-royalty relative to fixed-royalty bidding is specific to Louisiana's allocation rule. To investigate this question, we consider the quasi-linear scoring rule $S(a, b) = b - \omega p / a^\rho$, increasing in both a and b, where $\omega > 0$ is a weight on the royalty component and $\rho > 0$ defines the curvature with respect to it. A higher ρ increasingly discourages low-end and high-end royalty rates. We also allow the royalty component to carry more weight with oil price p. We simulate the outcome of scoring auctions for values of $\rho = 1, 2, \ldots, 10$. For each ρ, we perform a grid search for the optimal weight ω maximizing expected government revenue.

The first row of Table 4 displays $\mathrm{E}[(b - \underline{b})/(s - \underline{s})]$, the expected portion of the score that is due to the cash payment b, where $\underline{b}$ and $\underline{s}$ are the minimum of cash and score values, respectively. This portion ranges from 0.35 to 0.51. We simulate scoring auction outcomes using a second-score mechanism for computational ease. Correspondingly, we use a second-price auction to simulate the fixed-royalty auction for comparison. With a quasi-linear score, Asker and Cantillon (2008) show that the second-score auction induces the same royalty choice as in a first-score auction because the royalty choice is

[24] See the Online Appendix for an assessment of the allocative performance of fixed-royalty auctions.

independent of the score a bidder is trying to achieve. We compute the optimal royalty bid and cash bid for each bidder. The last two rows of Table 4 show the mean and median royalty bids given each ρ. For all simulated values of ρ, these scoring auctions would lead to higher royalty rates on average than those observed in Louisiana.

Figure 8 displays the variance of royalty bids in a dashed curve and the government's total revenue per acre in a solid curve with ρ on the x-axis. As the curvature ρ increases, the variance of royalties decreases and the government's revenue increases to approach that of the optimal fixed-royalty auction. These results confirm a consistent pattern: Reducing or eliminating bidder-driven royalty variance improves revenue for the government. Indeed, the fixed-royalty auction achieves a better outcome even though the weights ω were chosen optimally in the scoring auctions. Table 5 provides more details on fixed-royalty versus scoring auctions with $\rho = 1$. In the former, the optimal royalty is about 30%. The scoring auction would lead to a loss of \$799 per acre in royalty revenue that is not recouped in cash revenue which increases by only \$654. This leads to a 5% decrease in total government revenue, while yielding 38% higher or \$356 of additional information rents to firms. Thus, adverse selection outweighs the potential benefits of royalty flexibility in this class of scoring auctions. In contrast, Table 5 suggests that the scoring auction may induce a higher exercise probability and higher social surplus than the optimal fixed-royalty auction. Also, total government revenue from this scoring auction exceeds that of the Louisiana cash-royalty auction.

To summarize, fixed-royalty auctions would yield higher government revenue than the current cash-royalty format and quasi-linear scoring auctions. Since fixed-royalty auctions are straightforward to implement, fixing the royalty seems the more reliable way to auction lease contracts. This policy recommendation contrasts with conventional wisdom from price-quality auctions, providing new insights on incentive contracts and the merits of multi-attribute auctions. This is not to say that superior multidimensional mechanisms do not exist. They may just take more complex forms. Taking cues from Asker and Cantillon (2010), such a mechanism may have allocation rules that cannot be summarized by a scoring function. It could involve the principal designing a menu of contracts or revising the current contract form. What superior mechanisms look like are open questions given multidimensional private information.

5.3 Alternative Policies: Lease Duration and Timing

Some industry voices have argued that the rate of development would increase if leases were longer. The 3-year lease in Louisiana is shorter than the 5-year lease in New Mexico and the 10-year federal lease. We simulate fixed-royalty auctions for a 6-year lease thereby doubling the observed duration. We present results for the American option valuation to emphasize that firms can exercise their option at any time during the lease. Our method for valuing American options is discussed in Section 6.1.

Though a longer lease duration could increase the ex ante exercise probability, this is not guaranteed. First, at any point in time, an agent possessing an American option compares the payoff of exercising it today to the continuation value of waiting for a potentially higher price. This continuation value is increasing in the remaining duration of the option. Thus, the time-specific threshold for exercising the option is higher when the remaining duration is longer. Second, from an ex ante perspective, the later three years of a six-year lease starting today are not equivalent to a three-year lease starting today; they do not 'cancel out' in a comparison. The starting oil price for the latter is today's price, while the starting oil price for the former is uncertain today and has a log normal distribution. The combined effects of the above on exercise probability are ambiguous. At all fixed royalties between 0% and 50%, we find that increasing the lease duration decreases the ex ante exercise probability, which contradicts the popular belief. At a 23% fixed royalty, the decrease would be from 0.46 to 0.40. However, it does increase government revenue because it increases option values and hence cash bids. At a 23% fixed royalty, total revenue would increase by about 16%.

An alternative policy would be to exploit fluctuations in oil prices. As Louisiana has control over auction offerings, the DNR could withhold leases when oil prices are low and release them when oil prices are high. We simulate government revenue under fixed-royalty auctions had oil prices been 20% higher than what they were at the time of the auction. The resulting increase in government revenue is at least 46% depending on the fixed royalty. From Table 2, higher oil prices increase not only the royalty dollars conditional on option exercise but also the exercise probability, as well as option value and cash bids. This is a promising policy that states could pursue. The Online Appendix provides figures showing additional details from these simulations.

