Asset Price Booms and Macroeconomic Policy: a Risk-Shifting Approach*

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Abstract

This paper uses a risk-shifting model to analyze policy responses to asset price booms. We show risk shifting leads to inefficient asset and credit booms in which asset prices can exceed fundamentals. However, the inefficiencies associated with risk shifting arise independently of whether the asset is a bubble. Given evidence of risk-shifting, policymakers may not need to determine if assets are bubbles to justify intervention. We then show that some of the main candidate interventions against asset booms have ambiguous welfare implications: Tighter monetary policy can mitigate some inefficiencies but at a cost, while leverage restrictions may raise asset prices and lead to more leveraged speculation rather than less. Policy responses are more effective when they disproportionately discourage riskier investments.

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Introduction

Policymakers have long debated how to respond to asset booms and potential bubbles, i.e. situations where asset prices surge to levels that seem to exceed the value of dividends these assets are expected to yield. One view, summarized in Bernanke and Gertler (1999) and Gilchrist and Leahy (2002), holds that policymakers should hold off in these cases, acting only if asset prices collapse and drag down economic activity. An alternative view, summarized in Borio and Lowe (2002) and reinforced in later work by Jorda, Schularick, and Taylor (2015) and Mian, Sufi, and Verner (2017), argues these asset booms are likely to end in dire financial crises and recessions, especially when they coincide with credit booms. By intervening to dampen asset prices during booms, they reason, policymakers might mitigate the eventual crash.

The severity of the Global Financial Crisis in 2007 and the difficulty central banks faced in providing stimulus in its wake led many policymakers to lean toward a more proactive response to asset booms. This shifted the debate from *whether* to intervene to *how* to intervene. The leading proposals for intervention include monetary tightening and macroprudential regulation. Yet both approaches have also been criticized. For example, Svensson (2017) argues the costs of monetary tightening during asset booms exceed its benefits. In the opposite direction, Stein (2013) argues that even if regulatory policy could work in principle, in practice it is likely to be circumvented through clever financial engineering.

This paper revisits the question of how policy should respond to asset booms. It does so through the lens of a risk-shifting model, one in which the lenders who ultimately finance asset purchases cannot gauge the default risk they face from any individual borrower. We focus on risk shifting because asset booms often feature extensive lending against assets that are hard for lenders to evaluate, either because these assets are tied to new and imperfectly understood technologies (dot-com, tranched securities, blockchain) or because they are valued idiosyncratically, like housing, making it hard to distinguish committed buyers from speculators who will walk away if prices fall.[1] To be sure, there is a vast literature on asset booms and bubbles that abstracts from risk shifting, so this feature is not essential for booms or bubbles to arise. However, as we elaborate in the Conclusion, risk-shifting can naturally emerge in these alternative models. Our analysis should thus be viewed as complementary to alternative models of bubbles, not as a substitute.

The idea of risk shifting goes back at least to Jensen and Meckling (1976) and Myers (1977), and has become central to a large literature in finance. Some of the papers in this literature have proposed private solutions to the risk-shifting problem, including warrants as in Green (1984), managerial compensation as in John and John (1993), and unlimited liability as in Saunders and Wilson (1995). We consider the case where contracting frictions make these private solutions infeasible, raising the question of whether policy interventions such as monetary policy, credit limits, and leverage restrictions can help when alternative private remedies are unavailable. In using risk-shifting models to study macroeconomic phenomena such as

[1] While we describe situations where limited information is an exogenous feature of an asset, Asriyan, Laeven, and Martin (2018) argue asset booms can reduce the incentive to screen borrowers, so information about assets deteriorates endogenously.

asset booms rather than the decisions of individual firms, we follow Aikman, Haldane, and Nelson (2015) and Martinez-Miera and Repullo (2017) who use these models to analyze credit booms and banking crises.

Our model builds on existing work on risk-shifting and asset pricing. Allen and Gorton (1993) were the first to show that risk-shifting allows asset prices to exceed fundamentals, an idea further developed in Allen and Gale (2000), Barlevy (2014), Dow and Han (2015), Dubecq, Mojon, and Ragot (2015), and Bengui and Phan (2018). We contribute to this literature in two ways. First, we use a general equilibrium setup that can incorporate policy interventions absent in previous work. Second, we introduce costly default, allowing us to capture a fall in output when an asset boom ends. Hoggarth, Reis, and Saporta (2002) and Reinhart and Rogoff (2009) estimate that asset price crashes are associated with a fall in GDP per capita of 9-16%; Atkinson, Luttrell, and Rosenblum (2013) estimate even larger cumulative losses for the US in the recent crisis. Economists have identified various reasons for why output falls when asset prices collapse. For example, financial intermediaries who lent against assets may not be able to finance new investments when they face an overhang of debt against those assets. Alternatively, indebted households may delever when the assets they borrowed to buy fall in price, and with nominal price rigidity such deleveraging can reduce aggregate demand and output.[2] Default costs similarly leave agents poorer when asset prices collapse, although this is because lenders must use resources to recover their obligations. Allowing for such a contraction, even in this stylized way, has important implications for policy.

At the heart of our model is an information asymmetry in which borrowers know the risks of their investments better than lenders. This encourages them to borrow and gamble on risky assets, knowing lenders will bear the losses and default costs if the gamble fails. As speculators buy up assets, they drive up asset prices and drive down the expected return on assets. Our model gives rise to equilibria that are broadly consistent with historical asset booms: Asset prices appear excessive and can grow exponentially, the asset boom is accompanied by a credit boom, borrowing to buy risky assets is relatively cheap, and realized returns on assets during the boom are high. Finally, the boom can feature a bubble in the sense that asset prices exceed fundamentals. But the boom in our model is inefficient even if it does not give rise to a bubble. The goal of policy is not to push prices toward fundamentals, but to correct distortions that arise when those who buy assets do not bear the full consequences of their purchases, i.e., the default costs borne by lenders. In contrast to Bernanke and Gertler (2001) who argue policymakers should not intervene when they are unsure if they face a bubble or not, in our model policymakers don't need to know if asset booms represent bubbles in order to intervene, at least if they have evidence of underlying risk-shifting. This provides a microfoundation for the Borio and Lowe (2002) view on intervention.

While our model suggests a role for intervention, the main remedies policymakers have focused on have ambiguous welfare effects and do not fully resolve the underlying inefficiency in the model. This is because

[2] See Phillipon (2010) on debt overhang and Korinek and Simsek (2016) and Farhi and Werning (2016) on aggregate demand externalities and deleveraging. Rognlie, Shleifer, and Simsek (2018) suggest another channel involving investment overhang, whereby a glut of assets during the boom such as housing dampens the production of new assets after the crash. The latter is tricky to capture in our setup, since we assume assets are either endowed or created at date 0 but not thereafter.

agents in our model undertake two types of investments, speculation and a safe activity, and policy interventions affect both rather than just speculation. For example, tighter monetary policy alleviates excessive leverage but also discourages productive investment. We show this might be mitigated if policymakers promise to tighten only if the boom continues (and, by implication, ease if it ends) rather than tighten immediately. Likewise, leverage restrictions can be counterproductive by discouraging safe investments and shifting resources toward speculation. Unlike with monetary tightening, promising to restrict future leverage only makes things worse. Our takeaway is that policies for fighting booms must be carefully designed to disproportionately deter speculation. While monetary tightening and leverage restrictions can both increase welfare, the two are not equivalent: Tighter money raises interest rates, while leverage restrictions reduce demand for credit and lower interest rates. This offers a contrast to recent work by Caballero and Simsek (2019) that emphasizes the features monetary tightening and leverage restrictions share.

The paper is organized as follows. Section 1 introduces the basic setup assuming assets are riskless. We build on this framework in Section 2 to study risky assets, and show these give rise to asset booms and, in some cases, bubbles. Section 3 shows that the equilibrium of our model is constrained inefficient. Section 4 considers monetary policy and Section 5 considers leverage restrictions. Section 6 concludes.

1 Credit, Production, and Assets

Our analysis requires credit, production, and assets. We begin with the simple case where the asset is riskless. We build on this setup in Section 2 to allow for risky assets, which leads to credit and asset booms.

Consider an overlapping generations economy where agents live for two periods and only value consumption when old. That is, agents born at date t value consumption c_t and c_{t+1} at dates t and $t+1$ at

$$u\left(c_t, c_{t+1}\right) = c_{t+1} \tag{1}$$

There is a cohort of old agents at date 0 who wish to consume that period. New cohorts of agents arrive at each date $t = 0, 1, 2, ...$ Any new cohort consists of two types. The first, whom we call savers, are endowed with a total of e units of the good while young. They cannot produce or store goods. Given their preferences, they goal is to convert the endowment e when young into consumption when old. The second type, whom we call entrepreneurs, can convert a unit of the good at date t into $1+y$ units of the good at date $t+1$ where $y > 0$, but only up to a finite capacity of one unit of input. Each entrepreneur is endowed with $w < 1$ goods while young. As this is below their productive capacity, there is scope for savers to fund the intertemporal production of entrepreneurs.

In principle, w and y can vary across entrepreneurs. For most of the analysis, we assume $w = 0$ for all entrepreneurs. They must therefore borrow all of their inputs. As will become clear in Section 5 when we allow for $w > 0$, allowing entrepreneurs to have wealth greatly complicates the analysis without changing the qualitative results. We do assume y varies across entrepreneurs. Let $n(y)$ denote the density of entrepreneurs with productivity y. We assume $n(y) > 0$ for all $0 < y < \infty$ and

$$e < \int_0^\infty n(y)\,dy < \infty \tag{2}$$

Condition (2) implies entrepreneurs collectively require more inputs than savers are endowed with.

Finally, the old at date 0 are endowed with a mass 1 of assets. Assets yield a constant real dividend $d > 0$ per period. In the next section we consider the more interesting case where the dividend is stochastic.

Each period, savers can use their endowment e to buy assets and to fund production, either of which will allow them to consume when old. We assume trade between savers and entrepreneurs is subject to several frictions. At this point, with deterministic dividends, these frictions are largely irrelevant. They will matter once we allow stochastic dividends in Section 2.

1. **Transaction Costs**: Any agent who reaches out to savers to secure financing incurs a fixed utility cost ϕ, where we let $\phi \to 0$.

2. **Information Frictions**: Savers cannot monitor if the agents they fund buy assets or produce. They also cannot observe any wealth the agent has that is not associated with the project the lender finances.

3. **Contracting Frictions:** Trade is restricted to non-contingent debt contracts, i.e., for each unit of funding agents receive at date t they must pay a fixed amount $1 + R_t$ at date $t + 1$.

4. **Default Costs:** If borrowers fail to pay their obligation, lenders can collect any proceeds from the project borrowers invested in, but the seizure wastes Φ resources per unit invested in the project.

A positive transaction cost ϕ ensures agents will not borrow for ventures that will lead them to default with certainty. We take the limit as $\phi \to 0$ to avoid keeping track of this cost. This cost eliminates equilibria in which agents borrow for strictly unprofitable purposes out of sheer indifference.

The information frictions we assume imply that savers cannot prevent their borrowers from buying assets instead of producing. This will not be an issue with deterministic dividends given transaction costs make buying assets unprofitable in equilibrium. When we allow for stochastic dividends in the next section, though, some borrowers will choose to buy assets. While buying assets and production are distinct activities in our model, we view this as a metaphor for situations in which borrowers buy assets for different purposes and lenders cannot tell the risk they face from any given borrower. For example, with new technologies, lenders cannot distinguish workable applications of new assets from speculative ventures. Likewise, mortgage lenders cannot distinguish illiquid agents who value homeownership and earn a surplus from borrowing, much as entrepreneurs in our model earn $1+y$, and speculators who buy houses intending to default if prices fell.[3]

[3] Of course, the surplus for agents who value homeownership is not a constant and depends on the current price of housing. For an example of a proper risk-shifting model of housing, see Barlevy and Fisher (forthcoming).

Assuming wealth is unobservable implies borrowers face limited liability, since lenders can only go after the resources they know about. Essentially, agents are restricted to non-recourse loans, or, alternatively, borrowers can resort to shell entities to limit their liability to the returns on the project they borrow for.

The contracting frictions we assume are motivated by the empirical prevalence of non-contingent debt. However, this restriction on contracts plays an important role in preventing savers from screening borrowers who intend to speculate. When we allow for stochastic dividends in the next section, savers could discourage speculation by stipulating a higher repayment when the return on the asset is high and a lower one when the return is low. Imposing noncontingent debt rules out such arrangements. While we do not model the friction on contracting, we implicitly view it as a high cost to a third party of verifying contingencies.

Finally, our assumption that default costs are proportional to the scale of the project captures the idea that auditing a borrower requires inspecting their entire project. Although we model these as recovery costs, we view them as a stand-in for various mechanisms that lead output to fall when asset prices collapse.

An equilibrium in our economy consists of paths for asset prices $\{p_t\}_{t=0}^{\infty}$ and interest rates on loans $\{R_t\}_{t=0}^{\infty}$ that ensure asset and credit markets clear when agents optimize. To simplify the exposition, suppose p_t and R_t are deterministic. We confirm in Appendix A there are no equilibria where prices are stochastic. To solve for an equilibrium, we need supply and demand for assets and credit. These are easily characterized. Agents in their last period of life neither supply nor demand credit. They do own all assets, though, and will sell them if the asset price $p_t > 0$. Young savers are the only ones who can lend. They compare the return to lending $1 + R_t$ with the return to the asset $1 + r_t \equiv \frac{d+p_{t+1}}{p_t}$ and invest in whatever offers the highest return. Young entrepreneurs choose whether to borrow to produce, and all young agents choose whether to borrow to buy assets. Agents will borrow for any activity they expect to profit from. Entrepreneurs find it profitable to borrow to produce iff their productivity exceeds the cost of borrowing, i.e. iff $y \geq R_t + \phi$. In the limit as $\phi \to 0$ entrepreneurs will borrow to produce iff $y \geq R_t$.

Savers use their endowment to either buy assets or make loans. Their borrowers in turn either produce or buy assets. Hence, the endowment is ultimately used either to finance production or buy assets, implying

$$\int_{R_t}^{\infty} n(y)\, dy + p_t = e \tag{3}$$

Since we assume $n(y) > 0$ for all $y \geq 0$, there is a unique interest rate $R_t = \rho(p_t)$ that satisfies (3) for any asset price p_t, where $\rho(p_t)$ is increasing in p_t. Intuitively, a higher p_t reduces the amount of goods available for productive investment, so the interest rate on loans R_t must rise to lower demand from entrepreneurs.

Next, we argue the equilibrium interest rate $1 + R_t$ will equal the return on the asset $1 + r_t \equiv \frac{d+p_{t+1}}{p_t}$. If $R_t < r_t$, agents can earn profits by borrowing to buy assets at a large enough scale to cover the fixed cost ϕ. Demand for borrowing would then be infinite, yet the supply of credit is at most e, so the credit market would fail to clear. If $R_t > r_t$, savers earn more from lending than from buying the asset. They would thus not buy the asset. Nor would anyone borrow to buy the asset. But the old would want to sell their assets, since the equilibrium price p_t of a dividend-paying asset must be positive. Hence, the asset market would not clear if $R_t > r_t$. For both credit and asset markets to clear, we must have

$$1 + R_t = 1 + r_t = \frac{d + p_{t+1}}{p_t} \tag{4}$$

Note that (4) holds regardless of ϕ. When $\phi > 0$, no agent would borrow to buy assets that offer the same return as the interest rates on loans given the transaction costs involved. Taking the limit as $\phi \to 0$ selects the equilibrium where agents only borrow to produce. Substituting (3) into (4) implies

$$\begin{aligned} p_{t+1} &= (1 + \rho(p_t)) p_t - d \\ &\equiv \psi(p_t) \end{aligned} \tag{5}$$

where $\psi'(p_t) > 1$, $\psi(0) = -d < 0$, and $\lim_{p \to e} \psi(p) = \infty$. The graph of $\psi(p)$ is illustrated in Figure 1 together with the 45° line. The two intersect at the unique value p^d for which $p^d = \psi\left(p^d\right)$. For any initial condition, the law of motion $p_{t+1} = \psi(p_t)$ defines a unique path of asset prices. For any initial condition other than $p_0 = p^d$, the path will reach in finite time a value that is either negative or exceeds e, neither of which can be an equilibrium. Hence, the unique equilibrium is one where the economy is at its steady state, i.e., $p_t = p^d$ and $R_t = \rho\left(p^d\right) \equiv R^d$ for all t. Substituting $p_t = p_{t+1} = p^d$ in (5) implies

$$d = \rho\left(p^d\right) p^d$$

The right hand side is increasing in p^d. It follows that the equilibrium price p^d is increasing in d. Graphically, a larger d will shift the curve $p_{t+1} = \psi(p_t)$ in Figure 1 down, and so the steady state p^d will rise.

In Appendix A, we confirm there is no equilibrium with stochastic prices, implying the following:

Proposition 1 *When $d_t = d$ for all t, in the limit as $\phi \to 0$, the unique equilibrium features a constant price $p_t = p^d$ and constant interest rate $R_t = \rho(p^d) = R^d$. Only entrepreneurs with productivity $y \geq R^d$ produce, agents only borrow to produce and not to buy assets, and only savers hold assets.*

In equilibrium, the return on assets and loans are equal. Denote the common return to both activities by $R^d = \rho\left(p^d\right)$. Consider the present value of dividends discounted at this return. This is given by

$$f_t \equiv \sum_{j=1}^{\infty} \left(\tfrac{1}{1+R^d}\right)^j d = d/R^d = p^d$$

The value of dividends discounted at the return agents earn on their savings coincides with the price of the asset. When $d_t = d$ for all t, the asset will not be associated with a bubble. In short, when the asset is riskless, agents only borrow to produce and assets are properly priced. This will offer a contrast to what happens in the next section when we assume dividends are stochastic

Remark 1: While we assume a single asset, we can easily allow multiple riskless assets. Suppose there were J assets indexed $j = 1, ..., J$, each with fixed supply of 1 but with different fixed dividends d_j. Let p_{jt}

denote the price of the j-th asset at date t. Define $d \equiv \sum_{j=1}^{J} d_j$ as the total dividends from all J assets and $p_t \equiv \sum_{j=1}^{J} p_{jt}$ as the combined value of all J assets. Resources that don't finance production will be used to buy assets, so (3) continues to hold. In addition, the return on each asset $1 + r_{jt} \equiv \frac{d_j + p_{j,t+1}}{p_{jt}}$ must equal the interest rate on loans $1 + R_t$. Combining these equalities implies (4). Hence, the equilibrium conditions that govern p_t and R_t are the same as in the one asset case, but p_t now represents the total value of all assets, each of which offers the same return R_t. ■

Remark 2: With some changes, we can also allow the set of assets to grow over time. This will become more relevant in the next section when booms are possible and can be triggered by the arrival of new assets. Suppose each period's old receive an endowment of new assets of size 1. Assets start paying dividends one period after arrival. For aggregate dividends to remain constant, dividends on existing assets must decay over time. Let d_{st} denote the dividend at date t on assets that arrived at date s, and suppose

$$d_{st} = \begin{cases} (1-\theta)^{t-1} d & \text{if } s = 0 \\ (1-\theta)^{t-(s+1)} \theta d & \text{if } s = 1, 2, 3, ... \end{cases} \quad \text{for } t \geq s+1$$

where $\theta \in (0, 1)$. The initial dividend on new assets is θd, and the dividend on any asset decays exponentially towards 0. The dividends are set to ensure total dividends $\sum_{s=0}^{t-1} d_{st}$ each period sum to d. Let p_{st} denote the date-t price of the asset that arrived by date s, and set $p_t = \sum_{s=0}^{t} p_{st}$ as the total value of all assets that exist at date t. The market clearing condition (3) still holds. The return on each asset $1 + r_{st} \equiv \frac{d_{s,t+1} + p_{s,t+1}}{p_{st}}$ will equal the interest rate on loans $1 + R_t$. Aggregating over all assets at date t and remembering to subtract new assets at date $t+1$ from p_{t+1} yields the following alternative to (4):

$$1 + R_t = \frac{d + (p_{t+1} - p_{t+1,t+1})}{p_t}$$

The equilibrium value of all assets p_t will be constant and equal to $\frac{d}{R^d + \theta}$, where R^d denotes the equilibrium interest rate on loans. The price of any individual asset equals $p_{st} = \frac{d_{s,t+1}}{d} p_t = \frac{d_{s,t+1}}{R^d + \theta}$. ■

2 Risky Assets, Credit Booms, and Bubbles

We now turn to the case where dividends are stochastic. For this, we return to assuming there is a single asset. Let the dividend on this asset follow a regime-switching process such that the dividend d_t starts at $D > d$ when $t = 0$ and then switches to d with a constant probability $\pi \in (0, 1)$ each period if it has yet to switch. Once the dividend falls to d, it will remain equal to d forever.

An equilibrium still consists of paths for asset prices $\{p_t\}_{t=0}^{\infty}$ and loan rates $\{R_t\}_{t=0}^{\infty}$ that ensure asset and credit markets clear at all dates t and for both values of d_t. But since agents might now borrow both to buy assets and to produce, we also need to track the share of lending used to buy assets, $\{\alpha_t\}_{t=0}^{\infty}$.

In what follows, it will be convenient to distinguish for each date t whether d_t still equals D or switched to d. If $d_t = D$, agents will be unsure about the dividend at date $t+1$. If $d_t = d$, agents know the asset will pay a dividend of d at date $t+1$. Let $\left(p_t^D, R_t^D, \alpha_t^D\right)$ denote equilibrium values if $d_t = D$ and $\left(p_t^d, R_t^d, \alpha_t^d\right)$ denote equilibrium values if $d_t = d$. Once dividends fall, the equilibrium will be as in Section 1, with $p_t^d = p^d$, $R_t^d = R^d$, and $\alpha_t^d = 0$ for all t. We only need to solve for $\left\{\left(p_t^D, R_t^D, \alpha_t^D\right)\right\}_{t=0}^{\infty}$.

To solve for these values, observe that the endowment e is still used either to fund production or buy assets, so condition (3) still holds. We now look for an analog to condition (4). In Appendix A, we show that if $d_t = D$, the return $\frac{d_{t+1}+p_{t+1}}{p_t^D}$ will be higher if $d_{t+1} = D$ than if $d_{t+1} = d$, i.e. if the dividend remains high. We now argue that the interest rate on loans $1 + R_t^D$ equals this maximal return on the asset, i.e.,

$$1 + R_t^D = \frac{p_{t+1}^D + D}{p_t^D} \tag{6}$$

Suppose instead that $1+R_t^D < \frac{p_{t+1}^D+D}{p_t^D}$. Agents would earn positive profits if they borrowed to buy the asset and it turned out $d_{t+1} = D$ and nonnegative profits if $d_{t+1} = d$ given they can always default. Demand to borrow would then be infinite, but supply is finite. The credit market would fail to clear. Next, suppose $1 + R_t^D > \frac{p_{t+1}^D+D}{p_t^D}$. In this case, no agent would borrow to buy the asset knowing they would default. The only agents who borrow are entrepreneurs with productivity $y > R_t^D$, and they repay for sure. The return to lending is then $1 + R_t^D$, which exceeds the highest return on the asset. Savers should then prefer lending to buying the asset. But then the asset market wouldn't clear given nobody buys assets yet the old want to sell their holdings. For both the asset and credit market to clear, we need $1 + R_t^D = \frac{p_{t+1}^D+D}{p_t^D}$.