6 Extensions

We discuss (i) the robustness of our results to the American option in which firms can exercise the option at any time and (ii) a general extension of our method.[25]

6.1 Robustness Analysis: American Option

To ensure that our conclusions are not sensitive to the European-option specification, we use a numerical procedure known as a binomial tree to value the lease as an American option and reestimate (θ_1, θ_2) accordingly. Following Cox, Ross and Rubinstein (1979) and Rendleman and Bartter (1979), the binomial option pricing model begins with a single node and each node connects to two nodes in the next period, one representing the probability that the oil price will go up, the other the probability that the oil price will go down. At each node, the agent chooses whether to exercise the option given the node-specific oil price p. The agent exercises the option at that node if $(1-a)p\theta_1 - \theta_2$ exceeds the continuation value. The value of the node is $(1-a)p\theta_1 - \theta_2$ if the option is exercised and the continuation value otherwise. The value of the American option is then computed as the present-discounted expected value of all nodes that extend from the first node. The fineness of the binomial tree is adjustable by the number of steps in the tree. As the number of steps grows large, the value of the binomial tree converges to the value of the option. See Hull (2017) for details. We use a binomial tree with 100 steps to value $V(a; \theta_1, \theta_2)$ and then use constrained optimization to estimate (θ_1, θ_2) from (8)-(9).[26] The Online Appendix explains how each step of the binomial tree evolves as a function of volatility σ and the number of steps. Figure 4 displays in a dash-dotted line the estimated marginal density of the unit cost $\hat{\theta}_2/\hat{\theta}_1$ using this method. The European and American estimated densities closely superimpose each other suggesting that our results are robust to the American option specification. In addition, the Online Appendix provides the results of all the counterfactuals using the American option. These results are qualitatively identical and quantitatively very close to the European option results.

[25] The Online Appendix contains discussions on bidders' cash constraints, unobserved heterogeneity and common values.

[26] Using 100 steps means that 2^{100} possible price paths are considered over the 3-year duration of each option, and firms make a decision about whether to exercise the option every $3 \times 365/100 = 10.95$ days. We adapt code from Zagaglia's (2012) option pricing package to compute the option value.

6.2 Extension

Our methodology extends to a large class of auctions under multidimensional private information. It accommodates as special cases the scoring auctions used in construction procurements and oil lease sales among other examples. For simple exposition, we assume independent private information across bidders and omit auction covariates. Let $(\theta_1, \ldots, \theta_{K+1})$ be a $K+1$ vector of the bidder's private information distributed as $F(\cdot, \ldots, \cdot|n)$. Each bidder submits a $K+1$ vector of bid components $(b_1, \ldots, b_{K+1})$, where b_{K+1} is a cash component. The bidder's value for the auctioned object is $V(b_1, \ldots, b_K; \theta_1, \ldots, \theta_{K+1})$. The auctioned object is allocated to this bidder with probability $P(b_1, \ldots, b_{K+1}|n)$. Each bidder maximizes his expected profit $[V(b_1, \ldots, b_K; \theta_1, \ldots, \theta_{K+1}) - b_{K+1}]P(b_1, \ldots, b_{K+1}|n)$. The first-order conditions with respect to $(b_1, \ldots, b_{K+1})$ lead to a system of $K+1$ equations

$$
\begin{aligned}
V_k(b_1, \ldots, b_K; \theta_1, \ldots, \theta_{K+1}) &= -\frac{P_k(b_1, \ldots, b_{K+1}|n)}{P_{K+1}(b_1, \ldots, b_{K+1}|n)}, \; k = 1, \ldots, K \\
V(b_1, \ldots, b_K; \theta_1, \ldots, \theta_{K+1}) &= b_{K+1} + \frac{P(b_1, \ldots, b_{K+1}|n)}{P_{K+1}(b_1, \ldots, b_{K+1}|n)},
\end{aligned}
$$

where the index k refers to the derivative with respect to b_k. This extends the system (3)-(4) under independent private information. Our identification argument of Section 3.2 applies. For example, in the case of scoring procurement auctions, bidders bid a quality vector $(b_1, \ldots, b_K)$ and a cash component b_{K+1}. The bidder's value is given by the total cost $\theta_{K+1} + \theta_1 b_1 + \ldots + \theta_K b_K$, where θ_{K+1} is interpreted as a fixed cost and θ_k as the marginal cost of quality k. The bidder's expected profit is $[b_{K+1} - \theta_{K+1} - b_1\theta_1 - \ldots - \theta_K b_K]P(b_1, \ldots, b_{K+1}|n)$. Our method, allowing for general allocation rules, does not require a defined scoring rule, but accommodates them as a special case where bidder i's winning probability is $\Pr[S_i \geq S_j, \forall j \neq i|(b_1, \ldots, b_{K+1})_i] = G_S^{n-1}(S_i|n)$ with score $S_i = S[(b_1, \ldots, b_{K+1})_i]$ and score distribution $G_S(\cdot)$. Up to sign, the first-order conditions are as above. See Takahashi (2018), Hanazano, Hirose, Nakabayashi and Tsuruoka (2018) and Sant'Anna (2018) for recent contributions on scoring auctions with endogenous qualities.[27]

[27] Our methodology also extends to scale auctions that are used in the sale of timber and construction procurement auctions. See Bajari, Houghton and Tadelis (2014), Luo and Takahashi (2021) and Bolotnyy and Vasserman (2021). In such auctions, agent i bids a unit price b_{ik} on each of the $(K+1)$ items composing the auctioned object. His score is $S_i = \sum_{k=1}^{K} b_{ik} q_k^e + b_{iK+1}$, where q_k^e is the estimated quantity for item k. His ex post payment is based on realized quantities q_k as $\sum_{k=1}^{K} b_{ik} q_k + b_{iK+1}$. In a procurement setting, bidder i's cost is $\sum_{k=1}^{K} c_k \theta_{ik} q_k + c_{K+1}\theta_{iK+1}$, where $(\theta_{i1}, \ldots, \theta_{iK+1})$ is private

7 Conclusion

In this paper, we perform a structural analysis of bidding on contracts. Using oil lease data in Louisiana, our goal is to assess the impact of multi- versus single-attribute auctions on government revenue, allocation, information rents, development rates and social surplus. By allowing bidders with multidimensional private information to choose the most favorable combination of their multidimensional bid components, we account for adverse selection. Using option values, our model also accounts for moral hazard as firms can choose whether to execute the contract depending on their incentives. The latter in turn affects the bid components and vice versa. Our model and empirical methodology allow for a general contract value and allocation rule. We recover expected production volumes, costs and development rates without ex post observations.

In the case of Lousiana, we find that (i) cash-royalty bidding exacerbates adverse selection, (ii) a fixed-royalty auction would improve government revenue without harming development rates or social surplus and (iii) a fixed-royalty auction would also dominate a scoring auction. These findings contrast with what we have learned about multi-attribute auctions in the price-quality auction literature, where scoring auctions dominate one-dimensional fixed-quality auctions. Lastly, auctioning of incentive contracts with multidimensional private information is an area where the design space is rich but auction design recommendations and empirical work are sparse. We hope that this paper helps trigger new developments in this area.

information and $(c_1, \ldots, c_{K+1})$ are the known engineering cost estimates. To account for bid skewing patterns as first shown by Athey and Levin (2001), one introduces bidders' risk aversion. Maximization of expected profit gives first-order conditions similar to those above adjusted for the nonseparability of b_{K+1}. Risk aversion could be identified and estimated using Guerre, Perrigne and Vuong (2009) and Campo, Guerre, Perrigne and Vuong (2011).

Appendix

Verification of Assumption A1 for Option Value: In view of Table 2, we only need to show that $V_a(a, \theta_1, \theta_2)$ is decreasing in θ_1 and increasing in θ_2. Let $R \equiv (1-a)p\theta_1$. By the chain rule and product rule of derivatives, we have for $k = 1, 2$

$$\frac{\partial}{\partial \theta_k} V_a(a; \theta_1, \theta_2) = \frac{\partial}{\partial \theta_k}\left(\frac{\partial V}{\partial R}\frac{\partial R}{\partial a}\right) = \frac{\partial}{\partial \theta_k}\left(\frac{\partial V}{\partial R}\right)\frac{\partial R}{\partial a} + \frac{\partial}{\partial \theta_k}\left(\frac{\partial R}{\partial a}\right)\frac{\partial V}{\partial R}.$$

It is well known from the option pricing literature (see Hull (2017)) that $\partial V/\partial R = e^{-rt}\Phi(x)$. So using this and the definition of x in (2) leads to

$$\frac{\partial}{\partial \theta_1}\left(\frac{\partial V}{\partial R}\right) = e^{-rt}\phi(x)\frac{\partial x}{\partial \theta_1} = e^{-rt}\phi(x)\frac{1}{\theta_1\sigma\sqrt{t}},$$
$$\frac{\partial}{\partial \theta_2}\left(\frac{\partial V}{\partial R}\right) = e^{-rt}\phi(x)\frac{\partial x}{\partial \theta_2} = e^{-rt}\phi(x)\frac{1}{-\theta_2\sigma\sqrt{t}}.$$

Meanwhile, $\partial R/\partial a = -p\theta_1$, so $\partial\left(\partial R/\partial a\right)/\partial\theta_1 = -p$ and $\partial\left(\partial R/\partial a\right)/\partial\theta_2 = 0$. Plugging these into the above first equation, we have

$$\frac{\partial}{\partial \theta_1} V_a(a; \theta_1, \theta_2) = -e^{-rt}\phi(x)\frac{1}{\theta_1\sigma\sqrt{t}}p\theta_1 - pe^{-rt}\Phi(x) < 0,$$
$$\frac{\partial}{\partial \theta_2} V_a(a; \theta_1, \theta_2) = e^{-rt}\phi(x)\frac{1}{\theta_2\sigma\sqrt{t}}p\theta_1 + 0 > 0.$$

Proof of Proposition 1: Under assumptions A1–A4, the first-order conditions (3)-(4) become (8)-(9) as explained in the text. Thus, by the invertibility assumption A4, (8)-(9) has a unique solution in (θ_1, θ_2) given (a, b). Hence, the private information (θ_1, θ_2) is identified for each bidder from his bid (a, b) since the RHS of (8)-(9) is identified. It follows that the $2n$-dimensional joint distribution of types $F(\cdot, \ldots, \cdot|n)$ is identified.

Verification of Necessary Identification Condition for Option Value: A necessary condition for identification is that the system $V_a(a; \theta_1, \theta_2) = Y_1$ and $V(a; \theta_1, \theta_2) = Y_2$ has at most one solution in (θ_1, θ_2) given (a, Y_1, Y_2). Indeed, if this was not the case, then (8)-(9) would have more than one solution in (θ_1, θ_2). We now prove that the option value in (1)-(2) satisfies this condition. Since (2) is a monotonic function of the ratio θ_1/θ_2, we use a change of variables to solve (8)-(9) for θ_1 and x instead of θ_1 and θ_2. First, we use (8) to express θ_1 in terms of x. Second, we plug this expression for θ_1 into (9), yielding an equation with one unknown x. Third, we show that there cannot be more than one solution x to this equation. Finally, we give closed-form expressions for θ_1 and θ_2 as functions of the solution x.

First, we define $R \equiv (1-a)p\theta_1$. We have $V_a(a; \theta_1, \theta_2) = (\partial V/\partial R)(\partial R/\partial a) = -e^{-rt}p\theta_1\Phi(x)$ using $\partial V/\partial R = e^{-rt}\Phi(x)$. See Hull (2017). Substituting this into the left-hand side of (8) leads to $-e^{-rt}p\theta_1\Phi(x) = Y_1$. Hence, $\theta_1 = -Y_1/[e^{-rt}p\Phi(x)] = C_1/\Phi(x)$, where $C_1 = -Y_1/[e^{-rt}p] > 0$ is a known constant.