Condition (6) is identical to the equilibrium condition for an asset that offers a constant dividend $d_t = D$ for all t. From the previous section, we know there is a unique path $\left\{p_t^D, R_t^D\right\}_{t=0}^{\infty}$ that satisfies both this condition and (3). The equilibrium price p_t^D is thus constant and equal to p^D, where p^D solves

$$\rho\left(p^D\right) p^D = D$$

The interest rate on loans R_t^D is given by $\rho\left(p^D\right) \equiv R^D$. The asset thus trades as if $d_{t+1} = D$ with certainty, even though d_{t+1} may equal $d < D$ with probability π that can be arbitrarily close to 1.

The reason the asset is priced this way is as follows. Lenders cannot monitor borrowers, nor can they use contingent contracts to screen out speculators. This allows agents to blend in with entrepreneurs and borrow to buy risky assets. Since borrowers can default and shift any losses to creditors if the asset return is low, they only care about the maximal return on the asset. They will drive the price of the asset to the price consistent with the maximal dividend D, even if this realization is unlikely. We discuss below whether this implies the asset is a bubble in the sense that its price exceeds its fundamental value.

The only part we still need to solve is the share of lending used to buy assets, $\left\{\alpha_t^D\right\}_{t=0}^{\infty}$. We leave the derivation to Appendix A and briefly summarize the results. As we show in Figure 2, the value of α_t^D depends on the size of the default cost Φ. There exists a $\Phi^* > 0$ such that if $\Phi \leq \Phi^*$, only borrowers buy the asset while savers only lend, implying $\alpha_t^D = \frac{p^D}{e}$. Intuitively, when $\Phi = 0$, lending to a borrower who buys the asset is equivalent for the lender to buying the asset outright, while lending to a borrower who produces yields a sure return of $1 + R^D$ equal to the maximal return on the asset. Savers will thus strictly prefer lending to buying the asset outright. As Φ rises, lending becomes less and less profitable, until eventually savers turn indifferent between lending and buying the asset. In this case, savers buy some but not all of the asset, and α_t^D is below $\frac{p^D}{e}$ but still positive.

To see why $\alpha_t^D > 0$, note that if agents only borrowed to produce, no one would default. Since the equilibrium interest rate on loans R^D equals the maximum return on the asset, lending would be more profitable than buying the asset. But then nobody would buy the asset, which cannot be an equilibrium given the old want to sell all their assets. This logic also helps explain why the equilibrium value of α^D is unique even though agents are indifferent about borrowing to speculate. While agents earn zero from speculation, the amount of speculation in equilibrium must ensure that enough agents are willing to buy all the assets so the asset market clears, or else that savers are willing to both lend and buy the asset so both asset and credit markets clear. By contrast, who borrows to speculate in equilibrium is indeterminate. It could be less productive entrepreneurs with $y < R^D$ who don't produce, but it could equally be savers and productive entrepreneurs who can hide their wealth. In contrast with the equilibrium in Section 1, lending now finances both production and speculation. The equilibrium can be summarized as follows:

Proposition 2 *When d_t follows a regime-switching process, as $\phi \to 0$, the unique equilibrium is given by*

$$(p_t, R_t) = \begin{cases} \left(p^D, R^D\right) & \text{if } d_t = D \\ \left(p^d, R^d\right) & \text{if } d_t = d \end{cases}$$

The share of lending used to buy assets α_t when $d_t = d$ equals 0 and when $d_t = D$ is given by

$$\alpha_t = \alpha^D = \begin{cases} \frac{p^D}{e} & \text{if } \Phi \leq \Phi^* \\ \frac{D+p^D-d-p^d}{D+p^D-d-p^d+\Phi p^D} & \text{if } \Phi > \Phi^* \end{cases}$$

and $\Phi^* \equiv \left(\frac{e}{p^D} - 1\right) \frac{D+p^D-d-p^d}{p^D}$.

Remark 3: In our model, there are no safe assets when $d_t = D$. We can add a technology with return r^f to mimic a safe real asset. If r^f is below the expected equilibrium return on lending, the safe technology would go unused. If r^f exceeded the expected return to lending, savers would shift from lending to the technology, depressing the price of the risky asset p^D and increasing the expected return on loans. This mirrors what we find in Section 4 when we allow agents to make deposits with a central bank: If the central bank offered to pay a higher rate on (safe) deposits, the asset price p^D would fall. The fact that the boom occurs in the absence of safe assets or when the return to such assets is low is reminiscent of work by Aoki, Nakajima, and Nikolov (2014), Caballero and Farhi (2017), and Acharya and Dogra (2018) who shows these same conditions lead to bubbles. But the mechanism is different. In those papers, bubbles arise when the safe interest rate falls below the economy's growth rate. Here, a low r^f encourages savers to "search for yield" and tolerate risky lending, similarly to Martinez-Miera and Repullo (2017). ■

We now argue our model captures some key features of the booms documented in Borio and Lowe (2002), Jorda, Schularick, and Taylor (2015), and Mian, Sufi, and Verner (2017). That is, we show the equilibrium involves asset and credit booms, may involve bubbles, features high realized returns to savings even as borrowing is relatively cheap, and the asset boom ends with a crash and costly defaults.

Asset Booms: We begin with asset prices. The equilibrium price p_t^D while $d_t = D$ will be the same as in an economy in which dividends equal D forever. Recall that p^d is increasing in d, so $p^D > p^d$. Our economy thus features an initially high asset price that eventually collapses.

Actual asset booms, however, feature rapid asset price growth despite otherwise regular dividends, not high but]stable prices sustained by high dividends. We can generate a more realistic-looking boom in our model if we allowed dividends to rise over the course of the initial regime. Formally, suppose there were some finite date T such that dividends would start at d and jump to D only if we remained in the initial regime until date T, i.e., $d_t = d$ for $t < T$ and $d_t = D$ for $t \geq T$. Once we leave the initial regime, dividends will equal d forever.[4] This specification accords with how new technologies promise eventual rather than immediate profits, and how rents in boom markets are stable even as house prices surge. If we remained in the initial regime, the equilibrium from date T on would be as in Proposition 2. Between dates 0 and T, the equilibrium path of prices $\left\{p_t^D\right\}_{t=0}^T$ would satisfy the law of motion

$$p_{t+1}^D = \left(1 + \rho\left(p_t^D\right)\right) p_t^D - d \equiv \psi^d\left(p_t^D\right)$$

with the boundary condition that $p_T^D = p^D$. We can use Figure 1 to solve for the path consistent with this boundary condition. Essentially, since $p^D > p^d$, the price p_t^D must start above p^d at date 0 and rise towards p^D at date T. The trajectory for the price p_t^D conditional on staying in the initial regime is given in Figure 3. The asset price follows an explosive path that grows at an increasing rate even as dividends remain constant. We can confirm that the price grows faster than the expected return on savings, so asset price growth is not due to discounting. Rather, prices grow to compensate agents for the risk of capital losses should the initial regime end before date T. Our framework can thus generate booms with rising prices and constant dividends, but this would require us to solve the entire price path $\{p_t\}_{t=0}^T$. For analytical convenience, we will continue to assume d_t is constant within each regime.

Our setup also abstracts from how booms start. One might have thought we could start in the low dividend regime where $d_t = d$ and have a boom emerge when we transit stochastically to the regime where $d_t = D$. But in that case, agents would borrow to buy assets in the initial low regime, gambling that the high dividend regime would start next period. The asset would trade at p^D before we enter the high dividend regime, and the boom would be present at the beginning. This is reminiscent of the Diba and Grossman (1987) result that asset bubbles cannot emerge suddenly, but must be present from the very inception of the asset. Martin and Ventura (2012) show one can get around this result by allowing for the arrival of new assets that cannot be traded in advance and let bubbles emerge on these new assets. We could similarly allow for new assets as per Remark 2. Most new assets would pay a predictable dividend

[4] This setup is reminiscent of Zeira (1999). He assumed dividends grow until a stochastic date. In both his setup and ours, dividends rise more the longer the initial regime survives.

that decays over time, but new assets arrive periodically that pay temporarily high dividends.[5] With some modifications, then, our model can allow periodic booms. Again, we do not pursue this approach here.

Credit Booms: We now show that the asset boom coincides with a boom in borrowing against assets. When $d_t = D$, the amount agents borrow to buy assets is given by

$$\frac{\alpha^D}{1-\alpha^D}\int_{R^D}^{\infty} n(y)\,dy \tag{7}$$

By contrast, no one borrows to buy assets when $d_t = d$.

Since informational frictions imply that it is hard to distinguish between borrowing to buy assets and borrowing for productive purposes, arguably the relevant empirical measure is not borrowing against assets but total borrowing. The total amount agents borrow to buy assets or produce is given by

$$\frac{1}{1-\alpha_t}\int_{R_t}^{\infty} n(y)\,dy$$

Since $\alpha^D > 0 = \alpha^d$, the term $\frac{1}{1-\alpha_t}$ is higher during the boom. At the same time, with $R^D > R^d$, the integral $\int_{R_t}^{\infty} n(y)\,y$ is smaller when $d_t = D$ than when $d_t = d$. Total lending can therefore rise or fall when the boom ends. Recall we find there a cutoff Φ^* such that if $\Phi \leq \Phi^*$, savers strictly prefer lending. In that case, total lending will equal e when $d_t = D$ but $e - p^d$ when $d_t = d$, so total lending is higher in the boom. When $\Phi > \Phi^*$, total lending is less than e. If lending to entrepreneurs when the boom ends rises by more than what savers were lending to speculators during the boom, total lending would rise. The asset boom is thus associated with a boom in lending against assets, and, unless Φ is large, a boom in total lending.

Asset Bubbles: We next turn to whether the asset and credit boom in our model is associated with a bubble, in the sense that the asset's price exceeds the present expected discounted value of its dividends. Empirical booms are often suspected to be bubbles, even if this is hard if not impossible to verify. But in the model we can compute the fundamental value of the asset to see if the boom features a bubble.

To properly define the fundamental value of the asset, let us distinguish several rates of return when $d_t = D$. First is the interest rate on loans R^D that borrowers are asked to repay. Recall that $1 + R^D$ is equal to the maximal return on the asset, i.e.,

$$1 + R^D = 1 + \tfrac{D}{p^D} \tag{8}$$

This is not the rate lenders will expect to collect in the boom, since a fraction $\alpha^D > 0$ of lending is used to buy assets and may result in default. Lenders instead expect to earn $1 + \overline{R}^D$, defined as

$$1 + \overline{R}^D = \left(1 - \alpha^D \pi\right)\left(1 + \tfrac{D}{p^D}\right) + \alpha^D \pi \left(\tfrac{d+p^d}{p^D} - \Phi\right) \tag{9}$$

[5] To ensure the return on assets is riskless outside of booms may require one-off changes in the dividends of existing assets if the assets that arrive are risky to ensure the return on existing assets is the same as it would be if the new assets were riskless.

Finally, the expected return to buying the asset is given by

$$1+\overline{r}^{D}=\tfrac{(1-\pi)\left(D+p^{D}\right)+\pi\left(d+p^{d}\right)}{p^{D}} \tag{10}$$

These three returns can be ranked, with $R^D > \overline{R}^D \geq \overline{r}^D$. The last inequality follows from the fact that if the expected return to buying the asset $\overline{r}^D$ exceeded $\overline{R}^D$, agents would prefer to buy assets than lend. But this is inconsistent with how demand for credit is always positive given the unbounded support for y.

We need to take a stand on which rate to discount dividends when defining the fundamental value. If an agent had a unit of resources, the best she can expect to earn on it is $\overline{R}^D$, since in equilibrium she can do no better than lending out her unit. This suggests using $\overline{R}^D$ as the discount rate. Since the equilibrium is stationary, the fundamental value of the asset f^D satisfies the recursive equation

$$f^{D}=\tfrac{(1-\pi)\left(D+f^{D}\right)+\pi\left(d+p^{d}\right)}{1+\bar{R}^{D}} \tag{11}$$

Equation (11) discounts dividends at rate $1+\overline{R}^D$, and uses the fact there $p^d = f^d = d/R^d$ as we showed in Section 1. Rearranging (11) implies

$$1+\overline{R}^{D}=\tfrac{(1-\pi)\left(D+f^{D}\right)+\pi\left(d+p^{d}\right)}{f^{D}} \tag{12}$$

Comparing (12) with (10) shows that $p^D > f^D$ whenever $\overline{R}^D > \overline{r}^D$ and $p^D = f^D$ whenever $\overline{R}^D = \overline{r}^D$. Recall that when the cost of default Φ falls below Φ^*, savers strictly prefer lending to buying the asset, which means $\overline{R}^D > \overline{r}^D$. When $\Phi \geq \Phi^*$, savers are indifferent between lending and buying the asset, which implies $\overline{R}^D = \overline{r}^D$. Whether the asset price p^D exceeds fundamentals f^D thus depends on Φ. Solving for p^D from (10) and f^D in (12) yields an expression for the bubble term $b^D = p^D - f^D$.

Proposition 3 *Let f^D denote the value of dividends discounted at the expected return on loans $\bar{R}_t$. Then the difference between the price of the asset and its fundamental value $b^D = p^D - f^D$ is*

$$b^{D}=\left((1-\pi)\,D+\pi\left(d+p^{d}\right)\right)\left[\frac{1}{\pi+\bar{r}^{D}}-\frac{1}{\pi+\bar{R}^{D}}\right] \tag{13}$$

The bubble term b^D is positive when $\Phi < \Phi^*$ but equal to 0 when $\Phi \geq \Phi^*$.

The fact that the asset is priced as if $d_{t+1} = D$ with certainty does not on its own imply the price exceeds fundamentals. Intuitively, bubbles arise in our model when leveraged agents who only care about the upside potential of the asset pay more for the asset than its expected return. When Φ is small, only leveraged agents buy assets. In that case, the price of the asset will exceed fundamentals. When Φ is large, lending against the entire stock of assets is too costly, and in equilibrium savers will have to buy some of the assets. But savers who invest their own funds will refuse to pay more than fundamentals, so a bubble cannot occur. Since previous work on risk-shifting ignored default costs, it tended to conflate risk-shifting with bubbles.

Although bubbles only arise when $\Phi < \Phi^*$, there is a sense in which the price of the asset is too high regardless of Φ. To see this, note that since $R^D > \overline{R}^D \geq \overline{r}^D$, the return R^D that the marginal entrepreneur can earn exceeds the expected return $\bar{r}^D$ on the asset, regardless of whether $\overline{R}^D > \bar{r}^D$ or $\overline{R}^D = \bar{r}^D$. The asset thus yields a lower return than what the marginal entrepreneur can earn. The asset price is too high not in the sense that it exceeds the present discounted value of future earnings, but that the return on the asset is low relative to what the economy could achieve. Comparing price and fundamentals is not the only sense in which an asset can be viewed as overpriced.

Realized Returns and Interest Rates: We next consider rates of return during the boom. Since $R^D > R^d$, the realized return on investment, both for those who buy assets and for those who lend, will be higher while the boom lasts. A boom will appear to be a good time for savers.

But even as *realized* returns must be higher during the boom, *expected* returns can be lower. The expected return to lending is $\overline{R}^D$ during the boom and R^d after the boom. The expected return $\overline{R}^D$ defined in (9) is a weighted average of $1 + \frac{D}{p^D}$ and $\frac{d+p^d}{p^D} - \Phi$. Since $D/p^D = R^D > R^d$ and $R^d = d/p^d > d/p^D$, we have

$$1 + \tfrac{D}{p^D} > 1 + R^d > \tfrac{d+p^d}{p^D}$$

If the weighted average of $1 + \frac{D}{p^D}$ and $\frac{d+p^d}{p^D}$ gives enough weight to the latter, e.g. if π is close to 1, the expected return to lending will be below $1 + R^d$ even before accounting for default costs. Asset booms can therefore be times of high realized returns but low expected returns.

Although realized returns to savers during the boom are high, there is an important sense in which the interest rate for borrowing to buy risky assets is low. When lenders cannot distinguish safe and risky borrowers, the former end up cross-subsidizing the latter. We can formalize this intuition by comparing the equilibrium in Proposition 2 to a hypothetical full-information benchmark. With complete information, lenders would charge those who buy assets an interest rate at least as high as the maximum return on the asset, $1 + D/\widehat{p}^D$, where a hat denotes the asset price with full information. At the same time, lenders would charge entrepreneurs an interest rate equal to the expected return on the asset $(1-\pi)\left(1 + D/\widehat{p}^D\right) + \pi\left(d + p^d\right)/\widehat{p}^D$ that lenders can earn on their own. If $\widehat{p}^D$ were equal to p^D, entrepreneurs would be charged a lower interest under full information. But then more entrepreneurs would borrow to produce, leaving fewer resources to spend on the asset. In equilibrium, the asset price $\widehat{p}^D$ must be lower than p^D, implying

$$R^D = \tfrac{D}{p^D} < \tfrac{D}{\widehat{p}^D}$$

Agents who borrow to buy risky assets are charged less than they would be to buy equally risky assets under full information. The interest rate R^D doesn't fully reflect the risk of the assets speculators buy.

Fallout from the Crash: Finally, we turn to how booms end in our model. When the initial regime ends and d_t falls, agents who previously borrowed to buy assets will be forced to default. This imposes a cost of Φp^D on lenders. The collapse in asset prices leaves this cohort with fewer resources to consume, above and beyond the decline in the dividend income they earn. By construction, the fall in available resources is proportional to the price of assets p^D during the boom. A larger boom thus implies a larger loss once the boom ends. This is because recovery costs are larger when agents borrow more resources to spend on assets. As we noted above, we view default costs as a stand-in for other channels in which a fall in asset prices would lead to lower output, e.g. debt overhang and or deleveraging. In all of these mechanisms, the decline in output after a crash would also increase in the size of the run-up in asset prices during the boom.

Our model thus replicates key features of the asset booms and busts we see in practice. In the remainder of the paper, we examine whether there is a reason to intervene against these booms in our model, and whether the particular interventions policymakers have focused on can in fact improve welfare.

3 Inefficiency of Equilibria

An equilibrium is *constrained inefficient* if, starting from the equilibrium allocation, a planner facing the same markets and constraints as private agents can intervene to make some agents better off without making any agents worse off. The relevant constraints in our model are the informational and contractual restrictions in Section 1. We now show that the equilibrium in Proposition 2 is constrained inefficient.

The intervention we consider involves a quota on total lending. Such a restriction does not require private information on what borrowers do with the funds they secure. Nor does it violate the restriction on the type of contracts agents can use. To study the effect of a quota, we need to be more specific on how credit markets operate and allow for the possibility of rationing. We continue to assume that there is a single interest rate on loans R_t that borrowers and lenders take as given. However, we now assume savers seeking to lend line up sequentially according to some pre-specified order. If the total amount of lending reaches the quota, savers who still want to lend will be turned away. These savers can still buy assets, though. Since there is no quota on how much agents can borrow, either individually or in total, there is no need to specify the order in which borrowers show up to the market as we do with savers.

Suppose we impose a quota that total lending at date 0 cannot exceed $(e - p^D)$, where p^D is the equilibrium price of the asset in Proposition 2. Without a quota at dates $t \geq 1$, the equilibrium after date 0 will be the same as in Proposition 2.

As for date 0, entrepreneurs whose productivity y exceeds the interest rate on loans R_0^D at date 0 will borrow to produce. Any resources not used to finance production will be used to buy the asset. Hence, the market clearing condition (3) still holds at date 0. We need an equilibrium condition for R_0^D that is analogous to (6). The interest rate on loans R_0^D at date 0 must still be at least as large as the maximal return on the asset, $\frac{p^D+D}{p_0^D}$, to ensure finite demand for borrowing. In the Appendix, we show $1 + R_0^D$ cannot exceed $\frac{p^D+D}{p_0^D}$. The equilibrium interest rate condition is thus identical to (6), implying $p_0^D = p^D$ and $R_0^D = R^D$ just as when there is no quota. However, given savers can lend at most $\left(e - p^D\right)$, the quota forces savers to spend at least p^D on the asset. Since the asset is worth p^D in equilibrium, no borrowers can buy the asset, i.e. $\alpha_0^D = 0$. Even though agents are indifferent to speculating, the asset market clears only when none of them do so. While the quota restricts total lending rather than not what borrowers do, only speculation gets crowded out in equilibrium. The amount lent for production remains unchanged.

We can verify this equilibrium is a Pareto improvement. The quota only affects agents alive at date 0. The old can still sell their assets at price p^D and so are unaffected. Young entrepreneurs face the same interest rate R^D and are also unaffected. Agents who would have borrowed to buy assets without the quota would have earned zero, and so are no worse now that none of them speculate. Finally, if $\Phi > 0$, the total return for savers is higher given they avoid default costs. We can therefore make all savers better off. Savers may complain about being forced to buy assets when the return to lending R^D exceeds the expected return on the asset. But committing themselves not to lend makes savers better off. In the Appendix, we show that extending the quota of $(e - p^D)$ for as long as $d_t = D$ leaves p_t^D and R_t^D unchanged while forcing $\alpha_t^D = 0$ for all t, making all cohorts born while $d_t = D$ better off. This implies the following:

Proposition 4 *If $\Phi > 0$, the equilibrium described in Proposition 2 is constrained inefficient while $d_t = D$.*

Intuitively, constrained inefficiency arises because without a quota, there is scope for lenders and speculators to renegotiate and have the lender buy assets directly to avoid default but compensate the speculator for what she would have earned. This is not feasible without a quota, since paying borrowers not to buy assets would lead to infinite demand for credit. A quota lets savers commit to buying assets *a priori*.[6]

Our constrained inefficiency result bears some similarity to work by Korinek and Simsek (2016) and Farhi and Werning (2016) on the benefits of macroprudential policy under rigid pricing. In those models, credit is associated with a negative externality whereby if there is a negative shock, leveraged agents demand fewer goods, and, given sticky prices, output falls. Restricting lending can therefore increase welfare. Our setting features rigid contracting rather than prices. Increasing lending worsens the pool of borrowers by drawing in speculators in addition to entrepreneurs. If there is a negative shock, these agents default. The output available for agents to consume falls, not because of lower aggregate demand but because resources are wasted on recovery. Limiting the amount agents can lend again increases welfare.