Second, plugging this expression for θ_1 in (1) gives

$$\begin{aligned} V(a;\theta_1,\theta_2) &= e^{-rt}(1-a)p\theta_1\left[\Phi(x)-\theta_2\Phi(x-\sigma\sqrt{t})/((1-a)p\theta_1)\right] \\ &= e^{-rt}(1-a)p\,(C_1/\Phi(x))\left[\Phi(x)-\theta_2\Phi(x-\sigma\sqrt{t})/((1-a)p\theta_1)\right] \\ &= e^{-rt}(1-a)pC_1\left[1-\theta_2\Phi(x-\sigma\sqrt{t})/\left((1-a)p\theta_1\Phi(x)\right)\right] \\ &= e^{-rt}(1-a)pC_1\left[1-e^{-\sigma\sqrt{t}x}e^{\sigma^2t/2}\Phi(x-\sigma\sqrt{t})/\Phi(x)\right], \end{aligned}$$

where the last equality follows from $\theta_2/\left[(1-a)p\theta_1\right] = \exp(-\sigma\sqrt{t}x+\sigma^2t/2)$ by (2). Using this expression for $V(a;\theta_1,\theta_2)$ in (9) gives

$$e^{-rt}(1-a)pC_1\left(1-e^{-\sigma\sqrt{t}x}e^{\sigma^2t/2}\frac{\Phi(x-\sigma\sqrt{t})}{\Phi(x)}\right) = Y_2.$$

Collecting x on the left-hand side yields

$$e^{-\sigma\sqrt{t}x}\frac{\Phi(x-\sigma\sqrt{t})}{\Phi(x)} = e^{-\sigma^2t/2}\left(1-\frac{Y_2}{e^{-rt}(1-a)pC_1}\right). \tag{A.1}$$

Third, the right-hand side of (A.1) is known and denoted C_2. We show that the left-hand side is strictly monotonic in x, so that there cannot be more than one value of x satisfying (A.1). Taking the derivative of the left-hand side with respect to x gives

$$\begin{aligned} \frac{\partial}{\partial x}\left(e^{-\sigma\sqrt{t}x}\frac{\Phi(x-\sigma\sqrt{t})}{\Phi(x)}\right) &= -\sigma\sqrt{t}e^{-\sigma\sqrt{t}x}\frac{\Phi(x-\sigma\sqrt{t})}{\Phi(x)}+e^{-\sigma\sqrt{t}x}\left(\frac{\phi(x-\sigma\sqrt{t})\Phi(x)-\Phi(x-\sigma\sqrt{t})\phi(x)}{\Phi(x)^2}\right) \\ &= \left(e^{-\sigma\sqrt{t}x}/\Phi(x)\right)\left(-\sigma\sqrt{t}\Phi(x-\sigma\sqrt{t})+\phi(x-\sigma\sqrt{t})-\frac{\phi(x)}{\Phi(x)}\Phi(x-\sigma\sqrt{t})\right) \\ &= \left(e^{-\sigma\sqrt{t}x}\phi(x-\sigma\sqrt{t})/\Phi(x)\right)\left(-\sigma\sqrt{t}\frac{\Phi(x-\sigma\sqrt{t})}{\phi(x-\sigma\sqrt{t})}+1-\frac{\phi(x)}{\Phi(x)}\frac{\Phi(x-\sigma\sqrt{t})}{\phi(x-\sigma\sqrt{t})}\right) \\ &= \left(e^{-\sigma\sqrt{t}x}\phi(x-\sigma\sqrt{t})/\Phi(x)\right)\left(1-\frac{\Phi(x-\sigma\sqrt{t})}{\phi(x-\sigma\sqrt{t})}\left(\sigma\sqrt{t}+\frac{\phi(x)}{\Phi(x)}\right)\right). \end{aligned}$$

But $h'(x) > -1$, where $h(x) \equiv \frac{\phi(x)}{\Phi(x)}$.[28] Thus

$$\sigma\sqrt{t}+\frac{\phi(x)}{\Phi(x)} > \frac{\phi(x-\sigma\sqrt{t})}{\Phi(x-\sigma\sqrt{t})} \Rightarrow 1-\frac{\Phi(x-\sigma\sqrt{t})}{\phi(x-\sigma\sqrt{t})}\left(\sigma\sqrt{t}+\frac{\phi(x)}{\Phi(x)}\right) < 0,$$

for any value of $\sigma\sqrt{t}$. Thus from the above derivative, it follows that $\frac{\partial}{\partial x}\left(e^{-\sigma\sqrt{t}x}\frac{\Phi(x-\sigma\sqrt{t})}{\Phi(x)}\right) < 0$, showing that the left-hand side of (A.1) is strictly decreasing in x. Thus, there is at most one solution in x to (A.1), which can be obtained numerically. Finally, as functions of the solution x, $\theta_1 = P_a(a,b|a,b,n)/(P_b(a,b|a,b,n)e^{-rt}p\Phi(x))$ and $\theta_2 = (1-a)p\theta_1\exp(-\sigma\sqrt{t}x+\sigma^2t/2)$.

[28] To prove this, we adapt Sampford (1953)'s proof about the derivative of $\frac{\phi(x)}{1-\Phi(x)}$. Specifically, consider a standard normal distribution which is top-truncated at x. Its variance is $1-\frac{x\phi(x)}{\Phi(x)}-\left(\frac{\phi(x)}{\Phi(x)}\right)^2 = 1-xh(x)-h(x)^2 > 0$. Because $h'(x) = -xh(x)-h(x)^2$, then $h'(x) > -1$.

References

Allen, J., R. Clark, B. Hickman and **E. Richert** (2019): "Resolving Failed Banks: Uncertainty, Multiple Bidding and Auction Design," Bank of Canada Staff, Working Paper.

Aradillas-Lopez, A., **P. Haile**, **K. Hendricks** and **R. Porter** (2018): "Testing Competition in U.S. Offshore Oil and Gas Lease Auctions," Working Paper, University of Wisconsin.

Armstrong, M. (1996): "Multiproduct Nonlinear Pricing," *Econometrica*, 64, 51-76.

Andreyanov, P. (2018): "Mechanism Choice in Scoring Auctions," Working Paper, UCLA.

Asker, J. and **E. Cantillon** (2008): "Properties of Scoring Auctions," *Rand Journal of Economics*, 39, 69-85.

Asker, J. and **E. Cantillon** (2010): "Procurement When Price and Quality Matter," *Rand Journal of Economics*, 41, 1-34.

Athey, S. and **P. Haile** (2007): "Nonparametric Approaches to Auctions," in *Handbook of Econometrics, Volume VI*, J. Heckman and E. Leamer, eds, Amsterdam: North Holland.