Since the quota we consider has no effect on prices, it is still the case that the expected return on the asset is lower than what the marginal entrepreneur can earn. Young savers could consume more if they could coordinate to redirect some of the resources they spend on assets to be used for production. Since the old would earn less from selling their assets, this reallocation would not constitute a Pareto improvement, a point highlighted in Grossman and Yanagawa (1993) in a related model.[7] While the high asset price matters

[6] Allowing for contingent contracts can also eliminate speculation, although this would violate the constraints on private contracting. If lenders could charge a lower interest rate when $d_{t+1} = d$, lending to entrepreneurs and speculators could be designed to be equally profitable in equilibrium, and one can show that α^D would necessarily equal 0.

[7] In a previous version of this paper, we argued that redirecting resources to production could make all agents better off if assets were produced rather than endowed. Policymakers could then raise welfare by taxing the production of the asset, even if they can't observe whether those who buy the asset are leveraged. We omit this extension of the model for the sake of brevity.

for welfare, the case for intervention in our model stems from eliminating speculation and the externalities it involves rather than correcting asset prices. The fact that the equilibrium is constrained inefficient for any $\Phi > 0$ while bubbles only arise if $\Phi < \Phi^*$ further confirms that whether the asset is a bubble is unrelated to constrained inefficiency. As long as policymakers know that there is risk-shifting, they do not need to know if there is an asset bubble to determine whether to intervene.

Although a lending cap is Pareto improving in our model, the policy debate on how to respond to asset booms has focused on monetary policy and leverage restrictions. In the remainder of the paper, we study how these interventions affect outcomes and welfare in our model. We will need to modify some of our simplifying assumptions to study these. But a cap on lending remains Pareto improving even with these changes, and an asset boom features too much lending given the speculation and default it encourages.

To capture the effects of monetary policy, we need to relax our assumption that each cohort is endowed with an exogenously fixed supply of goods e. While this assumption is convenient, models of monetary policy often rely on price rigidities that allow economic activity to expand or contract when the monetary authority moves. In the next section, we allow the initial income e of savers to be endogenous.

To capture the effect of leverage restrictions, we need to relax our assumption that entrepreneurs have no initial wealth. When borrowers have no resources, there is no way to restrict leverage other than to cut off credit altogether. In Section 5, we return to assuming savers are endowed with an exogenous income, but we assume entrepreneurs also have some initial endowment. Whereas penniless entrepreneurs must take on infinite leverage, those with wealth can choose how much leverage to take on. This introduces a complication we were able to avoid thus far, namely that we need a continuum of markets to span all possible choices of leverage agents might entertain. By contrast, so far we had all credit intermediated in a single market. We discuss how to deal with the complication of a continuum of markets and then study the effect of restrictions on the amount of leverage borrowers can take on.

4 Monetary Policy

This section explores monetary policy. As we noted above, we need to drop our simplifying assumption that savers are endowed with goods and consider a production economy. Our approach follows Galí (2014), who also studies monetary policy in an overlapping generations economy with assets. We assume savers are endowed with labor that can be used to produce goods rather than endowed directly with goods. We then introduce a monetary authority and monopolistic competition so producers who hire labor set the prices of their goods. We assume the monetary authority moves after goods producers set their prices but before they hire labor. This allows the real wage – and consequently output – to respond to monetary policy.

We leave the detailed analysis to Appendix B and sketch the results here. Our assumptions imply labor supply only depends on the real wage. In the absence of money, the equilibrium real wage will be constant over time and independent of the return on savings and hence independent of d_t. The reduced-form representation of our production economy is the same as the endowment economy we studied up to now: Each cohort of savers has a fixed budget e to allocate between entrepreneurial activity and assets.

Next, we introduce a monetary authority that can announce a nominal interest rate at which it is willing to borrow and lend. As in Galí (2014), we consider an equilibrium in which money doesn't circulate. This requires inflation to adjust so that the real value of the nominal rate set by the monetary authority equals the real return agents earn elsewhere, leaving agents indifferent to holding money. At the beginning of each period, producers set the prices of the goods they expect to sell. The monetary authority then sets a nominal interest rate. Finally, producers hire workers and produce goods. If producers could perfectly anticipate what the monetary authority will do, the nominal interest rate would have no effect on the real wage or any other real variable: Producers would set their prices as a markup over the nominal wage they know will prevail, the real wage would not depend on monetary policy, and neither would earnings e.

If producers cannot perfectly anticipate what the monetary authority will do, producers will set their price as a markup over the *expected* nominal wage that will prevail after the monetary authority moves. If the nominal interest rate this period turns out to be higher (lower) than expected, the nominal and real wage can be higher (lower) than expected. Essentially, an unanticipated move by the monetary authority allows a self-fulfilling fall in demand for goods. Lower demand for goods means producers don't need to hire as much labor, the real wage falls, and since agents earn less, demand for goods will indeed be lower. A surprise move by the monetary authority at date 0 can thus change earnings e_0, just as an income tax or subsidy would. Since producers set prices at the beginning of each period, an intervention at date 0 will not affect real variables beyond date 0. We can therefore deduce the effects of such an intervention using comparative statics on e_0 in our original endowment economy holding $e_t = e$ at all other dates. The next proposition, based on our analysis in Appendix B, summarizes these effects.

Proposition 5 *An unanticipated monetary contraction that reduces earnings e_0 at date 0 leads to a lower asset price p_0^D and a higher real interest rate on loans R_0^D at date 0 as compared to no intervention.*

A surprise intervention at date 0 will have no impact on real variables beyond date 0, so only the agents alive at that date are affected. Begin with the young. A contractionary policy induces them to work less. Since they value leisure, this on its own makes them better off. But their utility also depends on consumption. Suppose $\Phi < \Phi^*$, so absent intervention savers would lend all of their earnings e_0 and only borrowers buy the asset. The expected total consumption of young agents would then be

$$\int_{R_0^D}^{\infty} (1+y)\, n(y)\, dy + \left[(1-\pi)\left(D + p_1^D\right) + \pi\left(d + p_1^d\right)\right] - \pi\Phi p_0^D \tag{14}$$

The first term in (14) represents output produced for date 1 by entrepreneurs. The second term represents the expected payout on the asset at date 1. The last term represents expected default costs. A contractionary monetary policy increases R_0^D and reduces the amount entrepreneurs produce captured by the first term in (14). This policy has no effect on the payout on the asset at date 1. Finally, it lowers expected default costs by reducing spending on assets p_0^D.[8] Contractionary policy thus has ambiguous effects on the total consumption of those born at date 0: With fewer resources they can fund less entrepreneurial activity, but they also face smaller deadweight losses from default. If Φ is sufficiently large, this cohort will consume more. Since they also enjoy more leisure, they will be better off with tighter monetary policy.[9]

While contractionary policy can make the young better off for large Φ, it will make the old at date 0 worse off by lowering the price at which they sell their assets. To ensure no agent is worse off, we need to transfer resources from savers to the old. In Appendix B, we confirm that even though this redistribution further increases R_0^D and decreases p_0^D, it will be possible to fully reimburse the old but leave the young better off. Contractionary monetary policy can thus be Pareto improving. However, it is a relatively inefficient intervention: If the policymaker merely imposed a lump-sum tax on savers and transferred it to the old, it could still lower p_0^D and default costs $\pi\Phi p_0^D$ without distorting labor or depressing the income e_0 savers earn. A cap on lending as in Section 3 would be better still, since it can eliminate all default costs without depressing either e_0 or entrepreneurial activity, eliminating speculation without changing p_0^D or R_0^D.

Our finding that contractionary monetary policy offers a costly way to improve welfare has parallels in other papers. Svensson (2017) and Gourio, Kashyap, and Sim (2018) also discuss the costs of monetary policy. In their models, tighter monetary policy reduces the odds of a financial crisis rather than mitigates the severity of the output decline when the boom ends as in our model. Farhi and Werning (2020) introduce assets into the model of leverage and rigid prices in Farhi and Werning (2016). They find that there may be scope for tighter monetary policy if the agents who borrow are also overly optimistic about asset returns. In our model, speculators who create negative externalities are also overly eager to buy assets, not because of distorted beliefs but because of distorted incentives. Our finding that monetary policy is an imperfect substitute for lending restrictions mirrors theirs.

Finally, when π is close to 1 and the boom is likely to end quickly, there may be a better way to use monetary policy during the boom. Rather than tightening at date 0, suppose the monetary authority promises to tighten at date 1 if $d_1 = D$ and the boom continues. For this, we assume d_t is revealed after producers set their prices at the beginning of date t but before the monetary authority moves. Since producers at date 1 set prices based on the expected nominal wage, monetary policy is contractionary when $d_1 = D$ only if it is also expansionary when $d_1 = d$. The monetary authority will thus set $e_1^d > e$ if $d_1 = d$ and $e_1^D < e$ if $d_1 = D$. Per Proposition 5, the fact that $e_1^D < e$ will depress p_1^D and increase R_1^D, and likewise the fact that $e_1^d > e$ increases p_1^d and decreases R_1^d. Income e_0 will not change, but as we show in Appendix B, the promise of such future intervention will lower both p_0^D and R_0^D at date 0.

[8] In principle, expected default costs might also fall if a contractionary policy induced savers to buy some of the asset directly. It turns out that in our model, contractionary policy if anything discourages savers from buying assets directly.

[9] If $\Phi \geq \Phi^*$ so that absent intervention savers directly buy some of the assets, tighter monetary policy is even more ambiguous. Although it lowers p_0^D, it also increases the share of assets bought by borrowers. This increases expected default costs. Eventually, though, all assets will be bought by borrowers, and the only effect of policy would be to lower p_0^D.

Proposition 6 *A commitment by the monetary authority at date* 0 *to set* $e_1^d > e > e_1^D$ *leads to a lower asset price* p_0^D *and a lower interest rate on loans* R_0^D *at date* 0 *than would have prevailed absent intervention.*

Promising to intervene at date 1 can only affect cohorts who are alive at dates 0 and 1. Consider first the cohort born at date 1. If $d_1 = D$, tightening at date 1 means savers in this cohort enjoy more leisure, earn a lower income, and expect to incur smaller default costs $\pi\Phi p_1^D$ when they lend. Just as with tightening at date 0, the effect on this cohort is ambiguous and depends on how higher leisure and lower default costs compare to lower earnings. If $d_1 = d$, easing at date 1 means this cohort works more, earns more, and, just as with no easing, incurs no default costs. Given we assume firms are monopolists and set prices, easing helps mitigate the monopoly distortion, meaning the marginal gains from higher consumption exceed the losses from less leisure. When π is close to 1 and the monetary authority will almost surely ease, this cohort will be better off ex ante. Next, consider the cohort born at $t = 0$. Their labor supply is unaffected, while their consumption is given by (14). Since R_0^D is lower, they fund more entrepreneurial activity. Since p_0^D is lower, they face lower expected default costs. The expected payout on the asset is ambiguous given the intervention lowers p_1^D and raises p_1^d. For π close to 1, though, the expected payout will be higher, and this cohort will be better off, including better off than under a contractionary policy at date 0. Finally, the cohort that is old at date 0 will be worse off given they earn less from selling assets at a lower price p_0^D. But once again we can use a lump-sum tax on the young at date 0 to leave them no worse off.[10]

Essentially, when π is close to 1, policymakers can promise to tighten in an unlikely state of the world. Such a threat remains useful, since speculation is driven by the maximum return on the asset regardless of how likely it is. By its nature, this intervention is expansionary in some states and contractionary in others. But if π is close to 1, these effects of monetary policy are largely anticipated and inconsequential. The main effect of the threat is to discourage speculation by making it less profitable.

5 Leverage Restrictions

We now turn to leverage restrictions. This intervention in credit markets is different from the quota in Section 3. A quota limits the total amount savers can lend. A leverage restriction imposes no direct limits on how much savers can lend, and instead forces borrowers to complement the funds they borrow with their own funds. Given our assumption so far that entrepreneurs have no resources, any requirement that borrowers provide their own funds would kill all credit. For leverage restrictions to have a non-trivial effect, entrepreneurs must have some initial wealth. However, this modification introduces complications. When agents have nothing, they have to be infinitely levered. If they have wealth, they must choose how much leverage to take on. This requires multiple markets to accommodate all possible degrees of leverage.

[10] If there were a subsidy to correct the underlying monopoly distortion, the cohort born at date 1 would be worse off if $d_t = d$. However, if π is close to 1, they would not be much worse off given d_t is largely anticipated and the stimulus is small. The cohort born at date 0 would have to compensate the cohort born at date 1, but the compensation would vanish as π tended to 1. Since contractionary policy at date 0 is always costly, those born at date 0 would prefer delayed intervention.

Let us briefly preview our results. Leverage restrictions reduce demand for credit, which lowers interest rates. This in turn tends to raise asset prices, especially if, as in our model, credit demand among entrepreneurs falls, freeing up resources for speculation. Tighter leverage restrictions thus have the opposite effect on interest rates and asset prices as tighter monetary policy. But, like the cap on lending we considered in Section 3, leverage restrictions can lower the fraction of assets purchased by borrowers. The effect of these restrictions on speculation and welfare is ambiguous: Borrowers buy a smaller share of assets, but they spend more on the assets they buy. Default costs may actually rise, unlike what we saw with a lending cap or tighter monetary policy. Our results thus offer a contrast to Caballero and Simsek (2019), who describe an economy where leverage restrictions and tighter monetary policy are welfare equivalent.

For tractability, we return to assuming agents are endowed with goods rather than labor. Cohorts still consist of unproductive savers endowed with e goods and entrepreneurs who can convert goods at date t into goods at date $t+1$. Rather than assume entrepreneurs are all endowed with $w = 0$ and differ in productivity y, we turn to the opposite case where entrepreneurs differ in w and share the same productivity y^*. We discuss the case where both w and y vary across entrepreneurs at the end of this section.

We assume the wealth of entrepreneurs w is distributed uniformly. Specifically, for each $w \in [0, 1]$, there is a constant density $2\varphi e$ of entrepreneurs with wealth w, where e is the endowment of savers and φ is a constant such that $0 < \varphi < 1$. The combined endowment of all entrepreneurs is therefore

$$\int_0^1 w\,(2\varphi e)\,dw = \varphi e$$

The combined wealth of savers and entrepreneurs is $(1 + \varphi)\,e$. To produce at capacity, entrepreneurs need

$$\int_0^1 (1 - w)\,(2\varphi e)\,dw = \varphi e$$

Since $\varphi < 1$, entrepreneurs require fewer resources than savers have, in contrast to what we assumed in (2).

We assume the common productivity y^* is large enough to exceed the maximal return on the asset. To establish that this maximal return is finite, observe that the asset price p_t is bounded above by $(1 + \varphi)\,e$, the most each cohort has to spend on the asset, and is bounded below by $(1 - \varphi)\,e$, the amount of resources left to spend on the asset if all entrepreneurs produce at capacity. The maximal return on the asset occurs when $d_{t+1} = D$, the price of the asset at date t assumes its lowest value $(1 - \varphi)\,e$, and the price at $t + 1$ assumes its maximum value $(1 + \varphi)\,e$. We assume $1 + y^*$ exceeds this return, i.e.,

$$1 + y^* > \frac{D + (1 + \varphi)\,e - (1 - \varphi)\,e}{(1 - \varphi)\,e} = \frac{D + 2\varphi e}{(1 - \varphi)\,e} \tag{15}$$

Assumption (15) ensures the return on production exceeds the return on the asset, so all entrepreneurs will want to produce at capacity in equilibrium. This allows us to avoid solving for the endogenous fraction of entrepreneurs funded in each of a continuum of markets, which greatly simplifies the analysis.

Now that entrepreneurs have wealth, they can fund their investments. Lenders still cannot observe what borrowers invest in, but they can observe the resources borrowers invest in the same project. Effectively, we let borrowers set up a shell entity and choose how much of their wealth to endow the entity with. By helping finance her investment, a borrower discloses some of her wealth to the lender, which in turn allows the lender to seize that wealth in case of default. The borrower's remaining wealth stays hidden.

Formally, borrowers choose the fraction $\lambda \in [0,1)$ of their investment to self-finance. We model this as a continuum of markets indexed by $\lambda \in [0,1)$. An agent who borrows in market λ receives $\frac{1-\lambda}{\lambda}$ units for each unit of her own wealth that she invests. She can thus leverage her endowment of w to finance an investment of size $\frac{w}{\lambda}$. When $w > 0$, the choice of leverage is non-trivial. By going to a market with a lower λ, an entrepreneur can borrow more and produce at a larger scale. But this will leave their lender with a smaller cushion to go after in case of default.[11] Back when we assumed all entrepreneurs had no wealth, agents had no choice. They could only borrow in market $\lambda = 0$ and choose infinite leverage. This allowed us to consider only a single market. Now that agents have wealth, we need a market for each $\lambda \in [0,1)$ to accommodate any leverage they might choose, meaning we must allow a continuum of markets.

We now define and solve the equilibrium with a continuum of markets. To anticipate where we are going, we study an equilibrium where entrepreneurs with wealth $w \in [0,1]$ go to market $\lambda = w$ and invest their entire endowment in production, borrowing $1 - w$ to attain capacity. Entrepreneurs with different wealth thus sort into different markets. As before, when $d_t = D$, some agents will borrow to buy assets and speculate. However, they will only borrow in markets with low λ. This motivates us to consider macroprudential regulations that shut down markets where λ is below some floor $\underline{\lambda}$.

5.1 Equilibrium with Multiple Markets

An equilibrium in our economy still consists of a path of asset prices $\{p_t\}_{t=0}^{\infty}$ and a path of interest rates, but the latter now consists of a path of interest rates $\{R_t(\lambda)\}_{t=0}^{\infty}$ for each market $\lambda \in [0,1)$. In addition, let $f_t^a(\lambda)$ and $f_t^p(\lambda)$ denote the density of borrowing in market λ used to buy assets and to produce, respectively, and $f_t(\lambda) \equiv f_t^a(\lambda) + f_t^p(\lambda)$ denote the density of total borrowing in market λ. We can integrate these densities to obtain the total amounts borrowed in all markets, $\int_0^1 f_t^a(\lambda)\,d\lambda$ and $\int_0^1 f_t^p(\lambda)\,d\lambda$. Although we refer to the density of borrowing, we do not require agents to borrow infinitesimal amounts in all markets. Indeed, once we introduce leverage restrictions, there will be a market that will attract a positive mass of borrowers. We discuss how to deal with this formally in Appendix C, but, loosely, such markets can be viewed as having infinite borrowing rates. We refer to market λ as *inactive* if $f_t(\lambda) = 0$ and *active* if $f_t(\lambda) > 0$. The price p_t, interest rates $R_t(\lambda)$, and amounts borrowed $f_t^a(\lambda)$ and $f_t^p(\lambda)$ must ensure markets clear when agents acts optimally, just as with a single credit market.

To determine if lenders are optimizing, we need to know what they expect to earn from lending in any

[11] A market with smaller λ means fewer resources for the lender to seize in case of deafult. But wth fewer assets to review, default costs $\frac{\Phi}{1-\lambda}$ per unit borrowed are actually smaller for more leveraged buyers. Below we show that having less to seize is the dominant factor, and in equilibrium more leveraged buyers will have to pay higher interest rates.

market $\lambda \in [0,1)$. Building on our previous notation, let $\overline{R}_t(\lambda)$ denote the *expected* return to lending at date t in market λ. If market λ is active, we can deduce $\overline{R}_t(\lambda)$ from the interest rate $R_t(\lambda)$ in market λ together with the amounts $f_t^a(\lambda)$ and $f_t^p(\lambda)$ that agents borrow to buy assets and to produce, respectively. If market λ is inactive, there is nothing to guide lenders on what to expect if they were to lend to a market where no borrowers show up. Instead, we need to assign an expected return $\overline{R}_t(\lambda)$ to each inactive market as part of our definition of an equilibrium. In what follows, we first look for an equilibrium in which all markets are active to avoid the question of how to assign $\overline{R}_t(\lambda)$ in inactive markets. We then discuss equilibria in which markets can be inactive. This naturally leads into our analysis of regulatory interventions in which some markets are inactive by decree rather than because of what agents believe.

We begin with the case where $d_t = d$ for all t. As in Section 1, we proceed as if equilibrium prices are deterministic and verify this is the case in Appendix C. In this case there will be no default, and so the expected return to lending $\overline{R}_t(\lambda)$ will equal the interest rate on loans $R_t(\lambda)$ in each active market λ. The expected return in all active markets must be the same for lenders to agree to lend in all of these markets, and so $R_t(\lambda)$ must be the same in all active markets. Moreover, this common interest rate must equal the return on the asset $1 + r_t \equiv \frac{d+p_{t+1}}{p_t}$ to ensure savers agree both to buy assets and to lend in active markets. That is, $R_t(\lambda) = r_t$ in all active markets λ. At these interest rates, borrowing to buy assets is unprofitable given $\phi > 0$. Since (15) ensures y^* exceeds r_t, all entrepreneurs will want to borrow in order to produce at capacity. Given $R_t(\lambda)$ is the same for all λ, entrepreneurs will be indifferent as to which market they borrow in, as long as they borrow enough to reach capacity. This includes the case where those with wealth w borrow $1 - w$ in market $\lambda = w$, an arrangement that ensures all markets are active.

Since all entrepreneurs produce at capacity, they will use $2\varphi e$ units of input to produce. Any part of the total wealth $(1+\varphi)e$ of each cohort not used to produce will be spent on the asset. This implies

$$p_t + 2\varphi e = (1+\varphi)e \tag{16}$$

It follows that $p_t = (1-\varphi)e$ for all t. The return to buying the asset r_t and the interest rate on loans $R_t(\lambda)$ in all markets λ will then be $\frac{d}{(1-\varphi)e}$. This leads to the following analog to our earlier Proposition 1:

Proposition 7 *When $d_t = d$ for all t, there exists an equilibrium in which all markets are active. In any such equilibrium, $p_t = (1-\varphi)e \equiv p^d$ for all t, $R_t(\lambda) = \frac{d}{(1-\varphi)e} \equiv R^d$ for all $\lambda \in [0,1)$ and all t, all entrepreneurs borrow and produce at capacity, no agent borrows to buy assets, and only savers hold assets.*

Next, we turn to the case where $d_t = D$ at date 0 and permanently switches to d with constant probability π per period. We again use a superscript D to refer to an equilibrium object at date t when $d_t = D$. We begin by solving for equilibrium interest rates. For each active market λ where $f_t(\lambda) > 0$, either agents borrow to buy assets, i.e., $f_t^a(\lambda) > 0$, or they do not, i.e., $f_t^a(\lambda) = 0$. In the latter case, there will be no default and the expected return to lending $\overline{R}_t^D(\lambda)$ will equal the interest rate on loans $R_t^D(\lambda)$. In equilibrium, $\overline{R}_t^D(\lambda)$ must be the same in all active markets for lenders to agree to lend in these markets. Denote this common expected return by $\overline{R}_t^D$. Then $R_t^D(\lambda) = \overline{R}_t^D$ in any active market λ in which $f_t^a(\lambda) = 0$.

Consider next an active market λ in which agents do borrow to buy assets, i.e., $f_t^a(\lambda) > 0$. Agents would only borrow to buy assets if they intend to default. Borrowing to buy assets and never defaulting cannot be profitable, since lenders would not lend at an interest rate below the expected return they could earn from buying the assets themselves, and for $\phi > 0$ this would be unprofitable. The expected payoff per unit invested from borrowing in market λ to buy assets and defaulting if $d_{t+1} = d$ is given by

$$(1-\pi)\left[\frac{p_{t+1}^D + D}{p_t^D} - (1-\lambda)\left(1 + R_t^D(\lambda)\right)\right] \tag{17}$$

For each unit of resources agents invest in buying assets, a fraction λ must come from their own wealth. If they had lent out their own wealth instead, they would have earned $(1+\overline{R}_t^D)\lambda$. We now argue that in equilibrium, this payoff must equal (17). If $(1+\overline{R}_t^D)\lambda$ exceeded (17), nobody would borrow to buy assets in market λ given they could earn more from lending, contradicting the fact that $f_t^a(\lambda) > 0$. Conversely, if $(1+\overline{R}_t^D)\lambda$ were lower than (17), no agent would be willing to lend in any market given they can borrow in market λ to buy assets, again contradicting the fact that $f_t^a(\lambda) > 0$. Equating the two payoffs yields an expression for the interest rate on loans $R_t^D(\lambda)$ in any active market λ in which $f_t^a(\lambda) > 0$:

$$1 + R_t^D(\lambda) = \frac{1}{1-\lambda}\left[\frac{p_{t+1}^D + D}{p_t^D} - \frac{\lambda\left(1+\overline{R}_t^D\right)}{1-\pi}\right] \tag{18}$$

Thus, we have expressions for the interest rate $R_t^D(\lambda)$ if $f_t^a(\lambda) = 0$ and if $f_t^a(\lambda) > 0$, respectively. The next lemma, derived in Appendix C, shows there exists a cutoff $\Lambda_t^D \in [0,1)$ such that $R_t(\lambda)$ is given by (18) in markets $\lambda < \Lambda_t^D$ but is equal to $\overline{R}_t^D$ in markets $\lambda \geq \Lambda_t^D$.

Lemma: *If all markets are active, then there exists a cutoff* $\Lambda_t^D \in [0,1)$ *such that*

$$1 + R_t^D(\lambda) = \begin{cases} \frac{1}{1-\lambda}\left[\frac{p_{t+1}^D + D}{p_t^D} - \frac{\lambda\left(1+\overline{R}_t^D\right)}{1-\pi}\right] & \text{if } \lambda \in [0, \Lambda_t^D) \\ 1 + \overline{R}_t^D & \text{if } \lambda \in [\Lambda_t^D, 1) \end{cases} \tag{19}$$

Figure 4 plots the schedule of interest rates from (19). In market $\lambda = 0$, where agents are infinitely levered, the interest rate $R_t^D(0)$ equals the maximal return on the asset, $\frac{p_{t+1}^D + D}{p_t^D}$. This is the same as in Section 2, where $\lambda = 0$ was the only possible market. The logic is the same: When agents put no resources down, they must hand over all returns from the asset to the lender to ensure they earn no profits. For $0 < \lambda \leq \Lambda_t^D$, the interest rate $R_t(\lambda)$ decreases with λ. We prove this formally in Appendix C, but intuitively, when the borrower pledges more resources, the lender need not charge as much interest to make speculation unprofitable. Finally, for $\lambda \geq \Lambda_t^D$ the interest rate $R_t(\lambda)$ is constant and equal to $\overline{R}_t^D$. Credit markets thus fall into two distinct groups: In markets with $\lambda < \Lambda_t^D$ some agents borrow to buy assets and the interest rate $R_t^D(\lambda)$ exceeds the expected return $\overline{R}_t^D$, while in markets with $\lambda \geq \Lambda_t^D$ agents only borrow to produce and the interest rate $R_t^D(\lambda)$ is the same as the expected return $\overline{R}_t^D$. Intuitively, borrowers become reluctant to speculate once they have enough skin in the game and invest their own wealth.

Once again, equilibrium interest rates ensure agents cannot profit from borrowing to speculate so that demand to borrow is finite. As before, this means interest rates depend only on the payoff to the borrower and not the default costs Φ lenders incur. One noteworthy difference is that while speculators never earn a profit if they borrow in market $\lambda = 0$, in markets with $\lambda > 0$ they invest their own wealth in assets and must earn strict profits if $d_{t+1} = D$ to offset their losses if $d_{t+1} = d$.

Given the equilibrium interest rates schedule (19), we can solve for what entrepreneurs do. Recall that (15) implies y^* exceeds the maximal return on the asset. We just argued $R_t^D(0)$ is equal to this maximal return, and that $R_t^D(0)$ exceeds $\overline{R}_t^D$, the expected return to lending. $\overline{R}_t^D$ is also the most agents can expect to earn by leveraging their wealth in some market λ to buy assets. Entrepreneurs should thus use their endowment w for production, which yields the highest return. The question is which market $\lambda \in [0,1]$ they should borrow to scale up their production, where $\lambda = 1$ denotes no borrowing.

Consider an entrepreneur with wealth $w > \Lambda_t^D$. If she borrowed in market $\lambda = w$, she could borrow up to $1 - w$ at an interest rate of $\overline{R}_t^D$, the lowest available interest rate on loans. If she borrowed in some market $\lambda < w$, she could borrow more than $1 - w$. But there is no benefit to this extra borrowing given her capacity. Moreover, the interest rate in this market would be the same or higher than $\overline{R}_t^D$. So there is no advantage to going to markets $\lambda < w$ over going to market $\lambda = w$. If she borrowed in some market $\lambda > w$, she would have to borrow less than $1 - w$, and she would face the same interest rate $\overline{R}_t^D$. This too offers no benefit over going to market $\lambda = w$. The best this entrepreneur can do is go to market $\lambda = w$ to borrow $1 - w$, although she could also achieve the same payoff going to any market $\lambda \in \left[\Lambda_t^D, w\right]$.

Next, consider an entrepreneur with wealth $w \leq \Lambda_t^D$. If she borrowed in market $\lambda = w$, she could borrow up to $1 - w$ at an interest rate of $R_t^D(w)$. If she borrowed in some market $\lambda < w$, she would be able to borrow more than $1 - w$, but she has no use for this extra borrowing. Moreover, the interest rate in this market would be higher than $R_t^D(w)$. If she borrowed in some market $\lambda > w$, she would have to borrow less than $1 - w$. But she would face a lower interest rate. The question is whether it is worth reducing capacity to obtain a lower rate. Her payoff from borrowing in market $\lambda \in [w, \Lambda_t^D]$ would be $\frac{w}{\lambda}\left[1 + y^* - (1-\lambda)(1 + R_t(\lambda))\right]$. Substituting in from (19), this is equal to

$$\frac{w}{\lambda}\left[1 + y^* - \frac{p_{t+1}^D + D}{p_t^D} + \frac{\lambda\left(1+\overline{R}_t^D\right)}{1-\pi}\right]$$

This payoff is decreasing in λ, so there is no advantage to borrowing in these markets instead of $\lambda = w$. Borrowing in any market $\lambda \in (\Lambda_t^D, 1)$ is dominated by borrowing in market $\lambda = \Lambda_t^D$, which we already argued was worse than borrowing in $\lambda = w$. So borrowing $1 - w$ in market $\lambda = w$ is uniquely optimal.

In any equilibrium where all markets are active, then, entrepreneurs with wealth $w \in [0, \Lambda_t^D)$ will borrow in market $\lambda = w$, while those with wealth $w \geq \Lambda_t^D$ would borrow in some market between Λ_t^D and w. This implies $f_t^p(\lambda) = 2\varphi e$ for $\lambda \in [0, \Lambda_t^D)$ while $f_t^p(\lambda)$ is indeterminate for $\lambda \in [\Lambda_t^D, 1)$. But this indeterminacy is irrelevant for allocations or welfare, since in any such equilibrium we know agents with wealth $w \geq \Lambda_t^D$ borrow $1 - w$ at an interest rate of $1 + \overline{R}_t^D$. Just as before, we can ensure all markets are active by assuming entrepreneurs with wealth $w \geq \Lambda_t^D$ also borrow $1 - w$ in market $\lambda = w$.

We now turn to the equilibrium price p_t^D. Since resources not used for production are spent on the asset,

$$p_t^D + 2\varphi e = (1+\varphi) e \tag{20}$$

It follows that $p_t^D = (1-\varphi) e$ for all t. This is the same price as when $d_t = d$. Although the price is the same, the expected return to buying the asset when $d_t = D$ is higher, with $1+\overline{r}^D = \frac{(1-\pi)D+\pi d}{(1-\varphi)e}$.

As in Section 2, we were able to solve for the equilibrium price p_t^D and the interest rates $R_t^D(\lambda)$ for all markets λ without solving for the amounts people borrow to buy assets. We now solve for the amounts agents borrow to buy assets in each market, $f_t^a(\lambda)$. Recall that the expected return to lending in all active markets λ is given by $\overline{R}_t^D(\lambda) = \overline{R}_t^D$. Let $\alpha_t(\lambda) \equiv f_t^a(\lambda)/f_t(\lambda)$ denote the fraction of lending in any active market λ that is used to buy assets. With probability $1-\pi\alpha_t(\lambda)$, the borrower can afford to pay the lender back $R_t^D(\lambda)$ in full. With remaining probability $\pi\alpha_t(\lambda)$ the borrower will be a speculator and $d_{t+1} = d$, leading the borrower to default. In this case, the lender will seize $\frac{1}{1-\lambda}$ assets per unit borrowed from the borrower, and incurs a default cost to seize these assets. Since this expected return must equal $\overline{R}_t^D$, we have

$$(1-\pi\alpha_t(\lambda)) R_t^D(\lambda) + \frac{\pi\alpha_t(\lambda)}{1-\lambda}\left[\frac{d}{(1-\varphi)e} - \Phi\right] = \overline{R}_t^D \tag{21}$$

Given the value of $R_t^D(\lambda)$ in (19), we can solve for $\alpha_t(\lambda)$. Since $f_t^p(\lambda) = 2\varphi e$, we can use $\alpha_t(\lambda)$ to solve for $f_t^a(\lambda)$. Since $R_t^D(\lambda) > \overline{R}_t^D$ for $\lambda < \Lambda_t^D$, we have $\alpha_t(\lambda) > 0$, i.e., some borrowers in these markets speculate.

Earlier we established that agents do not borrow to buy assets in markets $\lambda \geq \Lambda_t^D$. Hence, $f_t^a(\lambda) = 0$ for $\lambda \in [\Lambda_t^D, 1)$, and so $f_t^a(\lambda)$ is uniquely determined for all $\lambda \in [0,1)$ in any equilibrium in which all markets are active. We can also say something about who engages in speculation. In Section 2, which agents borrowed to buy assets was indeterminate. This is still true for market $\lambda = 0$. But in markets $\lambda > 0$, borrowers must invest their own wealth to speculate. Entrepreneurs with $w < \Lambda_t^D$ invest all of their wealth w in production. So it must be savers and wealthy entrepreneurs who borrow to buy assets in markets $\lambda \in (0, \Lambda_t^D)$.

We have now solved for the equilibrium price p_t^D, and, for all λ, the interest rate on loans $R_t^D(\lambda)$ and the rates of borrowing $f_t^a(\lambda)$ and $f_t^p(\lambda)$ to buy assets and to produce. These are all defined in terms of the expected return to saving $\overline{R}_t^D$, which we have yet to derive. We leave the details on solving $\overline{R}_t^D$ to Appendix C, where we show it is constant over time, i.e. $\overline{R}_t^D = \overline{R}^D$ for all t. Given a value for $\overline{R}^D$, the analog to Proposition 2 can be summarized as follows:

Proposition 8 *There exists an equilibrium in which all markets are active while $d_t = D$. In any such equilibrium, the asset price is given by*

$$p_t^D = (1-\varphi)e \equiv p^D$$

and, in the limit as $\phi \to 0$, the interest rates on loans in different markets are given by

$$1 + R_t^D(\lambda) = \max\left\{1+\bar{R}^D, \tfrac{1}{1-\lambda}\left[1+\tfrac{D}{p^D} - \tfrac{\lambda(1+\bar{R}^D)}{1-\pi}\right]\right\}$$

where $\bar{R}^D$ can be solved separately. The rate of borrowing in market λ for production is given by $f_t^p(\lambda) = 2\varphi e$ for $\lambda \in [0, \Lambda^D)$ and the for buying assets $f_t^a(\lambda)$ is given by (21).

As in Proposition 2, there must be some agents who borrow to buy assets while $d_t = D$. They do this in markets with high leverage, although not just in market $\lambda = 0$ where leverage is infinite. The high dividend regime still gives rise to credit booms and, if Φ isn't too large, bubbles. Borrowing to buy assets is still socially wasteful when $\Phi > 0$, and an intervention that discouraged speculation can be Pareto improving.

So far, we have only considered equilibria in which all markets are active. But for any λ, we can always construct an equilibrium in which market λ is inactive by setting the interest rate on loans $R_t(\lambda)$ above y^* to ensure no agent would want to borrow in that market, and the expected return $\overline{R}_t(\lambda)$ to be arbitrarily low to ensure no one would want to lend in market λ. Such equilibria are essentially coordination failures where markets that could sustain trade are instead inactive. Inactivity in some markets will generally affect prices and interest rates in remaining active markets, and so characterizing equilibria with inactive markets would require us to solve again for interest rates, asset prices, and amounts borrowed. We will not try to characterize all such equilibria. However, we will now turn to studying interventions that shut down markets with low λ. This is equivalent to studying equilibria in which markets with low λ are inactive because of what agents believe rather than because they were shut down by fiat. The reason markets are inactive is irrelevant for how inactivity affects other markets. Given our interest in the effect of restricting markets that would otherwise trade, it seems natural to focus on equilibria in which markets are maximally active.

5.2 The Effect of Leverage Restrictions

Proposition 8 implies speculators only borrow in markets with low λ. A natural way to intervene against speculation, then, is to shut down all markets λ below some floor $\underline{\lambda}$, or, alternatively, to cap the leverage agents can take on. Agents with wealth $w < \underline{\lambda}$ can only undertake projects of size at most $w/\underline{\lambda} < 1$. We mostly consider a permanent floor at all dates, although later we also consider a floor only while $d_t = D$.

We restrict attention to equilibria in which all markets $\lambda \geq \underline{\lambda}$ are active. The equilibria in Proposition 8 is then a special case where $\underline{\lambda} = 0$. When $\underline{\lambda} > 0$, the same arguments imply interest rates $R_t(\lambda)$ are given by (19) when $d_t = D$. That is, interest rates $R_t^D(\lambda)$ given the expected return $\overline{R}_t^D$ is unchanged. However, changing $\underline{\lambda}$ may affect $\overline{R}_t^D$. Given (19), entrepreneurs will still prefer to invest all of their wealth w and borrow $1 - w$ to produce at full capacity. But entrepreneurs with $w < \underline{\lambda}$ cannot do so. Since their profits are decreasing with λ for $\lambda > w$, they will all flock to market $\underline{\lambda}$ and produce at scale $w/\underline{\lambda} < 1$. The total inputs entrepreneurs will use to produce is then

$$\begin{aligned}\int_{w=0}^{\underline{\lambda}} 2\varphi e\left(\frac{w}{\underline{\lambda}}\right) dw + \int_{w=\underline{\lambda}}^{1} (2\varphi e)\, dw &= \frac{\varphi e}{\underline{\lambda}} w^2 \Big/_{0}^{\underline{\lambda}} + 2\varphi e\,(1-\underline{\lambda}) \\ &= \underline{\lambda}\varphi e + (1-\underline{\lambda})\, 2\varphi e\end{aligned}$$

The amount that remains to spend on the asset is $(1+\varphi)\, e$ minus the above, which pins down its price:

$$p_t^D = (1 - \varphi\,(1-\underline{\lambda}))\, e \tag{22}$$

Increasing $\underline{\lambda}$ will lead to a higher asset price. Intuitively, leverage restrictions force poor entrepreneurs to operate at a smaller scale. Since savers want to save a fixed amount e regardless of $\underline{\lambda}$, the decline in production will release resources to buy assets, pushing p_t^D up. If the floor $\underline{\lambda}$ is imposed permanently, we similarly have $p_t^d = (1 - \varphi(1 - \underline{\lambda}))\, e$. The expected return on the asset when $d_t = D$ will then be

$$1 + \overline{r}_t^D = \frac{(1-\pi)\left(D + p_{t+1}^D\right) + \pi\left(d + p_{t+1}^d\right)}{p_t^D} = 1 + \frac{(1-\pi)\, D + \pi d}{(1 - \varphi(1 - \underline{\lambda}))\, e}$$

Increasing $\underline{\lambda}$ thus reduces the expected return to buying the asset $\overline{r}_t^D$. As for how increasing $\underline{\lambda}$ affects the schedule of interest rates $R_t(\lambda)$, this is hard to summarize. But we show in Appendix C that the expected return to lending $\overline{R}^D$ declines with $\underline{\lambda}$. Intuitively, increasing $\underline{\lambda}$ depresses demand for credit, and so should lower interest rates. Summarizing, a permanent increase in $\underline{\lambda}$ has the following effects:

Proposition 9 *Consider a permanent floor $\underline{\lambda}$. The asset price $p_t^D = (1-\varphi(1-\underline{\lambda}))e$ increases with $\underline{\lambda}$, while the expected returns on the asset $\bar{r}^D = \frac{(1-\pi)D+\pi d}{(1-\varphi(1-\underline{\lambda}))e}$ and the expected return to lending $\bar{R}^D$ decrease with $\underline{\lambda}$.*

Both the leverage restrictions above and the contractionary monetary policy in the previous section depress entrepreneurial production. Tighter monetary policy does so by reducing the resources e_0 agents can invest, while leverage restrictions force poor entrepreneurs to operate below capacity. As a result, the two interventions have the opposite effects on asset prices and the return to savings. Nevertheless, both tighter monetary policy and leverage restrictions can help discourage speculation. Let γ^D denote the share of assets purchased with leverage rather than directly by savers. Although leverage restrictions increase the asset price p^D, they also tend to reduce the share of assets purchased with debt γ^D. Indeed, setting $\underline{\lambda}$ above Λ^D will drive γ^D to 0 given that no agent will borrow to buy assets in markets $\lambda \geq \Lambda^D$. More generally, expected default costs are equal to $\pi\Phi\gamma^D p^D$. Whether increasing $\underline{\lambda}$ raises the deadweight loss from default depends on how increasing $\underline{\lambda}$ affects γ^D and p^D, respectively. Our next result shows that under certain conditions, increasing $\underline{\lambda}$ will increase p^D without changing γ^D. Specifically, this will be the case if the floor $\underline{\lambda}$ is already low and Φ isn't too large so that $\gamma^D = 1$, as well as if $\underline{\lambda}$ is already high enough to exceed Λ^D so $\gamma^D = 0$. In these cases, raising $\underline{\lambda}$ will make agents worse off. But we also argue there exists an intermediate value of $\underline{\lambda}$ for which increasing $\underline{\lambda}$ will decrease γ^D enough to lower expected default costs $\pi\Phi\gamma^D p^D$. Increasing $\underline{\lambda}$ is thus ambiguous, and can in principle either increase or decrease welfare.

Proposition 10 *There exist cutoffs $0 \leq \Lambda_0 < \Lambda_1 < 1$ in $[0, 1)$ such that*

1. *If $\underline{\lambda} < \Lambda_0$, increasing $\underline{\lambda}$ leaves $\gamma^D = 1$, increases expected default costs $\pi\gamma^D\Phi p^D$, and leaves fewer goods for cohorts to consume from date $t = 1$ on.*
2. *If $\underline{\lambda} \geq \Lambda_1$, $\gamma^D = 0$ and there is no default. Increasing $\underline{\lambda}$ then leaves fewer goods for cohorts to consume from date $t = 1$ on.*
3. *If $\Lambda_0 < \underline{\lambda} < \Lambda_1$, there exist values of $\underline{\lambda}$ at which increasing $\underline{\lambda}$ lowers γ^D and expected default costs $\pi\gamma^D\Phi p^D$. In this case, increasing $\underline{\lambda}$ while $d_t = D$ can be Pareto improving for large Φ.*

Our model thus shows that leverage restrictions can be counterproductive: They increase the total amount leveraged speculators spend on the asset as well as expected default costs. Kim and Santomero (1988) previously argued that leverage restrictions can lead to greater risk-taking. In their model, this was because such restrictions encourage borrowers to opt for even riskier projects. Our model does not feature a range of risky projects. Instead, our result stems from the fact in risk-shifting models there must be some investment activity that cross-subsidizes speculation. If this other investment is particularly sensitive to leverage restrictions, intervention may end up redirecting resources toward speculation. We anticipate that the same would be true in risk-shifting models where speculators and less risky investors buy the same asset. For example, if demand for housing by liquidity constrained home buyers was particularly sensitive to leverage restrictions while the funds available for mortgage lending is relatively inelastic with respect to interest rates, leverage restrictions might very well encourage more speculative activity.[12]

While leverage restrictions have ambiguous welfare effects, in our model a threat to restrict leverage in the future will unambiguously make things worse today. Recall that tighter monetary policy if $d_{t+1} = D$ will lower p^D_{t+1}, discouraging speculation at date t. By contrast, raising $\underline{\lambda}$ at date $t+1$ will increase p^D_{t+1}. Regardless of how it affects γ^D_{t+1}, a higher p^D_{t+1} encourages speculation at date t. This contrast highlights how the two interventions affect asset prices and interest rates in opposite ways.

That said, we should be clear that while tighter leverage restrictions generally reduce demand for credit and lead to lower interest rates, the prediction of our model that this always leads to higher asset prices will not as naturally generalize. Suppose we let the wealth and productivity of entrepreneurs follow a general distribution $n(w, y)$. Entrepreneurs with low productivity would act like savers while entrepreneurs with high productivity would borrow to produce. An increase in $\underline{\lambda}$ that lowers the return to saving could induce some entrepreneurs who are on the margin to switch from lending out their wealth to borrowing in order to produce. If enough entrepreneurs switch from lending to producing, the fall in lending and the increase in demand for borrowing to produce could leave fewer resources to spend on the asset, and its price will fall. We confirm numerically that there exist distributions $n(w, y)$ for which increasing $\underline{\lambda}$ reduces p^D_t.[13]

While increasing $\underline{\lambda}$ can in principle dampen asset prices, in our model this will only be possible if there is risk-shifting. When $d_t = d$ and there is no risk shifting, increasing $\underline{\lambda}$ will raise p^d_t regardless of the distribution $n(w, y)$. To see this, recall when $d_t = d$, the interest rate $R^d_t(\lambda)$ is the same R^d_t for all λ. This common rate R^d_t and the asset price p^d_t satisfy two equilibrium conditions similar to (3) and (4). First, since

[12] Two recent papers propose still other ways in which central bank policies intended to cool activity might be counterproductive. Hachem and Song (2018) show that forcing banks to hold more liquidity may paradoxically lead to more interbank lending as large banks hold fewer reserves to hurt the small banks they compete with. Chen, Rhen, and Zha (2018) argue contractionary monetary policy in China led to an increase in lending by shadow banks as a fall in deposits encouraged banks to lend more to shadow banks to avoid liquidity coverage requirements.