Athey, S. and **J. Levin** (2001): "Information and Competition in U.S. Forest Service Timber Auctions," *Journal of Political Economy*, 109, 375-417.

Bajari, P., **S. Houghton** and **S. Tadelis** (2014): "Bidding for Incomplete Contracts: An Empirical Analysis of Adaptation Costs," *American Economic Review*, 104, 1288-1319.

Bhattacharya, V., **A. Ordin** and **J. Roberts** (2021): "Bidding and Drilling Under Uncertainty: An Empirical Analysis of Contingent Payment Auctions," Working Paper, Duke University.

Black, F. (1976): "The Pricing of Commodity Contracts," *Journal of Financial Economics*, 3, 167-179.

Black, F. and **M. Scholes** (1973): "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, 81, 637-654.

Board, S. (2007): "Selling Options," *Journal of Economic Theory*, 136, 324-340.

Bolotnyy, V. and **S. Vasserman** (2021): "Scaling Auctions as Insurance: A Case Study in Infrastructure Procurement," Working Paper, Stanford University.

Campo, S., **E. Guerre**, **I. Perrigne** and **Q. Vuong** (2011): "Semiparametric Estimation of First-Price Auctions with Risk Averse Bidders," *Review of Economic Studies*, 78, 112-147.

Campo, S., **I. Perrigne** and **Q. Vuong** (2003): "Asymmetry in First-Price Auctions with Affiliated Private Values," *Journal of Applied Econometrics*, 18, 179-207.

Carroll, G. (2017): "Robustness and Separation in Multidimensional Screening," *Econometrica*, 85, 453-488.

Che, Y.K. (1993): "Design Competition Through Multidimensional Auctions," *Rand Journal of Economics*, 24, 668-680.

Che, Y.K. and **J. Kim** (2010): "Bidding with Securities: Comment," *American Economic Review*, 100, 1929-1935.

Cong, L.W. (2020): "Timing of Auctions of Real Options," *Management Science*, 66, 3799-4358.

Cox J.C., **S.A. Ross** and **M. Rubinstein** (1979): "Option Pricing: A Simplified Approach," *Journal of Financial Economics*, 7, 229-263.

DeMarzo, P., **M. Kremer** and **A. Skrzypacz** (2005): "Bidding with Securities: Auctions and Security Design," *American Economic Review*, 95, 936-959.

Genest, C., **K. Ghoudi** and **L.P. Rivest** (1995): "A Semiparametric Estimation Procedure of Dependence Parameters in Multivariate Families of Distributions," *Biometrika*, 82, 543-552.

Guerre, E., **I. Perrigne** and **Q. Vuong** (2000): "Optimal Nonparametric Estimation of First-Price Auctions," *Econometrica*, 68, 525-574.

Guerre, E., **I. Perrigne** and **Q. Vuong** (2009): "Nonparametric Identification of Risk Aversion in First-Price Auctions under Exclusion Restrictions," *Econometrica*, 77, 1193-1227.

Haile, P., **K. Hendricks** and **R. Porter** (2010): "Recent U.S. Offshore Oil and Gas Lease Bidding: A Progress Report," *International Journal of Industrial Organization*, 28, 390-396.

Haile P., **H. Hong** and **M. Shum** (2005): "Nonparametric Tests for Common Value in First-Price Auctions," Working Paper, Yale University.

Hanazano, M., **Y. Hirose**, **J. Nakabayashi** and **M Tsuruoka** (2018): "Theory, Identification and Estimation of Scoring Auctions," Working Paper, Meiji Gakuin University.

Hansen, R.G. (1985): "Auctions with Contingent Payments," *Americian Economic Review*, 75, 862-865.

Hendricks, K. and **R. Porter** (1988): "An Empirical Study of an Auction with Asymmetric Information," *American Economic Review*, 78, 865-883.

Hendricks, K. and **R. Porter** (1996): "The Timing and Incidence of Exploratory Drilling on Offshore Wildcat Tracts," *American Economic Review*, 86, 388-407.

Hendricks, K., **R. Porter** and **B. Boudreau** (1987): "Information, Returns and Bidding Behavior in OCS Auctions: 1954-1969," *Journal of Industrial Economics*, 35, 517-542.

Herrnstadt, E., **R. Kellogg** and **E. Lewis** (2020): "The Economics of Time-Limited Development Options: The Case of Oil and Gas Leases," Working Paper, University of Chicago.

Hull, J.C. (2017): *Options, Futures and Other Derivatives*, New York: Pearson.

Kong, Y. (2020): "Not Knowing the Competition: Evidence and Implications for Auction Design," *Rand Journal of Economics*, 51, 840-867.

Kong, Y. (2021): "Sequential Auctions with Synergy and Affiliation Across Auctions," *Journal of Political Economy*, 129, 148-181.

Krasnokutskaya, E., **K. Song** and **X. Tang** (2020): "The Role of Quality in Internet Service Markets," *Journal of Political Economy*, 128, 75-117.

Krishna, V. (2009): *Auction Theory*, Oxford: Elsevier.

Laffont, J.J., **I. Perrigne**, **M. Simioni** and **Q. Vuong** (2020): "Econometrics of Scoring Auctions," *Advances in Econometrics*, 41, 287-322.

Laffont, J.J. and **J. Tirole** (1987): "Auctioning Incentive Contracts," *Journal of Political Economy*, 95, 921-937.

Laffont, J.J. and **Q. Vuong** (1996): "Structural Analysis of Auction Data," *American Economic Review* 86, 414-420.

Lewis, G. and **P. Bajari** (2011): "Procurement Contracting with Time Incentives: Theory and Evidence," *The Quarterly Journal of Economics*, 126, 1173-1211.

Lewis, G. and **P. Bajari** (2014): "Moral Hazard, Incentive Contracts, and Risk: Evidence from Procurement," *Review of Economic Studies*, 81, 1201-1228.

Li, T., **I. Perrigne** and **Q. Vuong** (2002): "Structural Estimation of the Affiliated Private Value Model," *Rand Journal of Economics*, 33, 171-193.