[13] Even without relying on a more general distribution $n(w, y)$, our results are in part due to our assumption that savers only like to consume when old, and so their saving is inelastic with respect to the interest rate. If we modified this, tighter leverage constraints that reduce the returns to savings could lead agents to save less and asset prices would fall.

all resources must be used to produce or buy the asset, we have

$$\int_{R^d}^{\infty} \int_0^1 \min\left\{1, \frac{w}{\underline{\lambda}}\right\} n\left(w, y\right)\, dw\, dy + p_t^d = \int_0^{\infty} \int_0^1 wn\left(w, y\right)\, dw\, dy + e \tag{23}$$

This defines R_t^d as a function $\rho_{\underline{\lambda}}\left(p_t^d\right)$ of the price p_t^d which is increasing in p_t^d for a fixed $\underline{\lambda}$ and decreasing in $\underline{\lambda}$ for a fixed p_t^d. Second, the interest rate on loans must equal the return on the asset, and so

$$\left(1 + R_t^d\right) p_t^d = d + p_{t+1}^d \tag{24}$$

Substituting in $R_t^d = \rho_{\underline{\lambda}}\left(p_t^d\right)$ implies $p_{t+1}^d = \rho_{\underline{\lambda}}\left(p_t^d\right) - d$. Figure 5 illustrates the effect of increasing $\underline{\lambda}$ graphically and shows that the steady state p^d will rise. Intuitively, increasing $\underline{\lambda}$ requires the interest rate on loans to fall so that credit markets continue to clear even as demand for credit falls. Without risk, the interest rate on loans and the return on the asset are equal, so the latter must fall. A lower return on the asset implies its price is higher. With risk shifting, the interest rate on loans and the return on the asset can differ, so it will be possible for interest rates on loans to fall while the return on the asset rises.

6 Conclusion

This paper analyzes policy in a risk-shifting model of asset prices. As in previous work on risk-shifting, our model can capture many observable features of asset and credit booms and busts. The general equilibrium framework we use allows us to go beyond this and analyze policy and welfare. We show that risk-shifting leads to excessive lending that finances socially costly speculative activity, creating a role for intervention. We then study some of the proposed remedies against booms such as contractionary monetary policy and leverage restrictions. In our model, tighter monetary policy increases interest rates, lowers asset prices, and lowers the amount spent on assets. Leverage restrictions have the opposite effect, lowering interest rates and increasing asset prices. But they also discourage borrowing against assets. Both policies turn out to have ambiguous welfare implications. Whether a policy improves welfare depends on how it affects speculators vis-a-vis the productive activities that cross-subsidize them. Neither intervention is optimal in our setting. Both reduce the output available for agents, in contrast to cap on total lending that eliminates speculation without affecting output. Finally, we find that when default costs are large, risk shifting can occur without giving rise to bubbles. This reveals that, given evidence of risk-shifting, policymakers contemplating intervening against asset booms might not need to determine if asset prices exceed fundamentals to justify their intervention.

We focus on risk shifting because asset booms often feature opaque assets that make it is difficult for lenders to judge the risks from any given borrower. However, a large literature has analyzed asset booms and bubbles without risk shifting. These models should not be viewed as competing explanations, since the mechanisms they consider are complementary to the risk-shifting we study. For example, there is a large literature showing bubbles can arise with fully rational agents because of dynamic inefficiency as in Galí (2014, 2017) or binding credit market frictions as in Martin and Ventura (2012), Hirano and Yanagawa (2017), and Miao and Wang (2018). These papers all consider bubbles that burst stochastically. Since they feature a risk that asset prices collapse, these models can potentially give rise to risk-shifting. In particular, this would occur if lenders are unsure what their borrowers are doing. For example, Bengui and Phan (2018) combine risk-shifting and dynamic inefficiency by assuming loans are pooled and lenders cannot monitor individual borrowers. One can similarly combine risk-shifting and models of bubbles based on borrowing constraints. The distortions we emphasize in our model would then have to be balanced against the fact that overvalued assets may help relax borrowing constraints. A separate literature derives bubbles by assuming agents disagree about the risky returns on assets, e.g. Scheinkman and Xiong (2003), Hong, Scheinkman, and Xiong (2006), Simsek (2013), and Barberis, Greenwood, Jin, and Shleifer (2018). Such differences in beliefs are again compatible with uncertainty about the risks lenders are exposed to. For example, we can allow savers in our model to hold different beliefs about the asset so that the most optimistic agents buy assets while the least optimistic prefer to lend. Whether risk-shifting interacts with disagreement in interesting ways remains an open question.

Finally, our model suggests several directions for future research on risk-shifting models of asset booms. First, we assumed lenders suffer a cost Φ when their borrowers default. In practice, the costs of collapsing asset prices also depend on how agents respond when asset prices fall. To get at these channels would require introducing financial intermediaries or borrowing constraints. These may have important implications for how well interventions work. In terms of applications, we have described how our setup might have analogs in the housing market. However, cross-subsidization in the housing market works differently given all borrowers buy housing. By contrast, in our model the safe activity does not involve buying an asset. This raises the question of whether an intervention that shifts resources from illiquid home buyers to speculators still drives house prices up as in our setting. Assuming speculators and safe investors both buy assets would also make it easier to study the effect of policies that affect the supply of assets, e.g. changing zoning restrictions in hot housing markets or using taxes to discourage construction. Finally, it is not obvious how the policies we study fare in more general environments. One example is open economies. While we argued a contractionary monetary policy raises interest rates and dampens asset prices, higher real rates may attract larger capital inflows. In that case, it is not clear whether asset prices would still fall. One could consider an open economy version of our model along the lines of Galí and Monacelli (2005) to explore such issues. Another issue is whether aggressive monetary easing when asset prices crash might exacerbate risk-shifting by bailing out lenders. In our simple framework, the only relevant issue is how an intervention affects the maximal return on the asset. But in a richer model in which lenders can choose how much to monitor and what contracts to offer, the way that policy interventions affect asset prices after a crash may lead lenders to behave in a way that encourages risk-shifting.

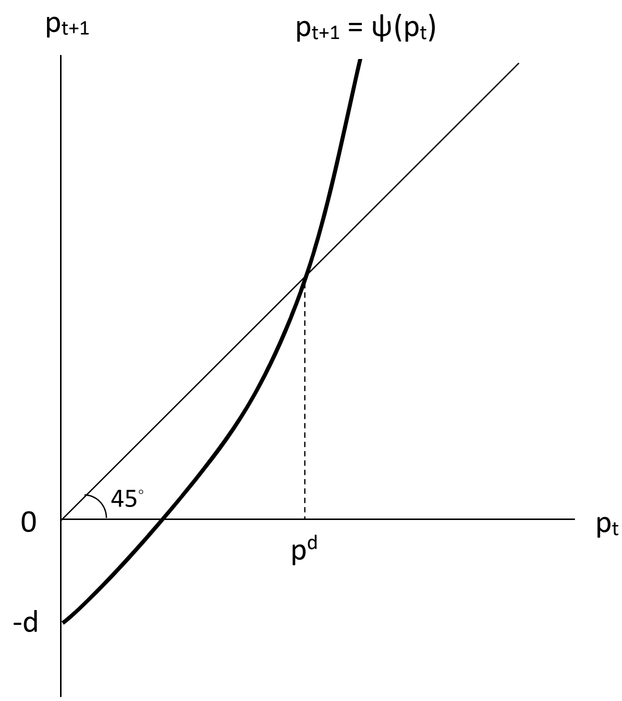

Figure 1: Determination of equilibrium price p^d with deterministic dividends

The value p^d denotes the steady state for the dynamical system $p_{t+1} = \psi(p_t)$. Any path which begins away from p^d leads either to a negative price or a price above e, neither of which can occur in equilibrium. Hence, the unique equilibrium is for the price to equal the steady state value p^d at all dates.

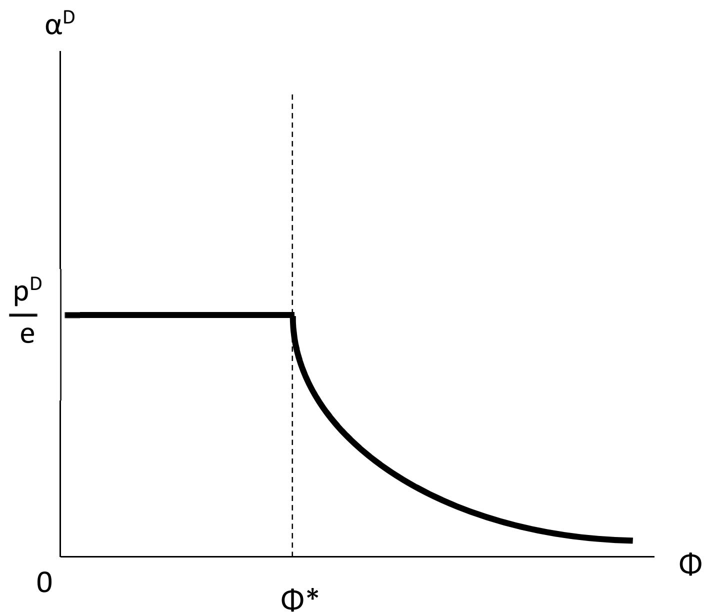

Figure 2: The share of loans used for speculation α_t^D as a function of default costs Φ

When the cost of default falls below Φ*, all assets are purchased by speculators who borrow, and the share of loans used for speculation α^d is equal to p^D/e. When the cost of default exceeds Φ*, savers purchase some of the asset directly, and the share of lending used for speculation falls, tending towards zero as the cost Φ tends to infinity.

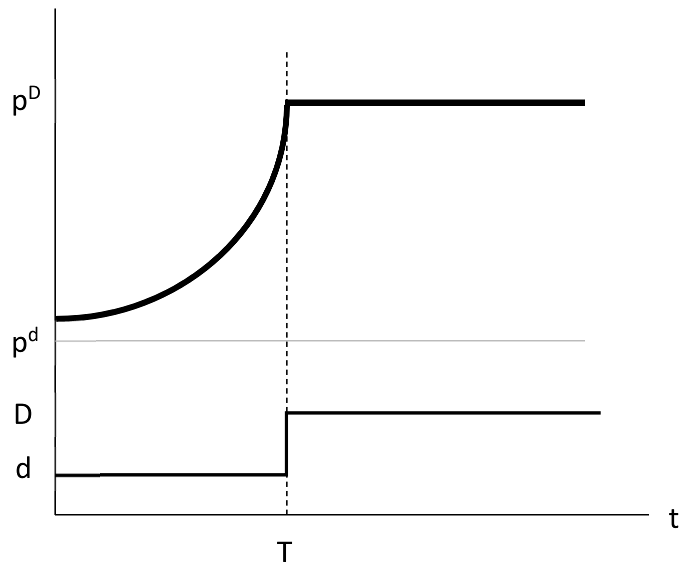

Figure 3: Equilibrium prices p_t^D with delayed dividend increase

The figure depicts the path of dividends and asset prices if dividends started out equal to d and jump to D at date T as long as we stay in the high regime. Prices follow an explosive path until date T even as dividends remain unchanged. From date T on, the price will equal p^D for as long as we remain in the high regime. When the state leaves the high regime, the price of the asset falls to p^d.

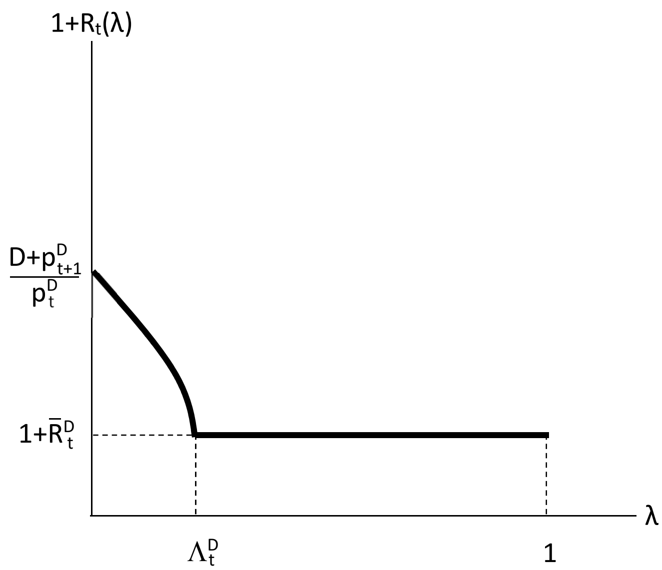

Figure 4: Interest rates as a function of share λ of investment that borrowers pay

The figure depicts the equilibrium schedule of interest rates across different markets. Interest rates are declining in the share λ of their projects that borrowers finance. For $\lambda < \Lambda_t^D$ the interest rate is falling in λ, and for $\lambda \geq \Lambda_t^D$ it is constant.

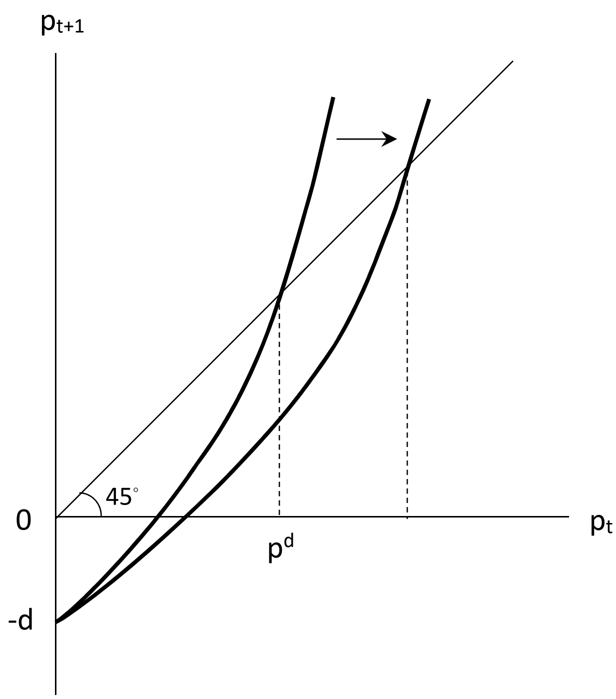

Figure 5: Effect of increasing floor $\underline{\lambda}$ with deterministic dividends

Appendix A: Proofs of Propositions

Proof of Proposition 1: In the text, we showed there is a unique deterministic equilibrium. Here we allow for stochastic equilibrium paths for $\{p_t, R_t\}_{t=0}^{\infty}$ and confirm that the equilibrium is in fact deterministic.

First, note that for any date t, in equilibrium it must be the case that $0 < p_t \leq e$. If the price $p_t \leq 0$ there would be infinite demand for the asset given its dividend $d > 0$ and there is free disposal. But the supply of assets is finite, so this cannot be an equilibrium. At the same time, the most any cohort can spend to buy the assets is e. Let z_t denote the return to buying the asset, i.e., $z_t \equiv \frac{d+p_{t+1}}{p_t}$. This can be random if p_{t+1} is random. Let $G_t(z)$ denote the (possibly degenerate) distribution of the return z_t. Since $0 < p_t \leq e$ for all t, the maximum return $z_t^{\max}$ is finite, since $z_t^{\max} = \frac{d+p_{t+1}^{\max}}{p_t} \leq \frac{d+e}{p_t} < \infty$, where $p_{t+1}^{\max}$ is the maximum possible realization of the price at date $t+1$.

The equilibrium satisfies two conditions. First, as in (3), all resources will be used either to buy assets or to initiate production:

$$\int_{R_t}^{\infty} n(y)\, dy + p_t = e \tag{25}$$

The implies $R_t = \rho(p_t)$ where $\rho'(\cdot) > 0$. Second, the interest rate on loans R_t must satisfy

$$(1 + R_t)\, p_t = d + p_{t+1}^{\max} \tag{26}$$

If the interest rate on loans $1 + R_t$ exceeded $\frac{d+p_{t+1}^{\max}}{p_t}$, no agent would want to buy assets, which cannot be an equilibrium. If interest rate on loans $1 + R_t$ exceeded $\frac{d+p_{t+1}^{\max}}{p_t}$, agents could earn positive profits from borrowing, so demand for credit would be infinite. Substituting $R_t = \rho(p_t)$ into (26) implies

$$p_{t+1}^{\max} = (1 + \rho(p_t))\, p_t - d$$

Suppose $p_t > p^d$. Consider the sequence $\{\widetilde{p}_\tau\}_{\tau=t}^{\infty}$ that comprises the upper support of prices at each date given the history of previous prices, starting from p_t. Formally, set $\widetilde{p}_t = p_t$ and define

$$\widetilde{p}_{\tau+1} = (1 + \rho(\widetilde{p}_\tau))\, \widetilde{p}_\tau - d$$

Since $p_t > p^d$, the sequence $\widetilde{p}_t$ would shoot off to infinity and would exceed e in finite time. This means there is a state of the world in which the price exceeds e, which cannot be an equilibrium. So $p_t \leq p^d$.

Next, suppose $p_t < p^d$. Again, we can construct the sequence $\{\widetilde{p}_\tau\}_{\tau=t}^{\infty}$ that comprises the upper support of prices at each date given the history of previous prices, starting from p_t. That is, we set $\widetilde{p}_t = p_t$ and then

$$\widetilde{p}_{\tau+1} = (1 + \rho(\widetilde{p}_\tau))\, \widetilde{p}_\tau - d$$

Since $p_t < p^d$, the sequence $\widetilde{p}_t$ would turn negative. Hence, there is a state of the world in which the price is negative, which cannot be an equilibrium. The distribution of the price at date t is degenerate with full support at p^d. From (25), $R_t = \rho(p_t)$ is uniquely determined as well. ■

Proof of Proposition 2: Below we fill in some of the missing steps from the discussion in the text.

First, we need to show that at any date t in which $d_t = D$, the return on the asset will be higher if $d_{t+1} = D$. That is, we need to show that

$$p_{t+1}^D + D > p^d + d$$

Suppose $p_{t+1}^D + D \leq p^d + d$. Since $D > d$, this requires $p_{t+1}^D < p^d$. From (3), we know the equilibrium interest rate on loans R_{t+1}^D must equal $\rho\left(p_{t+1}^D\right)$. If $p_{t+1}^D < p^d$, then since $\rho'(\cdot) > 0$, we have

$$R_{t+1}^D = \rho\left(p_{t+1}^D\right) < \rho\left(p^d\right) = R^d$$

But then we would have

$$\left(1 + R_{t+1}^D\right) p_{t+1}^D < \left(1 + R^d\right) p^d = p^d + d.$$

This means that if $d_{t+1} = D$, an agent who borrows to buy assets at date $t+1$ can make positive profits if $d_{t+2} = d$. But then there would be infinite demand for borrowing to buy assets, which cannot be an equilibrium given supply of credit is finite. Since this is inconsistent with equilibrium, it follows that $p_{t+1}^D + D > p^d + d$.

The text establishes that the equilibrium interest rate on loans must equal the maximal return on the asset, and so $p_t^D = p^D$ and $R_t^D = \rho\left(p^D\right)$. The step that remains is to solve for $\left\{\alpha_t^D\right\}_{t=0}^{\infty}$. For this, we use the expected return to the asset, denoted $\overline{r}_t^D$, and the expected return to lending, denoted $\overline{R}_t^D$. The former is given by

$$1 + \overline{r}_t^D = (1-\pi)\left(1 + \frac{D}{p^D}\right) + \pi\left(\frac{d+p^d}{p^D}\right) \equiv 1 + \overline{r}^D \tag{27}$$

As for the expected return to lending, a fraction α_t^D of lending is used to buy assets and the rest finances production. Since all of the proceeds from asset purchases accrue to the lender, the expected return to these loans is just the expected return to buying an asset net of default costs, $1 + \overline{r}^D - \pi\Phi$. The remaining loans that finance production will be repaid in full, so the return on those loans is $1 + R^D$. This implies

$$\begin{aligned} 1 + \bar{R}_t^D &= \left(1 - \alpha_t^D\right)\left(1 + R^D\right) + \alpha_t^D\left(1 + \overline{r}^D - \pi\Phi\right) \\ &= \left(1 - \alpha_t^D\right)\left(1 + \frac{D}{p^D}\right) + \alpha_t^D\left(1 + \overline{r}^D - \pi\Phi\right) \end{aligned} \tag{28}$$

If $\overline{R}_t^D > \overline{r}^D$, savers would prefer lending over buying assets. The only agents who would buy assets would be those who borrow to do so, and so $\alpha_t^D = \frac{p^D}{e}$. If $\overline{R}_t^D = \overline{r}^D$, savers would be indifferent between buying assets and lending. This means α_t^D can assume any value between 0 and $\frac{p^D}{e}$. Finally, if $\overline{R}_t^D < \overline{r}^D$, savers would prefer buying assets over lending. No agent would borrow to buy assets, implying $\alpha_t^D = 0$. Hence, the expected return to lending $\overline{R}_t^D$ and the share of lending used to buy assets α_t^D are jointly determined.

To solve for $\overline{R}_t^D$ and α_t^D, consider first the case where $\alpha^D = \frac{p^D}{e}$. This can only be an equilibrium if $\overline{R}_t^D \geq \overline{r}^D$ when $\alpha_t^D = \frac{p^D}{e}$, i.e., only if

$$\left(1 - \frac{p^D}{e}\right)\frac{D}{p^D} + \frac{p^D}{e}\left(\overline{r}^D - \pi\Phi\right) \geq \overline{r}^D$$

Rearranging this equation and substituting in for $\overline{r}^D$ implies $\alpha_t^D = \frac{p^D}{e}$ is an equilibrium only if

$$\Phi \leq \left(\frac{e}{p^D} - 1\right)\left(\frac{D + p^D - d - p^d}{p^D}\right) \equiv \Phi^* \tag{29}$$

Next, consider the case where $\alpha_t^D \in \left(0, \frac{p^D}{e}\right)$. This can only be an equilibrium if $\overline{R}_t^D = r^D$ when we evaluate $\overline{R}_t^D$ at the relevant α_t^D. Since $\overline{R}_t^D$ is decreasing in α_t^D, this requires that $\overline{R}_t^D < \overline{r}^D$ when $\alpha_t^D = \frac{p^D}{e}$, or

$$\Phi > \Phi^* \tag{30}$$

In this case, the equilibrium value of α_t^D is the one that equates $\overline{R}_t^D$ and $\overline{r}^D$, which implies

$$\alpha_t^D = \frac{D + p^D - d - p^d}{D + p^D - d - p^d + \Phi p^D} \tag{31}$$

Finally, there cannot be an equilibrium in which $\alpha_t^D = 0$. This would require $\overline{R}_t^D \leq \overline{r}^D$ when $\alpha_t^D = 0$. But $\alpha_t^D = 0$ implies $\overline{R}_t^D = \frac{D}{p^D} > \overline{r}^D$. Hence, the value of α_t^D is unique and is either equal to $\frac{p^D}{e}$ or some value between 0 and $\frac{p^D}{e}$, depending on the cost of default Φ. ■

Proof of Proposition 4: Here we fill the missing steps in deriving the equilibrium at date 0 when there is a quota. In the text, we argued that $1 + R_0^D \geq \frac{p^D + D}{p_0^D}$. Suppose $1 + R_0^D$ strictly exceeded $\frac{p^D + D}{p_0^D}$. Then no agent would borrow to buy assets knowing they would default. With agents only borrowing to produce, lending would be safe and would yield a higher return than the asset. Savers would prefer to lend, but under the quota can lend at most $e - p^D$ and must use the remaining p^D to buy assets. Since only they buy the asset, $p_0^D = p^D$. From the market clearing condition (3), we have $R_0^D = \rho\left(p^D\right) = R^D$. But we know $1 + R^D = 1 + \frac{D}{p^D}$, which contradicts our supposition that $1 + R_0^D > \frac{p^D + D}{p_0^D}$. It follows that $1 + R_0^D = \frac{p^D + D}{p_0^D}$. Combining this with (3) implies $R_0^D = R^D$ and $p_0^D = p^D$. Hence, imposing a lending cap of $e - p^D$ at date 0 will not change the price of the asset or the interest rate on loans relative to the equilibrium without a quota. Since savers spend at least p^D to buy assets under the quota and the value of assets is p^D, there can be no borrowers who buy the asset, so $\alpha_0^D = 0$.

A similar logic can be applied to a quota of $e - p^D$ at all dates as long as $d_t = D$. The market clearing condition (3) remains unchanged at all dates. First, the argument that $p_{t+1}^D > p^d$ for all t only relies on the market clearing condition (16), and is true even if we introduce a quota. Next, to ensure demand for borrowing is finite, we need $1 + R_t^D \geq \frac{p_{t+1}^D + D}{p_t^D}$. Suppose $1 + R_t^D > \frac{p_{t+1}^D + D}{p_t^D}$. In that case, no agent would borrow to buy the asset for any $\phi > 0$, and savers would strictly prefer lending to buying assets. Because of the quota, they would have to spend p^D on the asset. Hence, $p_t^D = p^D$, and from the market clearing condition, $R_t^D = \rho\left(p^D\right) = R^D$. This would imply $1 + R^D > \frac{p_{t+1}^D + D}{p^D}$. It follows that $p_{t+1}^D < p^D$. But this is impossible, since the quota would require savers spend at least p^D on the asset at date $t+1$ if $d_{t+1} = D$. The contradiction implies $1 + R_t^D = \frac{p_{t+1}^D + D}{p_t^D}$. The equilibrium conditions are therefore the same as in Proposition 2. The unique equilibrium is given by $\left(p_t^D, R_t^D\right) = \left(p^D, R^D\right)$ for all t. The same argument as above implies $\alpha_t^D = 0$ for all t. ■

Appendix B: Monetary Policy

This appendix introduces within-period production, a monetary authority, and nominal price rigidity into our setup as in our discussion in Section 4. We set up the model and derive the results that underlie Propositions 5 and 6 in the text.

B.1 Agent Types and Endowments

Our approach largely follows Galí (2014) in how we incorporate production, nominal price rigidity, and monetary policy into an overlapping generations economy with assets. As in our benchmark model, agents live two periods and care only about consumption when old. Each cohort still consists of two types – savers who are endowed with resources but cannot produce intertemporally and entrepreneurs endowed with no resources who can convert goods at date t into goods at date $t+1$. We continue to model entrepreneurs as in the benchmark model, but we now assume savers are endowed with the inputs to produce goods rather than with the goods themselves. This allows for an endogenous quantity of goods that can potentially vary with the stance of monetary policy.

More precisely, we assume two types of savers, each of mass 1. Half are workers, endowed with 1 unit of labor each who must choose how to allocate it. The other half are producers, endowed with the knowledge of how to convert labor into output but not with labor itself.[14] Producers set the price of the goods they produce and then hire the labor needed to satisfy their demand. Although producers and entrepreneurs both produce output, they differ in when and how they produce it. Producers born at date t convert labor into goods at date t. Entrepreneurs then convert the goods producers created at date t into goods at date $t+1$. Producers operate within the period; entrepreneurs operate across periods.

B.2 Production, Pricing, and Labor Supply

Workers allocate their one unit of labor to home and market production. Home production yields the same good as the market, but using a technology $h(\ell)$ that is concave in the amount of labor ℓ allocated to home production. We assume $h'(0) = 1$ and $h'(1) = 0$ for reasons that will become clear below.

Workers who sell their labor on the market earn a wage W_t per unit labor. Their labor services are hired by producers, whom we index by $i \in [0,1]$. Each producer can produce a distinct intermediate good according to a linear technology. In particular, if producer i hires n_{it} units of labor, she will produce $x_{it} = n_{it}$ units of intermediate good i. The different intermediate goods can then be combined to form final

[14] This setup borrows from Adam (2003) rather than Galí (2014). The latter assumes agents are homogeneous, selling labor when young and hiring labor when old. We want income to only accrue to the young as in our benchmark model.

consumption goods according to a constant elasticity of substitution (CES) production function available to all agents. That is, given x_{it} of each $i \in [0,1]$, the amount of final goods X_t that can be produced is

$$X_t = \left(\int_0^1 x_{it}^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} \tag{32}$$

with $\sigma > 1$. Let P_t denote the price of the final good and P_{it} denote the price of intermediate good i. At these prices, the x_{it} that maximize the profits of a final goods producer solve

$$\max_{x_{it}} P_t \left(\int_0^1 x_{it}^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} - \int_0^1 P_{it} x_{it} di$$

The first-order condition with respect to x_{it} yields

$$x_{it} = X_t \left(\frac{P_{it}}{P_t}\right)^{-\frac{1}{\sigma}} \tag{33}$$

If we set $X_t = 1$, we can compute the price of the cost of the optimal bundle of intermediate goods $x_{it} = \left(\frac{P_{it}}{P_t}\right)^{-1/\sigma}$ needed to produce one unit of the final good:

$$\int_0^1 P_{it} x_{it} di = \int_0^1 P_{it}^{1-\frac{1}{\sigma}} P_t^{\frac{1}{\sigma}} di$$

Since any agent can produce final goods, the price P_t must equal the per unit cost of producing a good in equilibrium. Equating the two yields the familiar CES price aggregator:

$$P_t = \left(\int_0^1 P_{it}^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \tag{34}$$

Each intermediate goods producer chooses their price P_{it} to maximize expected profits given demand (33) and wage W_t. To allow producers to move either before or after the monetary authority, we condition producer i's choice on their information Ω_{it} when choosing their price. Each producer will set P_{it} to solve

$$\max_{P_{it}} E\left[\left(P_{it} - W_t\right) X_t \left(\frac{P_{it}}{P_t}\right)^{-1/\sigma} \middle| \Omega_{it}\right]$$

The optimal price is then

$$P_{it} = \frac{E\left[W_t X_t | \Omega_t\right]}{(1-\sigma) E\left[X_t | \Omega_t\right]} \tag{35}$$

By symmetry, all producers will charge the same price, produce the same amount, and hire the same amount of labor, i.e., $n_{it} = n_t$ for all $i \in [0,1]$. The output of consumption goods is thus

$$X_t = \left(\int_0^1 n_t^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} = n_t$$

Workers receive $(W_t/P_t)\, n_t$ of these goods and producers get the remaining $(1 - W_t/P_t)\, n_t$. Workers also produce goods at home. Their income is thus $(W_t/P_t)\, n_t + h(1-n_t)$, which is maximized at

$$h'(1-n_t) = W_t/P_t \tag{36}$$

By contrast, the total resources available to young agents is $e_t = n_t + h(1-n_t)$, which is maximized at

$$h'(1-n_t) = 1$$

Our assumption that $h'(0) = 1$ implies total resources are maximized when $n_t = 1$ and all goods are produced in the market, and $e_t = n_t + h(1-n_t)$ is increasing in n_t for all $n_t \in [0,1]$.

B.3 Assets, Credit, and Money

Since agents want to consume when old, they will wish to save their earnings $e_t = n_t + h(1 - n_t)$. As in the benchmark model, they can buy assets and make loans. Without money, this specification would be equivalent to our benchmark model, the only difference being that the income of savers e_t which before was exogenous and fixed is now endogenous and potentially time-varying. Equilibrium in the asset and credit markets involves the same conditions as in the benchmark model. First, regardless of the income they earn, the young will spend all of their resources either funding entrepreneurs or buying assets, and so we still have

$$\int_{R_t}^{\infty} n(y)\, dy + p_t = e_t$$

where p_t is the real price of the asset and R_t is the real interest rate on loans. The interest rate R_t must still ensure agents cannot earn profits by borrowing and buying assets. When $d_t = d$, this requires

$$\left(1 + R_t^d\right) p_t^d = d + p_{t+1}^d$$

and when $d_t = D$, this requires

$$\left(1 + R_t^D\right) p_t^D = D + p_{t+1}^D$$

We can then use R_t and p_t to solve for the expected return on loans:

$$\overline{R}_t = \begin{cases} R_t^d & \text{if } d_t = d \\ \max\left\{\overline{r}_t^D, \left(1 - \frac{p_t^D}{e_t}\right) R_t^D + \frac{p_t^D}{e_t}\left(\overline{r}_t^D - \pi\Phi\right)\right\} & \text{if } d_t = D \end{cases} \tag{37}$$

where $\overline{r}_t^D$ is the expected real return to buying the asset. Below, we show that when prices are flexible or money is absent altogether, the equilibrium real wage W_t/P_t will be constant over time. Employment n_t and total earnings of all savers $e_t = n_t + h(1 - n_t)$ will then also be constant. The reduced form of our model in the absence of money thus coincides with our benchmark model.

To introduce money, we follow Galí (2014) in assuming money does not circulate in equilibrium. That is, money is the numeraire, and P_t and W_t denote the price of goods and labor relative to money. However, no agent actually holds money in equilibrium. The monetary authority announces a nominal interest rate i_t at each date t. The monetary authority commits to pay this rate at date $t+1$ to those who lend to it (with money it can always issue), and will charge i_t to those who borrow from it with full collateral. This is roughly in line with what central banks do in practice, paying interest on reserves and lending at the discount window against collateral. To ensure money doesn't circulate, the real return on lending to the monetary authority must equal the expected return on savings. Let $\Pi_t = P_{t+1}/P_t$ denote the gross inflation rate between dates t and $t+1$. Since agents always lend to entrepreneurs, the expected return on savings will equal $\overline{R}_t$, the expected return on loans. This implies

$$1 + i_t = \left(1 + \overline{R}_t\right) \Pi_t \tag{38}$$

When the monetary authority changes the nominal interest rate i_t, either inflation Π_t or the expected return $1 + \overline{R}_t$ or both will have to adjust to ensure agents will neither borrow nor lend to the monetary authority.

B.4 Defining an Equilibrium

Given a path of nominal interest rates $\{1+i_t\}_{t=0}^{\infty}$, an equilibrium consists of a path of prices $\{P_t, W_t, p_t, R_t\}_{t=0}^{\infty}$ and a path of employment $\{n_t\}_{t=0}^{\infty}$ such that agents behave optimally and markets clear. Collecting the relevant conditions from above yields the following five equations for these five variables:

(i) Optimal pricing: $P_t = \dfrac{E[W_t X_t|\Omega_t]}{(1-\sigma)\, E[X_t|\Omega_t]}$

(ii) Optimal labor supply: $h'(1-n_t) = W_t/P_t$

(iii) Optimal savings: $\int_{R_t}^{\infty} n(y)\, dy + p_t = e_t$

(iv) Credit market clearing: $1 + R_t = \begin{cases} \frac{D+p_{t+1}^D}{p_t^D} & \text{if } d_t = D \\ \frac{d+p_{t+1}^d}{p_t^d} & \text{if } d_t = d \end{cases}$

(v) Money market clearing: $\Pi_t = \dfrac{1+i_t}{1+\overline{R}_t}$

where the expected return on loans $\overline{R}_t$ in the last condition is given by (37).

B.5 Equilibrium with Flexible Prices

We begin with the case where producers set their prices P_{it} after observing the wage W_t. This corresponds to the case where prices are fully flexible, or alternatively where there is no money and so no sense in which nominal prices are set in advance. Producers can deduce what other producers will do and the labor workers will supply, they can perfectly anticipate total output X_t. Hence, their information set $\Omega_t = \{W_t, X_t\}$. It follows that $E[W_t X_t|\Omega_t] = W_t X_t$ and $E[X_t|\Omega_t] = X_t$. The optimal pricing rule (i) then implies

$$P_t = \frac{W_t}{1-\sigma}$$

The real wage is thus constant and equal to $1-\sigma$. Substituting this into (ii) yields

$$h'(1-n_t) = 1-\sigma \tag{39}$$

Since $h(\cdot)$ is concave, n_t is equal to some constant n^* for all t. It follows that $e_t = n^* + h(1-n^*)$ is also constant for all t. We can then use (iii) and (iv) to solve for p_t and R_t as in the benchmark model, and then use (37) to compute $\overline{R}_t$. Finally, given $\overline{R}_t$ we can use the implied Π_t from (v) to derive $\{P_t\}_{t=1}^{\infty}$ for any initial value for P_0. The initial price level P_0 is indeterminate, in line with the Sargent and Wallace (1975) result on the price level indeterminacy of pure interest rate rules. The nominal wage $W_t = (1-\sigma) P_t$.

B.6 Equilibria with Rigid Prices

We now turn to the case where producers set the price of their intermediate good P_{it} before the monetary authority moves. That is, producers set prices, the monetary authority sets $1+i_t$, and then producers hire workers at a nominal wage W_t. This formulation implies prices are only rigid for one period.

If monetary policy is deterministic, producers can perfectly anticipate the nominal interest rate and the equilibrium nominal wage W_t, and so $\Omega_t = \{W_t, X_t\}$ and $W_t/P_t = 1-\sigma$ as before.

Next, suppose monetary policy is contingent on some random variable, i.e., $i_t = i(\xi_t)$ where $\{\xi_t\}_{t=0}^{\infty}$ is a sequence of random variables. For simplicity, consider the case where ξ_t is only random at $t=0$, i.e.,

$$\xi_0 = \begin{cases} H & \text{w/prob } \chi \\ L & \text{w/prob } 1-\chi \end{cases}$$
$$\xi_t \text{ is deterministic for } t = 1, 2, ...$$

From date $t=1$ on, we know from the optimal price-setting condition (i) that $W_t/P_t = 1-\sigma$. It then follows that $n_t = n^*$ and $e_t = e^* \equiv n^* + h(1-n^*)$ for all $t \geq 1$, and we can determine p_t, R_t, and $\overline{R}_t$ for $t \geq 1$ just as in the case where prices are flexible. All we need is to solve for the equilibrium at date 0.

We use a superscript $\xi \in \{H, L\}$ to denote the value of a variable as for a given realization of ξ_0. Assume wlog that $i_0^H > i_0^L$. The optimal price setting condition (1) is now

$$\frac{\chi n_0^H \frac{W_0^H}{P_0} + (1-\chi) n_0^L \frac{W_0^L}{P_0}}{\chi n_0^H + (1-\chi) n_0^L} = 1-\sigma \tag{40}$$

That is, the output-weighted average real wage over the two values of ξ is equal to $1-\sigma$. Optimal labor supply (ii) then implies

$$\begin{aligned} h'\left(1-n_0^H\right) &= \min\left\{\frac{W_0^H}{P_0}, 1\right\} \\ h'\left(1-n_0^L\right) &= \min\left\{\frac{W_0^L}{P_0}, 1\right\} \end{aligned}$$

These are three equations for four unknowns, meaning the set of all equilibria can be parameterized by a single parameter. Wlog, we choose the real wage when $\xi = H$ to be this parameter. The three equations above yield values for W_0^L/P_0, n_0^H, and n_0^L given W_0^H/P_0. From these, we can deduce earnings $e_0^\xi = n_0^\xi + h\left(1-n_0^\xi\right)$ for each $\xi \in \{H, L\}$. We can then use (iii) and (iv) to derive p_0^ξ and R_0^ξ by solving

$$\int_{R_0^\xi}^{\infty} n(y)\,dy + p_0^\xi = e_0^\xi \tag{41}$$

$$\left(1+R_0^\xi\right) p_0^\xi = D + p^D \tag{42}$$

and then compute the expected return on loans $\overline{R}_0^\xi$ using (37), and, via (v), the inflation rate Π_0^ξ for each $\xi \in \{H, L\}$. As before, the price level P_0 is indeterminate. Optimal pricing only restricts the average real wage across states but not the real wage for each realization of ξ_0, introducing an indeterminacy. The equilibrium real wage can exceed $1-\sigma$ for one realization of ξ_0 if it falls below $1-\sigma$ for the other realization.

The case where monetary policy has no effect on real variables at date 0 remains an equilibrium. In this case, $W_0^H/P_0 = W_0^L/P_0 = 1-\sigma$. But price rigidity expands the set of equilibria to include ones in which real variables vary with the nominal interest rate. Since the nominal interest rate only serves as a signal to coordinate real activity but does not directly affect it, there are equilibria in which $W_0^H > W_0^L$ as well as equilibria in which $W_0^H < W_0^L$.[15] Since higher nominal interest rates seem to be contractionary in practice, we focus on equilibria in which $W_0^H/P_0 < 1-\sigma < W_0^L/P_0$, i.e., real wages are lower when the monetary authority unexpectedly raises the nominal interest rate. In this case, from condition (ii) we know that a higher nominal interest rate will be associated with lower employment ($n_0^H < n^* < n_0^L$) and hence lower earnings ($e_0^H < e^* < e_0^L$). From (41), we can infer that $R_0^\xi = \rho^\xi\left(p_0^\xi\right)$ where $\rho^H(x) > \rho^L(x)$ for the same value x. As is clear from Figure 1, this implies a higher nominal interest rate will be associated with a lower real asset price ($p_0^H < p^D < p_0^L$). This also implies a higher real interest rate on loans ($R_0^H > R^D > R_0^L$). The real expected return to buying assets will also be higher ($\overline{r}_0^H > \overline{r}^D > \overline{r}_0^L$). However, whether the real expected return to lending $\overline{R}_0^H$ will be higher is ambiguous. (37) implies $\overline{R}_0^\xi$ is either equal to $\overline{r}_0^\xi$ or to a weighted average of R_0^ξ and $\overline{r}_0^\xi$. In the latter case, although both terms are higher when $\xi = H$ the weight on $\overline{r}_0^\xi$, which is p_0^ξ/e_0^ξ, can be higher or lower for $\xi = H$. These results are summarized in Proposition 5 in the paper.

B.7 Redistribution and Welfare

We now argue that it will be possible to use redistribution to ensure that a monetary contraction is Pareto improving. To do this, ignore monetary policy temporarily and think about the effects of a lump sum tax τ_0 on savers at date 0 that is given to the old at that date. The wealth of savers is $e - \tau_0$. Our analysis above implies $\frac{dR_0^D}{d\tau_0} < 0$, i.e., impoverishing savers leads to a higher interest rate. From the market clearing condition, it follows that $0 < \frac{dp_0^D}{d\tau_0} < 1$. Hence, taxing savers and giving it to the old will make the old strictly better off. Since the derivatives $\frac{dp_0^D}{d\tau_0}$ and $\frac{dR_0^D}{d\tau_0}$ are independent of Φ, so the effect of the tax will be the same regardless of Φ. But from (14), when Φ is sufficiently large, a higher τ_0 will increase expected total consumption of the young. Thus, a redistribution from savers to the old will increase welfare for sufficiently large Φ. Intuitively, it is better to have the young give resources to the old directly than to lend them to speculators who use them to buy assets from the old.

Since a lump sum tax τ_0 makes both the old and the young better off, it will also make both sides better off if we discourage the young from working and reduce total output, as long as the fall in output is small. But this is exactly what contractionary monetary policy does. Hence, a redistribution combined with contractionary monetary policy can be Pareto improving.

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[15] One way to avoid such multiplicity is to assume dynamic monetary policy rules that are conditioned on past economic variables. This allows a central bank to take actions that are unsustainable if a high interest rate today leads to certain outcomes, eliminating equilibria with those outcomes. See Cochrane (2011) for a discussion of these issues.

B.8 Promises of Future Intervention

Our last point concerns the effects of a promise at date 0 to be contractionary at date 1 if the boom continues into that date. In this case, ξ_0 and ξ_t for $t \geq 2$ are deterministic, while $\xi_1 = d_1 \in \{d, D\}$. That is, we assume producers set prices each period before d_t is revealed. Solving for equilibrium at date 1 is identical to how we solved for the equilibrium at date 0 when we assumed ξ_0 was random. Consider equilibria in which the real wage is lower if the boom continues, so

$$W_1^D/P_1 < 1 - \sigma < W_1^d/P_1.$$

This implies $n_1^D < n^* < n_1^d$ and so $e_1^D < e < e_1^d$. In other words, if dividends fall and the boom ends, monetary policy must be expansionary. By the same logic as above, such a policy would imply $p_1^D < p^D$ and $p_1^d > p^d$, as well as $R_1^D > R^D$ and $R_1^d < R^d$. Turning back to date 0, conditions (iii) and (iv) imply

$$\begin{aligned} \int_{R_0^D}^{\infty} n(y)\, dy + p_0^D &= e \\ \left(1 + R_0^D\right) p_0^D &= D + p_1^D \end{aligned}$$

Comparative statics of this system with respect to p_1^D reveals that $p_0^D < p^D$ and $R_0^D < R^D$. That is, while contractionary monetary policy at date 0 dampens p_0^D but raises R_0^D at date 0, a threat to enact contractionary monetary policy at date 1 if dividends remain high will dampen both p_0^D and R_0^Dat date 0. These results are summarized in Proposition 6 in the paper.

Appendix C: Macroprudential Regulation

In this appendix, we define an equilibrium for an economy with multiple markets as in Section 5. We then show that for an equilibrium in which all markets are active, various aspects of the equilibrium are uniquely determined. We then discuss some comparative static results with respect to the set of active markets.