Luo, Y., **I. Perrigne** and **Q. Vuong** (2018): "Structural Analysis of Nonlinear Pricing," *Journal of Political Economy*, 126, 2523-2568.

Luo, Y. and **H. Takahashi** (2021): "Bidding for Contracts under Uncertain Demand: Skewed Bidding and Risk Sharing," Working Paper, University of Toronto.

Matzkin R. (2003): "Nonparametric Estimation of Nonadditive Random Functions," *Econometrica*, 71, 1339-1375.

McAfee, R.P. and **J. McMillan** (1987): "Competition for Agency Contracts," *Rand Journal of Economics*, 18, 296-307.

Merton, R.C. (1973): "The Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science*, 4, 141-183.

Milgrom, P. and **R. Weber** (1982): "A Theory of Auctions and Competitive Bidding," *Econometrica*, 50, 1089-1122.

Nakabayashi, J. (2013): "Small Business Set-Asides in Procurement Auctions," *Journal of Public Economics*, 100, 28-44.

Pakes, A. (1986): "Patents as Options: Some Estimates of the Value of Holding European Patent Stocks," *Econometrica*, 54, 755-784.

Rendleman, R.J. and **B.J. Bartter** (1979): "Two-State Option Pricing," *Journal of Finance*, 34, 1093-1110.

Rochet, J.C. and **L.A. Stole** (2003): "The Economics of Multidimensional Screening," in *Advances in Economic Theory: Eighth World Congress*, M. Dewatripont, L.P. Hansen, and S.J. Turnovsky, eds, Cambridge University Press.

Sampford, M.R. (1953): "Some Inequalities on Mill's Ratio and Related Functions," *Annals of Mathematical Statistics*, 24, 130-132.

Sant'Anna, M. (2018): "Empirical Analysis of Scoring Auctions for Oil and Gas Leases," Working Paper, FGV EPGE Brazilian School of Economics and Finance.

Skrzypacz, A. (2013): "Auctions with Contingent Payment- An Overview," *International Journal of Industrial Organization*, 31, 666-675.

Somaini, P. (2020): "Identification in Auction Models with Interdependent Costs," *Journal of Political Economy*, 128, 3820-3871.

Takahashi, H. (2018): "Strategic Design under Uncertain Evaluations: Structural Analysis of Design-Build Auctions," *Rand Journal of Economics*, 49, 594-618.

Yoganarasimhan, H. (2016): "Estimation of Beauty Contest Auctions," *Marketing Science*, 35, 27-54.

Zagaglia, P. (2012): *Option Pricing Package*, Matlab Central File Exchange.

Table 1: Bid Statistics

	mean	p10	p25	p50	p75	p90
cash per acre ($)	1,015	224	385	712	1,154	1,903
royalty	0.23	0.18	0.20	0.23	0.25	0.27
number of bids	2.3	2	2	2	2	3
acreage	327	44	72	157	400	750

Notes: Table shows summary statistics of bids in the estimation sample. Dollar amounts are expressed in 2009 dollars and a royalty rate of 0.23 corresponds to 23%. Observations are at the bid level in the upper two rows and auction level in the lower two rows.

Table 2: Effects on Option Value and Probability of Exercise

	$V(a;\theta_1,\theta_2)$	Pr(exercise)
θ_1	+	+
θ_2	−	−
a	−	−
p	+	+
σ	+	ambiguous
t	+	ambiguous

Notes: Table shows the sign of the partial derivative of option value $V(a;\theta_1,\theta_2)$ and the probability of option exercise with respect to each variable listed.

Table 3: Reduced-Form Analysis of Bids

	log(cash per acre)	royalty
Oil Price	0.002	0.001
	(0.004)	(0.000)
Interest Rate	3.131	0.305
	(2.371)	(0.126)
log(acreage)	-0.661	-0.004
	(0.215)	(0.011)
log(acreage)^2	0.058	0.000
	(0.020)	(0.001)
Royalty Recipient not DNR	-0.342	-0.013
	(0.068)	(0.004)
Township Production Index	0.000	0.000
	(0.005)	(0.000)
Heatmap log Cash Index	0.425	
	(0.040)	
Heatmap Royalty Index		0.178
		(0.031)
Constant	8.128	0.204
	(0.598)	(0.032)
Year fixed effects	Y	Y
Number of bidder fixed effects	Y	Y
Observations	1291	1291
R^2	0.204	0.137
Adjusted R^2	0.178	0.109

Notes: Table shows OLS regression of the log cash bid per acre and royalty bid, respectively, on covariates that describe the auctioned lease. Observations are at the bid level. Standard errors are in parentheses.

Table 4: Details of Quasi-Linear Scoring Auctions

ρ	1	2	3	4	5	6	7	8	9	10
$\mathrm{E}[(b-\underline{b})/(s-\underline{s})]$	0.51	0.35	0.41	0.42	0.44	0.46	0.49	0.48	0.45	0.47
mean royalty bid	0.37	0.41	0.31	0.32	0.33	0.31	0.30	0.31	0.32	0.32
median royalty bid	0.20	0.26	0.25	0.27	0.28	0.28	0.28	0.31	0.32	0.32

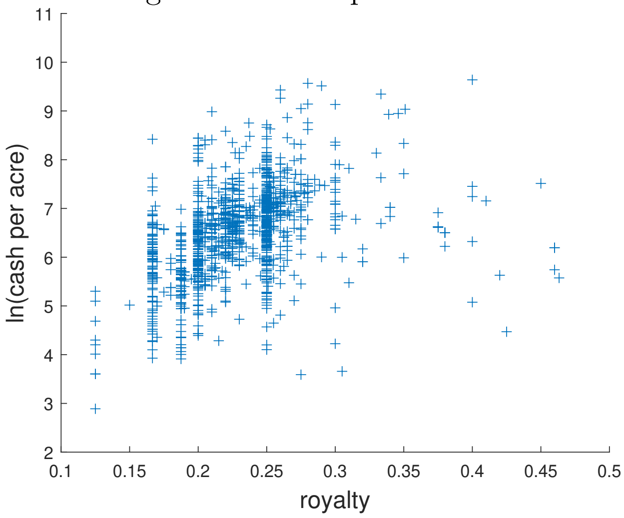
Notes: We counterfactually simulate second-score auctions with quasi-linear scoring rules of the form $S(a,b) = b - p(\omega/a^{\rho})$, where a and b are the royalty and cash components of the bid, respectively, p is the oil price, ω is a revenue-maximizing weight, and ρ determines the curvature of the scoring function. Table columns from left to right show auction outcomes associated with $\rho = 1, 2, \ldots, 10$. The first row shows the expected portion of the score that is due to the cash payment b, where $\underline{b}$ and $\underline{s}$ are the minimum of cash and score values, respectively.