C.1 Defining an Equilibrium

We begin with some notation. Let p_t denote the price of the asset at date t. Given asset prices, we can define the return to buying the asset at date t as

$$z_t \equiv \frac{d_{t+1}+p_{t+1}}{p_t}$$

The return z_t can be random both because d_{t+1} might be uncertain (if $d_t = D$) and because p_{t+1} might in principle be stochastic. Let $G_t(z)$ denote the (possibly degenerate) cumulative distribution of the return z_t, i.e., $G(z) \equiv \Pr(z_t \leq z)$. Let $1 + r_t^{\max}$ denote the maximum possible return on the asset. As discussed in the text, $1 + r_t^{\max}$ is finite, since $r_t^{\max} \leq \frac{D+2\varphi e}{(1-\varphi)e}$. We will use $\overline{r}_t$ to denote the expected return to buying the asset at date t, i.e.,

$$1 + \overline{r}_t \equiv \int_0^{1+r_t^{\max}} z_t dG_t(z)$$

We now define variables for the different markets $\lambda \in [0,1)$ agents can borrow in. Let $R_t(\lambda)$ denote the interest rate on loans in market λ, so an agent who agrees to pay a share λ of the project she undertakes will promise to pay back $1 + R_t(\lambda)$ for each unit she borrows. Since agents may default, let $\overline{R}_t(\lambda)$ denote what lenders expect to earn from lending in market λ given the possibility of default. Finally, we represent borrowing in markets with density functions $f_t^a(\lambda)$ and $f_t^p(\lambda)$ for all $\lambda \in [0,1)$ such that the total amount of resources borrowed to buy assets and to produce are given by $\int_A f_t^a(\lambda)\, d\lambda$ and $\int_A f_t^p(\lambda)\, d\lambda$, respectively. Let $f_t(\lambda) \equiv f_t^a(\lambda) + f_t^p(\lambda)$ denote the density of borrowing for any purpose in market λ.

Representing the quantities agents borrow in each market as a density function ignores the possibility that there may be equilibria in which agents borrow a positive mass of resources in certain markets. More generally, we can allow for a set $\Delta \subset [0,1)$ with countably many elements such that each market $\lambda \in \Delta$ is associated with a positive mass of borrowing $m_t^x(\lambda) > 0$. The amount borrowed in any market $\lambda \in [0,1)\backslash\Delta$ can still be represented with a density function. Heuristically, we can appeal to the Dirac-delta construction and represent the amount borrowed in any market as if it were a density. That is, for any $\lambda \in \Delta$, we set the density $f_t^x(\lambda) = m_t^x(\lambda)\,\delta_\lambda(\lambda)$, where $\delta_\lambda(q)$ is the Dirac-delta function defined so that $\delta_\lambda(q) = 0$ for $q \neq 0$ and $\int_0^1 \delta_\lambda(q)\, dq = 1$. This convention treats markets $\lambda \in \Delta$ as essentially having an infinite density. We will refer to a market λ as *inactive* if $f_t(\lambda) = 0$ and *active* if $f_t(\lambda) > 0$ or if $\lambda \in \Delta$.

Given these preliminaries, we define an equilibrium as a path $\left\{p_t, f_t^p(\lambda), f_t^a(\lambda), R_t(\lambda), \overline{R}_t(\lambda)\right\}_{t=0}^{\infty}$ that satisfies a series of conditions, (43)-(48), that ensure all markets clear when agents optimize.

The first few conditions we describe stipulate that all agents act optimally. We begin with lenders. Optimality requires that agents will only invest their wealth where the expected return is highest. Let $\overline{R}_t$ denote the maximal expected return to lending in any market λ, i.e.,

$$\overline{R}_t \equiv \sup_{\lambda \in [0,1)} \overline{R}_t(\lambda)$$

Optimal lending requires that agents lend in market λ' only if it they expect to earn $\overline{R}_t$ and if this rate exceeds the expected return to buying the asset, i.e.,

$$f_t(\lambda') > 0 \text{ only if } \overline{R}_t(\lambda') = \overline{R}_t \text{ and } \overline{R}_t \geq \overline{r}_t \tag{43}$$

Entrepreneurs must also act optimally. We first argue this means they should use their endowment to produce. Recall entrepreneurs have productivity y^* where $y^* > r_t^{\max} \geq \overline{r}_t$ from (15), so producing is better than buying assets. But y^* must also exceed the expected return to lending $\overline{R}_t$. For suppose $\overline{R}_t$ were higher than y^*. Since $y^* > r_t^{\max}$, then $\overline{R}_t$ must also exceed $r_t^{\max}$. In that case, no agent would use their endowment to buy assets, nor would any agent borrow to buy assets given the interest rate on loans in any active market must be at least $\overline{R}_t$. Yet assets must trade in equilibrium: Owners sell their assets whenever the asset price is positive, while demand for the asset would be infinite if its price were nonnegative. Since production offers the highest return, entrepreneurs should use their endowment w to produce.

Since entrepreneurs can leverage their endowment to produce at a larger capacity, we also need to characterize their borrowing. If they borrow in market λ where $\lambda < w$, they can borrow enough to reach full capacity. Optimality requires that there will be borrowing to produce in market λ' only if some entrepreneur finds it optimal to borrow in that market from all $\lambda \in [0,1]$, including $\lambda = 1$ for no borrowing. This implies

$$f_t^p(\lambda') > 0 \text{ only if } \lambda' \in \arg\max_{\lambda \in [0,1]} \left\{ \begin{array}{ll} [1 + y - (1-w)(1 + R_t(\lambda))] & \text{if } \lambda \leq w \\ \dfrac{w}{\lambda}[1 + y - (1-\lambda)(1 + R_t(\lambda))] & \text{if } \lambda > w \end{array} \right\} \text{ for some } w \tag{44}$$

Finally, agents who borrow to buy assets must act optimally. They will agree to borrow in market $\lambda \in [0,1)$ to buy assets only if doing so yields a higher expected return than lending out the same resources. Define

$$x_t(\lambda) \equiv (1 + R_t(\lambda))(1-\lambda)$$

The expected profits from borrowing in market λ to buy one consumption unit's worth of assets is

$$\int_{x_t(\lambda)}^{\infty} (z_t - x_t(\lambda))\, dG(z_t) \tag{45}$$

Agents will borrow in market λ to buy assets only if (45) equals $(1 + \overline{R}_t)\lambda$, the return they could have earned on any wealth that they use to buy assets. If (45) were lower than $(1 + \overline{R}_t)\lambda$, no agent would borrow to buy assets. If (45) were higher than $(1 + \overline{R}_t)\lambda$, then no one would ever lend given they can borrow in market λ', and so $f_t(\lambda') = 0$. But this contradicts the fact that $f_t^a(\lambda) > 0$. Optimality implies

$$f_t^a(\lambda') > 0 \text{ only if } \int_{x_t(\lambda')}^{\infty} (z_t - x_t(\lambda'))\, dG(z_t) = (1 + \overline{R}_t)\lambda' \tag{46}$$

Next, we require that savers use their entire endowment e to ensure consumption when old rather than go to waste. Since entrepreneurs prefer to use their endowment w for production, all the resources used to buy the asset must come from savers. This implies that e must be either lent to entrepreneurs to produce or be spent on assets:

$$\int_0^1 f_t^p(\lambda)\, d\lambda + p_t = e \tag{47}$$

Finally, we turn to equilibrium beliefs. In any active market λ', lenders must expect the return on lending $\overline{R}_t(\lambda')$ to conform with the actual fraction of borrowers who borrow in market λ' with the intent to produce and to buy assets, respectively. That is,

$$\overline{R}_t(\lambda') = \frac{f_t^p(\lambda')}{f_t(\lambda')} R_t(\lambda') + \frac{f_t^a(\lambda')}{f_t(\lambda')} E_t \min\left\{R_t(\lambda'), \frac{d_{t+1} + p_{t+1}}{p_t} - 1\right\} \text{ if } f_t(\lambda') > 0 \tag{48}$$

In a market $\lambda \in \Delta$ with a positive mass of borrowing, the expression $\frac{f_t^x(\lambda')}{f_t(\lambda')}$ will be replaced by $\frac{m_t^x(\lambda)}{m_t(\lambda)}$. Condition (48) does not impose any restrictions on expectations in inactive markets where $f_t(\lambda') = 0$.

C.2 Solving for Equilibrium

We now proceed to solve for an equilibrium. As in the text, we restrict attention to equilibria in which all markets $\lambda \in [0, 1)$ are active. Such equilibria are natural given we focus on the effects of interventions to shut down markets. Our first result characterizes the schedule of interest rates in such an equilibrium.

Proposition C1: *In an equilibrium where all markets are active, there exists a value $\Lambda_t \in [0, 1]$ such that the equilibrium interest rate schedule will be given by*

$$1 + R_t(\lambda) = \begin{cases} \frac{\widetilde{x}_t(\lambda)}{1-\lambda} & \text{if } \lambda \in [0, \Lambda_t) \\ 1 + \overline{R}_t & \text{if } \lambda \in [\Lambda_t, 1) \end{cases} \tag{49}$$

where $\widetilde{x}_t(\lambda)$ is the value of x that solves

$$\int_{z=x}^{1+r_t^{\max}} (z - x)\, dG_t(z) = \left(1 + \overline{R}_t\right) \lambda \tag{50}$$

The schedule of interest rates $R_t(\lambda)$ is a decreasing and continuous function of λ for $\lambda \in [0, \Lambda_t]$.

Proof of Proposition C1: Our proof proceeds as two lemmas.

Lemma C1: In an equilibrium where all markets are active, $1 + R_t(\lambda) = \max\left\{\frac{\widetilde{x}_t(\lambda)}{1-\lambda}, 1 + \overline{R}_t\right\}$, where $\widetilde{x}_t(\lambda)$ equals the x that solves (50) and $\overline{R}_t$ is the expected return to lending in any market λ.

Proof of Lemma C1: Equilibrium condition (46) holds that agents are either indifferent between lending their wealth and using it as a down payment in some market λ to buy assets, or else they strictly prefer to lend their wealth. That is, for all $\lambda \in [0,1)$, we have

$$\int_{z=x_t(\lambda)}^{1+r_t^{\max}} (z - x_t(\lambda))\, dG_t(z) \leq \left(1+\overline{R}_t\right)\lambda \tag{51}$$

In the latter case, since no agent borrows to buy the asset, we know that $R_t(\lambda) = \overline{R}_t$. This is one candidate for the interest rate in market λ. The other candidate is any value of $R_t(\lambda)$ which ensures (51) holds with equality. We now argue that is exactly one such candidate.

Consider the expression $\int_{z=x}^{1+r_t^{\max}} (z-x)\, dG_t(z)$. It is strictly decreasing in x, it tends to $+\infty$ as $x \to -\infty$ and to 0 as $x \to 1+r_t^{\max}$. Hence, for any $\lambda \in [0,1)$ and any $\overline{R}_t \geq 0$, there exists a unique $x \in (-\infty, 1+r_t^{\max}]$ for which

$$\int_{z=x}^{1+r_t^{\max}} (z-x)\, dG_t(z) = \left(1+\overline{R}_t\right)\lambda \tag{52}$$

Denote $\widetilde{x}_t(\lambda)$ as the unique solution to equation (52). If the LHS of (52) represents the payoff to borrowing to buy an asset, the expression $\widetilde{x}_t(\lambda)$ would correspond to the debt obligation of an agent who borrows in market λ, i.e. $\widetilde{x}_t(\lambda)$ would equal $(1+R_t(\lambda))(1-\lambda)$. Hence, the unique interest rate that ensures (51) holds with equality is given by

$$1 + R_t(\lambda) = \frac{\widetilde{x}_t(\lambda)}{1-\lambda}$$

Thus, there are two candidate expressions for the equilibrium interest rate in any market λ, namely $\frac{\widetilde{x}_t(\lambda)}{1-\lambda}$ and $\overline{R}_t$. To show that $1 + R_t(\lambda) = \max\left\{1+\overline{R}_t, \frac{\widetilde{x}_t(\lambda)}{1-\lambda}\right\}$, consider first a value of λ for which $\frac{\widetilde{x}_t(\lambda)}{1-\lambda} > 1+\overline{R}_t$. We want to argue that in this case, $\frac{\widetilde{x}_t(\lambda)}{1-\lambda}$ is the only possible equilibrium interest rate. Since $\int_{z=x}^{1+r_t^{\max}} (z-x)\, dG_t(z)$ is decreasing in x, it follows that

$$\int_{z=(1+\overline{R}_t)(1-\lambda)}^{1+r_t^{\max}} \left(z - (1+\overline{R}_t)(1-\lambda)\right) dG_t(z) > \int_{z=\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} (z - \widetilde{x}_t(\lambda))\, dG_t(z) = \left(1+\overline{R}_t\right)\lambda$$

Since the equilibrium interest rate $R_t(\lambda)$ must satisfy (51), we cannot $R_t(\lambda) = \overline{R}_t$. The only possible equilibrium for these values of λ is $1 + R_t(\lambda) = \frac{\widetilde{x}_t(\lambda)}{1-\lambda}$. In other words, for any λ such that $\frac{\widetilde{x}_t(\lambda)}{1-\lambda} > 1+\overline{R}_t$, the equilibrium interest rate must leave agents just indifferent leveraging their wealth and borrowing to speculate in market λ and lending out the same wealth and earning an expected return of $\overline{R}_t$.

Next, consider a value of λ for which $\frac{\widetilde{x}_t(\lambda)}{1-\lambda} < 1+\overline{R}_t$. In this case, $\frac{\widetilde{x}_t(\lambda)}{1-\lambda}$ cannot be an equilibrium interest rate for market λ, since it would mean the interest rate on loans in market is lower than the return lenders can earn elsewhere. That cannot be true in equilibrium. Hence, if $\frac{\widetilde{x}_t(\lambda)}{1-\lambda} < 1+\overline{R}_t$, the only one of the two candidates that can be an equilibrium is $R_t(\lambda) = \overline{R}_t$. Given that $\frac{\widetilde{x}_t(\lambda)}{1-\lambda} < 1+\overline{R}_t$, we can conclude that the expected payoff from borrowing to buy the asset and defaulting if the return is low is worse than lending at the safe rate $\overline{R}_t$, so no agent will borrow to buy assets in market λ. This establishes the lemma. ■

Our second lemma establishes that $\frac{\widetilde{x}_t(\lambda)}{1-\lambda}$ is a weakly decreasing and continuous function of λ. Combined with Lemma C1, this implies there exists a cutoff Λ_t such that $R_t(\lambda) = \overline{R}_t$ for $\lambda \geq \Lambda_t$.

Lemma C2: In any equilibrium where all markets are active, $\frac{\widetilde{x}_t(\lambda)}{1-\lambda}$ is nonincreasing and continuous in λ.

Proof of Lemma C2: The function $\widetilde{x}_t(\lambda)$ corresponds to the value of x which solves (50). Note that even though the distribution $G_t(z)$ can contain mass points, the integral $\int_{z=x}^{1+r_t^{\max}} (z-x)\, dG_t(z)$ is continuous in x and so $\widetilde{x}_t(\lambda)$ is a continuous function of λ. However, $\widetilde{x}_t(\lambda)$ may exhibit kinks. To show that $\widetilde{x}_t(\lambda)$ is decreasing, it will suffice to show that its directional derivatives are nonpositive for all $\lambda \in [0,1)$. Totally differentiating (50) with respect to λ implies

$$\frac{d\widetilde{x}_t(\lambda)}{d\lambda} = -\frac{1+\overline{R}_t}{\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z)}$$

For any λ where $\widetilde{x}_t(\lambda)$ is a mass point of $G_t(z)$, $\lim_{\lambda' \to \lambda^+} \int_{\widetilde{x}_t(\lambda')}^{1+r_t^{\max}} dG_t(z) \neq \lim_{\lambda' \to \lambda^-} \int_{\widetilde{x}_t(\lambda')}^{1+r_t^{\max}} dG_t(z)$. Nevertheless, both $\lim_{\lambda' \to \lambda^+} \frac{d\widetilde{x}_t(\lambda')}{d\lambda'}$ and $\lim_{\lambda' \to \lambda^-} \frac{d\widetilde{x}_t(\lambda')}{d\lambda'}$ are negative, so $\widetilde{x}_t(\lambda)$ is strictly decreasing in λ. But we want to show that $\frac{\widetilde{x}_t(\lambda)}{1-\lambda}$ is decreasing in λ and not just $\widetilde{x}_t(\lambda)$.

Define $\widetilde{R}_t(\lambda) \equiv \frac{\widetilde{x}_t(\lambda)}{1-\lambda} - 1$. By construction, $\widetilde{R}_t(\lambda)$ is continuous in λ with possible kink-points. Differentiating the equation $\widetilde{x}_t(\lambda) = (1-\lambda)(1+\widetilde{R}_t(\lambda))$ implies

$$\frac{d\widetilde{x}_t(\lambda)}{d\lambda} = -(1+\widetilde{R}_t(\lambda)) + (1-\lambda)\frac{d\widetilde{R}_t(\lambda)}{d\lambda}$$

Rearranging and using the expression for $\frac{d\widetilde{x}_t(\lambda)}{d\lambda}$ above yields

$$\begin{aligned} \frac{d\widetilde{R}_t(\lambda)}{d\lambda} &= \frac{1}{1-\lambda}\left[1+\widetilde{R}_t(\lambda) + \frac{d\widetilde{x}_t(\lambda)}{d\lambda}\right] \\ &= \frac{1}{1-\lambda}\left[1+\widetilde{R}_t(\lambda) - \frac{1+\overline{R}_t}{\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z)}\right] \\ &= \frac{1}{(1-\lambda)\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z)}\left[(1+\widetilde{R}_t(\lambda))\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z) - \left(1+\overline{R}_t\right)\right] \end{aligned} \quad (53)$$

We want to argue that the expression in brackets is negative. There are two possible cases. First, suppose $\widetilde{R}_t(\lambda) < \overline{R}_t$. Then

$$\begin{aligned} (1+\widetilde{R}_t(\lambda))\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z) &< \left(1+\overline{R}_t\right)\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z) \\ &\leq 1+\overline{R}_t \end{aligned}$$

In that case, we have $\frac{d\widetilde{R}_t(\lambda)}{d\lambda} < 0$ from (53) regardless of the direction we take the derivative.

Next, suppose $\widetilde{R}_t(\lambda) \geq \overline{R}_t$. Recall that $\widetilde{x}_t(\lambda)$ is the value of x that solves (52). Substituting in $\widetilde{x}_t(\lambda) = (1+\widetilde{R}_t(\lambda))(1-\lambda)$, we can rewrite (52) as

$$\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} \left[z - \left(1+\widetilde{R}_t(\lambda)\right)\right] dG_t(z) = \lambda\left[\left(1+\overline{R}_t\right) - \int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} \left(1+\widetilde{R}_t(\lambda)\right) dG_t(z)\right]$$

The RHS of the equation above has the opposite sign as $\frac{d\widetilde{R}_t(\lambda)}{d\lambda}$. Hence, we can establish that $\frac{d\widetilde{R}_t(\lambda)}{d\lambda} \leq 0$ if we can show that

$$\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} \left[z_t - \left(1 + \widetilde{R}_t(\lambda)\right)\right] dG_t(z) \geq 0$$

Here, we use the fact that $\widetilde{x}(\lambda) = (1 + \widetilde{R}(\lambda))(1 - \lambda)$ to rewrite the LHS of (52) evaluated at $x = \widetilde{x}_t(\lambda)$ as

$$\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} (z - \widetilde{x}_t(\lambda))\, dG(z) = \int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} (1-\lambda)\,[z_t - (1 + \widetilde{R}_t(\lambda))] dG(z) + \int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} \lambda z_t dG(z)$$

Note that when $\widetilde{R}(\lambda) > \overline{R}_t$, we must have $\widetilde{x}_t(\lambda) > 0$. When the equilibrium interest rate in market λ exceeds $\overline{R}_t$, some agents who borrow in market λ must default, since the only way the expected return to lending in market λ can equal $\overline{R}_t$ in this case is if some agents default. Hence, there must be some values of z for which an agent who borrows in market λ to buy assets defaults. But given the equilibrium price of the asset cannot be negative and the dividend $d > 0$, the lower support of z is bounded below by 0.

Armed with this observation, we can rewrite (52) as

$$\begin{aligned} \left(1 + \overline{R}_t\right)\lambda &= \int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} (1-\lambda)\left[z_t - (1 + \widetilde{R}_t(\lambda))\right] dG(z) + \int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} \lambda z_t dG(z) & (54)\\ &\leq \int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} (1-\lambda)\left[z_t - (1 + \widetilde{R}_t(\lambda))\right] dG(z) + \int_{0}^{1+r_t^{\max}} \lambda z_t dG(z) \\ &= \int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} (1-\lambda)\,[z_t - (1 + R_t)]\, dG(z) + (1 + \overline{r}_t)\,\lambda & (55) \end{aligned}$$

The inequality in the second row uses the fact that $\widetilde{x}_t(\lambda) \geq 0$ whenever $\widetilde{R}_t(\lambda) > \overline{R}_t$. But in an equilibrium where all markets are active, we must have $\overline{R}_t^D \geq \overline{r}_t^D$, i.e. since any saver can buy an asset, the return on savings $\overline{R}_t$ is at least as large as the return to buying an asset $\overline{r}_t$. This implies

$$0 \leq \left(\overline{R}_t - \overline{r}_t\right)\lambda \leq (1-\lambda)\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} (z_t - (1 + R_t))\, dG(z)$$

We have therefore confirmed that $\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} (z_t - (1 + R_t))\, dG(z) \geq 0$. It follows that all directional derivatives $\frac{d\widetilde{R}_t(\lambda)}{d\lambda}$ are nonnegative as claimed. ■

From Lemmas C1 and C2, define Λ_t as either 1 or the minimum value in $[0, 1]$ for which $R_t(\lambda) = \overline{R}_t$. It follows that $R_t(\lambda) > \overline{R}_t$ for $\lambda < \Lambda_t$ and $R_t(\lambda) = \overline{R}_t$ for all $\lambda \geq \Lambda_t$. This establishes the proposition. ■

We can use the schedule of interest rates in Proposition C1 to determine how much entrepreneurs should produce and in which markets to borrow if they do.

Proposition C2: *In an equilibrium where all markets are active, entrepreneurs with wealth w will borrow $1 - w$ units to produce, in a market with an interest rate equal to $R_t(w)$.*

Proof of Proposition C2: Consider an entrepreneur with wealth w. If she borrows in a market λ where $\lambda \leq w$, she can produce at full capacity and would only need to put down $\lambda\left(\frac{1-w}{1-\lambda}\right)$ resources to borrow $1-w$ to reach full capacity. This would earn her an expected profit of

$$1 + y^* - (1 + R_t(\lambda))(1 - w)$$

This value is maximized by choosing λ to minimize $R_t(\lambda)$. From Proposition C1, we know $R_t(\lambda)$ is weakly decreasing in λ and is therefore maximized at $\lambda = w$.