Table 5: Fixed-Royalty versus Scoring Auctions

	Fixed-royalty auction	Scoring auction, $\rho = 1$
Mean royalty	30%	37%
Median royalty	30%	20%
Total government revenue	\$2,889	\$2,743
Royalty revenue	\$1,995	\$1,196
Cash revenue	\$893	\$1,547
Firm information rents	\$944	\$1,300
Same allocation as fixed-royalty	–	0.97
Pr(option exercise)	0.41	0.45
Social surplus	\$3,832	\$4,042

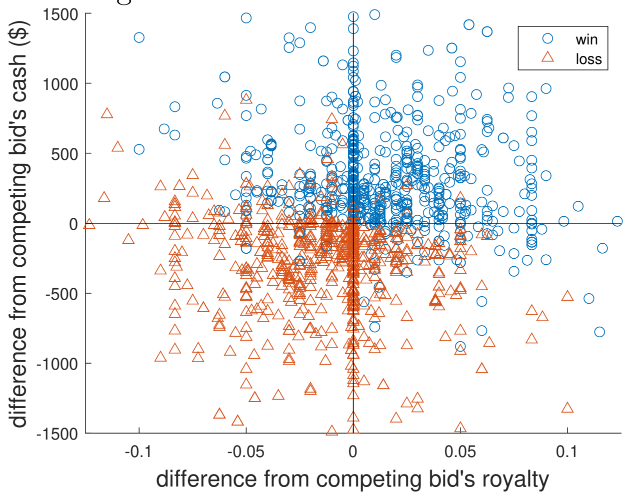
Notes: Table presents outcomes associated with counterfactual simulations of a second-price fixed-royalty auction in the first column, with revenue-maximizing fixed royalty of 30%, and a second-score scoring auction in the second column, which uses a quasi-linear scoring rule $S(a,b) = b - p(\omega/a^{\rho})$ with curvature $\rho = 1$ and revenue-maximizing weight ω. Dollars are expressed in 2009 dollars and per acre.

Figure 1: Scatterplot of Bids



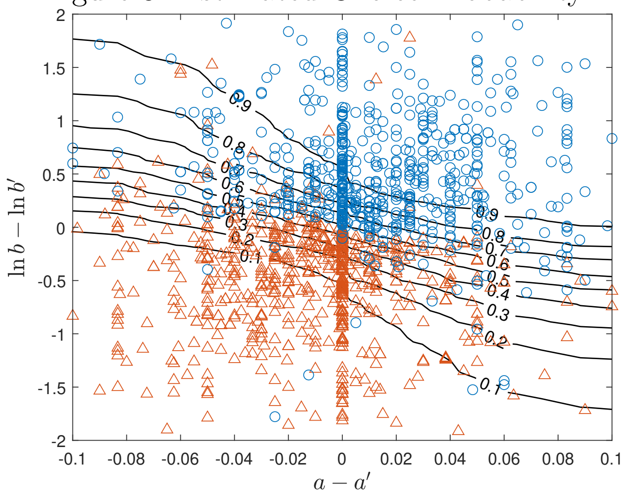
Notes: Figure displays a scatterplot of the cash-royalty bids observed in the data sample, of which the related statistics are provided in Table 1.

Figure 2: State's Choice Patterns



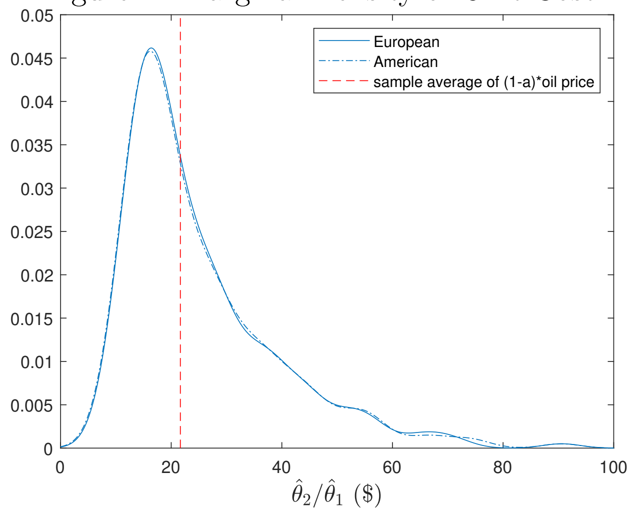
Notes: Figure visualizes the state's choice patterns. When a bid is chosen over the $n-1$ other bids in the auction, this correponds to $n-1$ observed pairwise choices. The winning and losing bids in each pairwise choice are plotted with a circle and a triangle, respectively. The x and y coordinates represent the royalty and cash components of each bid in relation to the competing bid in the pair.

Figure 3: Estimated Choice Probability



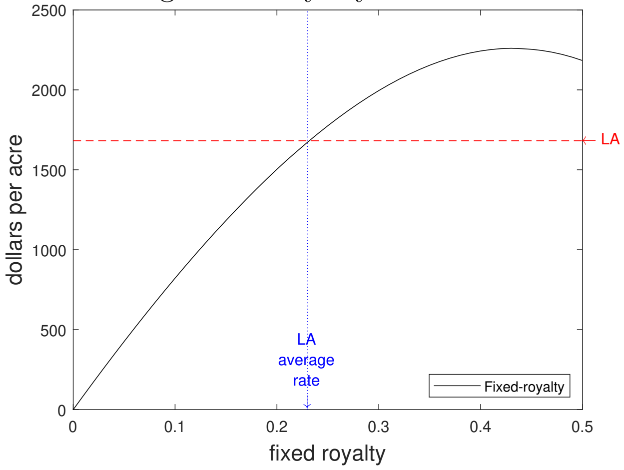
Notes: Solid lines plot level contours of the estimated probability that royalty-cash bid (a, b) is chosen over a competing bid (a', b'), as a function of $a - a'$ and $\ln b - \ln b'$, at the median value of the quality index $q(z)$. The contours range from 0.1, 0.2,..., 0.9. Bids in the observed pairwise choices used to estimate the probabilities are plotted in the background, with winning bids in circles and losing bids in triangles.

Figure 4: Marginal Density of Unit Cost



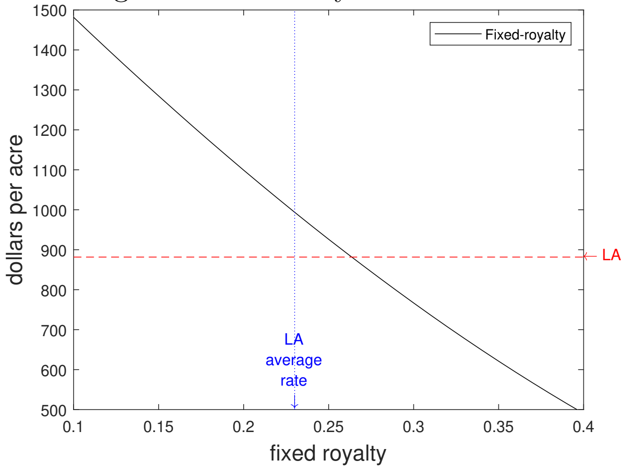
Notes: Figure plots kernel density of the estimated cost per unit of production, $\hat{\theta}_2/\hat{\theta}_1$. Solid line shows estimates from the European option model, and dash-dot line shows estimates from the American option model. Dashed vertical line marks \$21.7, the sample average of firms' per unit revenue after paying royalties.

Figure 5: Royalty Revenue



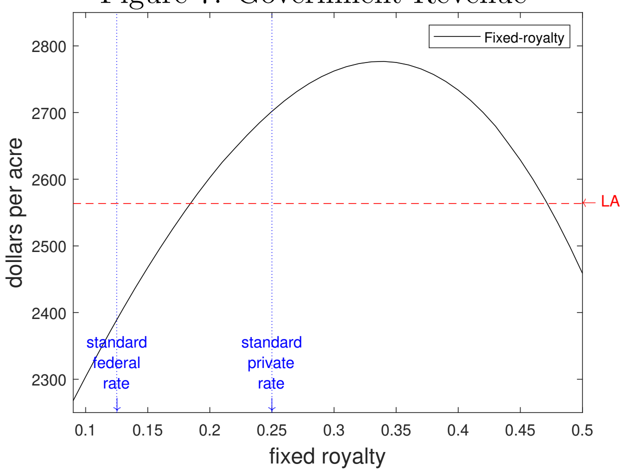
Notes: Solid line displays ex ante expected royalty revenue from counterfactual simulations of fixed-royalty auctions, as a function of the fixed royalty rate displayed on the x-axis. For comparison, the dashed horizontal line marks ex ante expected royalty revenue from observed bids in the Louisiana cash-royalty auction, and the vertical dotted line marks the average observed royalty rate resulting from that auction, 23%.

Figure 6: Cash Payment Revenue



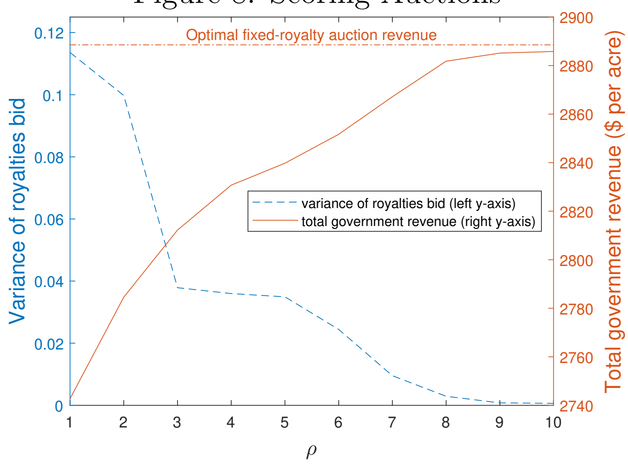
Notes: Solid line displays expected cash revenue from counterfactual simulations of fixed-royalty auctions, as a function of the fixed royalty rate displayed on the x-axis. For comparison, the dashed horizontal line marks cash revenue from observed bids in the Louisiana cash-royalty auction, and the vertical dotted line marks the average observed royalty rate resulting from that auction, 23%.

Figure 7: Government Revenue



Notes: Solid line displays ex ante expected total government revenue, which is the sum of cash and royalties, from counterfactual simulations of fixed-royalty auctions, as a function of the fixed royalty rate displayed on the x-axis. For comparison, the dashed horizontal line marks ex ante expected total government revenue from observed bids in the Louisiana cash-royalty auction. For reference, vertical dotted lines mark the standard royalty rate on federal leases, 12.5%, and the prevalent royalty rate on privately held lands, 25%.

Figure 8: Scoring Auctions



Notes: The solid line and dashed line plot simulated outcomes of a second-score scoring auction with quasi-linear scoring rule $S(a, b) = b - p(\omega/a^{\rho})$, as a function of curvature parameter ρ. Given each ρ, a revenue-maximizing weight ω is used. The dashed line is to be read by the left y-axis, and the solid line is to be read by the right y-axis. For comparison, the dash-dot horizontal line to be read by the right y-axis marks simulated revenue from a second-price fixed-royalty auction with revenue-maximizing fixed royalty (30%).