Next, suppose she borrows in a market λ where $\lambda > w$. In that case, she could not produce at full capacity. Since $y^* > r_t^{\max} = R_t(0) \geq R_t(\lambda)$ for all $\lambda \in [0, 1)$, it will be optimal to borrow enough to produce at the maximal capacity possible. For $\lambda > w$, this maximum is $\frac{w}{\lambda}$. Her profits would thus equal

$$\frac{w}{\lambda}(1 + y^* - x_t(\lambda)) \tag{56}$$

where recall $x_t(\lambda) = (1-\lambda)(1 + R_t(\lambda))$ is the amount a borrower is required to repay per each unit of resource she borrows. Since $R_t(\lambda) = \overline{R}_t$ for all $\lambda \in (\Lambda_t, 1)$, there would be no benefit to going to market $\lambda > \Lambda_t$: She would have to produce less at the same interest rate as in market Λ_t. The only case that remains is the interval of markets $\lambda \in [w, \Lambda_t]$. In that case, we can differentiate profits in (56) to get

$$\begin{aligned}
\frac{d}{d\lambda}\left(\frac{w}{\lambda}(1 + y^* - x_t(\lambda))\right) &= -\frac{w}{\lambda^2}\left[(1 + y^* - x_t(\lambda)) + \lambda\frac{dx_t(\lambda)}{d\lambda}\right] \\
&= -\frac{w}{\lambda^2}\left[(1 + y^* - x(\lambda)) - \frac{\lambda\left(1 + \overline{R}_t\right)}{\int_x^{1+r_t^{\max}} dG_t(z)}\right] \\
&= -\frac{w}{\lambda^2 \int_x^{1+r_t^{\max}} dG_t(z)}\left[\int_x^{1+r_t^{\max}} (1 + y^* - x_t(\lambda))\, dG_t(z) - \lambda\left(1 + \overline{R}_t\right)\right]
\end{aligned}$$

Since $y^* > \frac{D+2\varphi e}{(1-\varphi)e} > r_t^{\max}$, we have

$$\frac{d}{d\lambda}\left(\frac{w}{\lambda}(1 + y^* - x_t(\lambda))\right) < -\frac{w}{\lambda^2 \int_x^{1+r_t^{\max}} dG_t(z)}\left[\int_x^{1+r_t^{\max}} (z - x_t(\lambda))\, dG_t(z) - \lambda\left(1 + \overline{R}_t\right)\right]$$

But for $\lambda \leq \Lambda_t$, the expression in brackets is equal to 0. Hence, borrowing in a market with $\lambda > w$ will be strictly dominated by borrowing in the market with $\lambda = w$. At the optimum, each entrepreneur borrow $1-w$ at a rate of $R_t(w)$. ■

Proposition C3: *In an equilibrium where all markets are active, the equilibrium price of the asset will be given by* $p_t = (1-\varphi)e$

Proof of Proposition C3: Condition (47) implies that all the resources of the young in cohort t will be used to either produce or to buy assets. From Proposition C2, we know that all entrepreneurs will produce at capacity, so the total amount used to produce is given by

$$\int_0^1 (2\varphi e)\, dw = 2\varphi e$$

This implies

$$p_t + 2\varphi e = (1+\varphi)\,e$$

and so $p_t = (1-\varphi)\,e$ as claimed. ■

Propositions C1-C3 did not involve any restrictions on the distribution of the return $z_t = \frac{p_{t+1}+d_{t+1}}{p_t}$. But given Proposition 3 and the process for dividends, we can determine the distribution of this return to obtain a sharper characterization. When $d_t = d$, the return on the asset $1 + r_t$ will have a degenerate distribution with full mass at $\frac{d}{(1-\varphi)e}$. Substituting this into (50) reveals that $\widetilde{x}_t(\lambda) = (1-\lambda)\left(1+\frac{d}{(1-\varphi)e}\right)$ for all λ, that $\frac{d\widetilde{R}(\lambda)}{d\lambda} = 0$ for all λ, and the cutoff $\Lambda_t = 0$. Hence, when all markets are active, $R_t(\lambda) = \overline{R}_t = \frac{d}{(1-\varphi)e}$ for all $\lambda \in [0,1)$ as described in the text. One equilibrium in which all markets are active if it entrepreneurs with wealth w borrow in market $\lambda = w$. But other equilibria in which all markets are active also exist.

Next, when dividends d_t follow the regime-switching process between d and D and at date t we have $d_t = D$, the return z_t would have a two-point distribution:

$$z_t = \begin{cases} 1+\frac{D}{(1-\varphi)e} & \text{w/prob } 1-\pi \\ 1+\frac{d}{(1-\varphi)e} & \text{w/prob } \pi \end{cases}$$

In this case, equation (52) which defines $\widetilde{x}_t(\lambda)$ reduces to

$$(1-\pi)\left(1+\frac{D}{(1-\varphi)\,e} - \widetilde{x}_t(\lambda)\right) = \left(1+\overline{R}_t^D\right)\lambda$$

or

$$\widetilde{x}_t(\lambda) = 1 + \frac{D}{(1-\varphi)\,e} - \frac{1+\overline{R}_t^D}{1-\pi}\lambda \tag{57}$$

From this we can derive the implied interest rate $1+\widetilde{R}_t^D(\lambda)$ in market λ while $d_t = D$:

$$\begin{aligned} 1+\widetilde{R}_t^D(\lambda) &= \frac{1}{1-\lambda}\left[1+\frac{D}{(1-\varphi)\,e} - \frac{1+\overline{R}_t^D}{1-\pi}\lambda\right] \\ &\equiv \frac{1-\kappa\lambda}{1-\lambda} \end{aligned}$$

where $\kappa \equiv \frac{\left(1+\overline{R}_t^D\right)/(1-\pi)}{1+D/((1-\varphi)e)}$. Since the return on savings is at least as large as the return to buying the asset,

$$\begin{aligned} 1+\overline{R}_t^D &\geq 1+\overline{r}_t^D \\ &= 1+\frac{(1-\pi)\,D+\pi d}{(1-\varphi)\,e} \\ &> (1-\pi)\left(1+\frac{D}{(1-\varphi)\,e}\right) \end{aligned}$$

This means $\kappa > 1$, which in turns implies the interest rate on loans $\widetilde{R}_t^D(\lambda)$ is strictly decreasing in λ for $\lambda > 0$, in line with what we discuss in the text and depict in Figure 3.

Recall that, by definition, Λ_t^D is the minimum value of λ at which $\widetilde{R}_t^D(\lambda) = 1 + \overline{R}_t^D$. We can therefore solve for Λ_t^D by setting $\lambda = \Lambda_t^D$ in (57) and equating $\widetilde{x}_t\left(\Lambda_t^D\right)$ with $1 + \overline{R}_t^D$, i.e. by setting

$$\frac{1}{1-\Lambda_t^D}\left[1 + \frac{D}{(1-\varphi)e} - \left(1 + \overline{R}_t^D\right)\frac{\Lambda_t^D}{1-\pi}\right] = 1 + \overline{R}_t^D$$

Rearranging, we have

$$\Lambda_t^D = \frac{1-\pi}{\pi\left(1+\overline{R}_t^D\right)}\left(\frac{D}{(1-\varphi)e} - \overline{R}_t^D\right) \tag{58}$$

Since $R_t^D(\lambda)$ is decreasing in λ for $\lambda \in [0, \Lambda_t^D)$, Proposition C2 implies only borrowers with wealth w borrow in market $\lambda = w$ for $w \in [0, \Lambda_t^D)$. Hence, $f_t^p(\lambda) = 2\varphi e$ for $\lambda \in [0, \Lambda_t^D)$. By contrast, $f_t^p(\lambda)$ is indeterminate for $\lambda \in [\Lambda_t^D, 1)$. However, we know that $f_t^p\left(\Lambda_t^D\right) > 0$, since borrowers with wealth $w = \Lambda_t^D$ will have to borrow in this market to borrow $1 - w$. As for the amount borrowed to buy assets, $f_t^a(\lambda)$, we can deduce $f_t^a(\lambda)$ for $\lambda \in [0, \Lambda_t^D]$ from $R_t^D(\lambda)$, $\overline{R}_t^D$, and $f_t^p(\lambda)$ using (48). For $\lambda > \Lambda_t^D$, the fact that $\frac{dR_t^D(\lambda)}{d\lambda} < 0$ at $\lambda = \Lambda_t^D$, combined with the fact that $\frac{dR_t^D(\lambda)}{d\lambda} < 0$ for $\lambda > \Lambda_t^D$ from Lemma C2, implies that no agent would want to borrow to buy assets. So $f_t^a(\lambda) = 0$ for all $\lambda \geq \Lambda_t^D$.

Finally, we need to solve for $\overline{R}_t^D$. Consider the return on all forms of savings in this economy. First, savings are used to finance production by entrepreneurs, which yields savers a payoff of

$$\int_0^1 \left(1 + R_t^D(w)\right)(1-w)(2\varphi e)\,dw$$

Second, savings are used to buy assets, directly or indirectly through loans. The expected earnings from these investments equal $\left(1 + \overline{r}_t^D\right)p_t^D$. From this, we must net out expected default costs. We use γ_t^D to denote the fraction of spending on assets that is financed with some debt. These purchases will result in default if returns are low. Since default is proportional to the size of the borrower's project, expected default costs equal $\pi\gamma_t^D\Phi p_t^D = \pi\gamma_t^D\Phi(1-\varphi)e$. These payoffs must add up to $(1 + \overline{R}_t^D)e$, i.e.,

$$(1 + \overline{R}_t^D)e = \left[1 + \overline{r}^D - \pi\gamma_t^D\Phi\right](1-\varphi)e + \int_0^1 \left(1 + R_t^D(w)\right)(1-w)(2\varphi e)\,dw \tag{59}$$

We also need an equation to characterize γ_t^D. When the expected return to lending $\overline{R}_t^D$ exceeds the expected return to buying the asset $\overline{r}^D$, only agents who borrow will buy the asset. In that case, $\gamma_t^D = 1$, and we can solve for $1 + \overline{R}_t^D$ by plugging in $\gamma_t^D = 1$ in (59). When $\overline{R}_t^D = \overline{r}^D$, then γ_t^D would have to ensure that $\overline{R}_t^D$ is indeed equal to $\overline{r}^D$, where we know the latter is equal to $\frac{(1-\pi)D+\pi d}{(1-\varphi)e}$. We can combine these two conditions into a single equation:

$$1 + \overline{R}_t^D = \max\left\{1 + \overline{r}^D, \left[1 + \overline{r}^D - \pi\Phi\right](1-\varphi) + \int_0^1 \left(1 + R_t^D(w)\right)(1-w)(2\varphi)\,dw\right\} \tag{60}$$

Equation (60) ensures that when $\overline{R}_t^D > \overline{r}^D$, we must have $\gamma_t^D = 1$, and when $\overline{R}_t^D = \overline{r}^D$, the value of γ_t^D must equate the two returns. Since $\overline{r}^D$ is time invariant, the solutions to these equations, $\overline{R}^D$ and γ^D, are also time invariant. Given a value for $\overline{R}^D$, we can solve for the time invariant cutoff Λ^D as the smallest value of λ for which $R^D(\lambda) = \overline{R}^D$. This completes the characterization of an equilibrium when all markets are active.

C.3 Comparative Statics with a floor

Finally, we consider equilibria where all markets above some floor $\underline{\lambda}$ are active. These results correspond to Propositions 9 and 10 in the text. The first result concerns how the equilibrium changes with $\underline{\lambda}$.

Proof of Proposition 9: In the text, we derive the equilibrium values p^D and $\overline{r}^D$ are show that they are increasing and decreasing in $\underline{\lambda}$, respectively. Here, we show that $\overline{R}^D$ is decreasing in $\underline{\lambda}$. Recall that in equilibrium, $\overline{R}^D \geq \overline{r}^D$, i.e. the return on savings must be at least as high as the return agents can earn from buying the asset. We need to show that $\overline{R}^D$ is decreasing in $\underline{\lambda}$ when $\overline{R}^D > \overline{r}^D$.

When $\overline{R}^D > \overline{r}^D$, we have $\gamma^D = 1$, and the equilibrium conditions for $\overline{R}^D$ and Λ^D are given by two equations. First, since Λ^D corresponds to the minimum value of λ for which $R^D(\lambda) = \overline{R}^D$, we know from (57) that

$$1 + \overline{R}^D = \frac{1}{1-\Lambda^D}\left[1 + \tfrac{D}{(1-(1-\underline{\lambda})\varphi)e} - \left(1+\overline{R}^D\right)\tfrac{\Lambda^D}{1-\pi}\right] \tag{61}$$

Second, using the same approach to compute the return on savings as before, we have a similar equation for $\overline{R}^D$ as in (59):

$$\begin{aligned} 1+\overline{R}^D &= (1-\varphi(1-\underline{\lambda}))\left[1+\overline{r}^D - \pi\Phi\right] + \\ & 2\varphi\int_0^1 \left[\min\left\{\tfrac{w}{\underline{\lambda}}, 1\right\} - w\right]\left[1 + R^D(\max\{w, \underline{\lambda}\})\right] dw \end{aligned} \tag{62}$$

If $\overline{R}^D > \overline{r}^D$, we argue that the floor $\underline{\lambda}$ must be below the cutoff Λ^D. For suppose $\underline{\lambda} \geq \Lambda^D$. Then all markets where agents might default will be shut down. But without default, the expected return on lending and the expected return on the asset must be equal to ensure both the credit market and asset market clear. Since $\underline{\lambda} < \Lambda^D$, we can expand the integral term in (62) into the sum of three distinct terms:

$$\begin{aligned} \int_0^1 \left[\min\left\{\tfrac{w}{\underline{\lambda}}, 1\right\} - w\right]\left[1 + R^D(\max\{w, \underline{\lambda}\})\right] &= (1+R(\underline{\lambda}))\left(\frac{1}{\underline{\lambda}} - 1\right)\int_0^{\underline{\lambda}} w dw + \\ & \int_{\underline{\lambda}}^{\Lambda^D} (1+R(w))(1-w)\, dw + \left(1+\overline{R}^D\right)\int_{\Lambda^D}^1 (1-w)\, dw \end{aligned}$$

We then use the fact that $1 + R^D(\lambda) = \frac{1}{1-\lambda}\left[1 + \frac{D}{(1-(1-\underline{\lambda})\varphi)e} - \frac{\lambda\left(1+\overline{R}_t^D\right)}{1-\pi}\right]$ to evaluate the three terms above:

$$(1+R(\underline{\lambda}))\left(\frac{1}{\underline{\lambda}} - 1\right)\int_0^{\underline{\lambda}} w dw = \left[1 + \tfrac{D}{(1-(1-\underline{\lambda})\varphi)e} - \underline{\lambda}\left(\tfrac{1+\overline{R}^D}{1-\pi} - 1\right)\right]\frac{\underline{\lambda}}{2} \tag{63}$$

$$\int_{\underline{\lambda}}^{\Lambda^D} (1+R(w))(1-w)\, dw = \int_{\underline{\lambda}}^{\Lambda^D}\left[1 + \tfrac{D}{(1-(1-\underline{\lambda})\varphi)e} - \tfrac{w\left(1+\overline{R}_t^D\right)}{1-\pi}\right] dw \tag{64}$$

$$\left(1+\overline{R}^D\right)\int_{\Lambda^D}^1 (1-w)\, dw = \frac{1}{2}\left(1+\overline{R}^D\right)\left(1-\Lambda^D\right)^2 \tag{65}$$

We can write (61) and (62) more compactly as

$$\begin{aligned} h_1\left(\overline{R}^D, \Lambda^D\right) &= 0 \\ h_2\left(\overline{R}^D, \Lambda^D\right) &= 0 \end{aligned}$$

Totally differentiating this system of equations gives us the comparative statics of the equilibrium $\overline{R}^D$ and Λ^D with respect to any variable a as

$$\begin{bmatrix} \frac{\partial h_1}{\partial \overline{R}^D} & \frac{\partial h_1}{\partial \Lambda^D} \\ \frac{\partial h_2}{\partial \overline{R}^D} & \frac{\partial h_2}{\partial \Lambda^D} \end{bmatrix} \begin{bmatrix} d\overline{R}^D/da \\ d\Lambda^D/da \end{bmatrix} = \begin{bmatrix} -\frac{\partial h_1}{\partial a} \\ -\frac{\partial h_2}{\partial a} \end{bmatrix}$$

Differentiating (61) and (62) using expressions (63)-(65) yields

$$\begin{array}{ll} \frac{\partial h_1}{\partial \overline{R}^D} = 1 - \Lambda^D + \frac{\Lambda^D}{1-\pi} & \frac{\partial h_1}{\partial \Lambda^D} = \frac{\pi(1+\overline{R}^D)}{1-\pi} \\ \frac{\partial h_2}{\partial \overline{R}^D} = 1 + \varphi\left[\frac{1}{1-\pi}\left(\Lambda^D\right)^2 - \left(1-\Lambda^D\right)^2\right] & \frac{\partial h_2}{\partial \Lambda^D} = 0 \end{array}$$

When we evaluate comparative statics with respect to $\underline{\lambda}$, we now have

$$\begin{aligned} \begin{bmatrix} d\overline{R}^D/d\underline{\lambda} \\ d\lambda^D/d\underline{\lambda} \end{bmatrix} &= \begin{bmatrix} \frac{\partial h_1}{\partial \overline{R}^D} & \frac{\partial h_1}{\partial \Lambda^D} \\ \frac{\partial h_2}{\partial \overline{R}^D} & \frac{\partial h_2}{\partial \Lambda^D} \end{bmatrix}^{-1} \begin{bmatrix} \frac{dh_1}{d\underline{\lambda}} \\ \frac{dh_2}{d\underline{\lambda}} \end{bmatrix} \\ &= \frac{\varphi}{\kappa} \begin{bmatrix} 0 & \frac{\pi}{1-\pi}\left(1+\overline{R}^D\right) \\ 1+\varphi\frac{\left(\Lambda^D\right)^2}{1-\pi} - \varphi\left(1-\Lambda^D\right)^2 & -\left(1-\Lambda^D+\frac{\Lambda^D}{1-\pi}\right) \end{bmatrix} \begin{bmatrix} -\frac{D}{(1-(1-\underline{\lambda})\varphi)^2 e} \\ -\frac{2D\left(1+\Lambda^D\varphi\right)}{(1-(1-\underline{\lambda})\varphi)^2 e} - (1+\pi\Phi) \end{bmatrix} \end{aligned}$$

where $\kappa = \frac{\pi(1+\overline{R}^D)}{1-\pi}\left(1+\varphi\frac{\left(\Lambda^D\right)^2}{1-\pi} - \varphi\left(1-\Lambda^D\right)^2\right) > 0$. It follows that

$$\frac{d\overline{R}^D}{d\underline{\lambda}} = -\varphi\left(1+\varphi\left[\frac{1}{1-\pi}\left(\Lambda^D\right)^2 - \left(1-\Lambda^D\right)^2\right]\right)^{-1}\left[\frac{2D\left(1+\Lambda^D\varphi\right)}{\left(1-\left(1-\underline{\lambda}\right)\varphi\right)^2 e} + (1+\pi\Phi)\right] < 0$$

Since $\overline{R}^D$ is decreasing in $\underline{\lambda}$ whether $\overline{R}^D > \overline{r}^D$ or $\overline{R}^D = \overline{r}^D$, the claim follows. ■

Proposition 10 concerns how changing $\underline{\lambda}$ affects the expected costs of default $\gamma^D \Phi p^D$. Since we already know p^D is increasing in $\underline{\lambda}$, any changes in expected default costs occur entirely through γ^D. Our next result argues that there exists cutoffs Λ_0 and Λ_1 such that $d\gamma^D/d\underline{\lambda} = 0$ when $\underline{\lambda} < \Lambda_0$ or $\underline{\lambda} > \Lambda_1$. When $\Lambda_0 < \underline{\lambda} < \Lambda_1$, we only claim it must be decreasing for some $\underline{\lambda}$ in this interval.

Proof of Proposition 10: Define

$$\rho\left(\underline{\lambda}\right) = \frac{\overline{R}^D}{\left(1-\left(1-\underline{\lambda}\right)\varphi\right)}$$

Using the fact that $\frac{d\overline{R}^D}{d\underline{\lambda}} < 0$, we have

$$\frac{d\rho\left(\underline{\lambda}\right)}{d\underline{\lambda}} = \frac{d\overline{R}^D/d\underline{\lambda} - \varphi\rho\left(\underline{\lambda}\right)}{1-\left(1-\underline{\lambda}\right)\varphi} < 0$$

Since

$$\overline{R}^D/\overline{r}^D = \left[\left(1-\pi\right)D + \pi d\right]\rho\left(\underline{\lambda}\right)$$

it follows that the ratio $\overline{R}^D/\overline{r}^D$ is decreasing in $\underline{\lambda}$. Hence, there exists a value $\Lambda_0 \geq 0$ such that $\overline{R}^D > \overline{r}^D$ for $\underline{\lambda} < \Lambda_0$ and $\overline{R}^D = \overline{r}^D$ for $\underline{\lambda} \geq \Lambda_0$. Since $\overline{R}^D > \overline{r}^D$ when $\underline{\lambda} < \Lambda_0$, then $\gamma^D = 1$ for $\underline{\lambda} < \Lambda_0$. It follows that expected default costs $\pi\gamma^D\Phi p^D = \pi\Phi p^D$ are increasing in $\underline{\lambda}$ in this region. A higher $\underline{\lambda}$ for $\lambda < \Lambda_0$ reduces the amount entrepreneurs produce and increases the foregone output when dividends fall. Each cohort will therefore be left with fewer goods to consume.

We next turn to the case where $\underline{\lambda} \geq \Lambda_0$. Here, we know $\overline{R}^D = \overline{r}^D$. Substituting this into (61) yields

$$\left(1-\Lambda^D\right)\left(1+\overline{r}^D\right) = \left[1+\tfrac{D}{(1-(1-\underline{\lambda})\varphi)e} - \tfrac{\Lambda^D}{1-\pi}\left(1+\overline{r}^D\right)\right]$$

which, upon rearranging,

$$\Lambda^D = \tfrac{(1-\pi)(D-d)}{(1-(1-\underline{\lambda})\varphi)e+(1-\pi)D+\pi d}$$

From this, we can conclude that $\Lambda^D \geq \underline{\lambda}$ if

$$\tfrac{(1-\pi)(D-d)}{(1-(1-\underline{\lambda})\varphi)e+(1-\pi)D+\pi d} \geq \underline{\lambda}$$

or, upon rearranging, if

$$(1-\pi)(D-d) \geq \underline{\lambda}\left[(1-(1-\underline{\lambda})\varphi)e + (1-\pi)D + \pi d\right] \tag{66}$$

The RHS of (66) is a quadratic in $\underline{\lambda}$ with a positive coefficient on the quadratic term. The inequality is satisfied when $\underline{\lambda} = 0$ and violated when $\underline{\lambda} = 1$. Hence, there exists a cutoff $\Lambda_1 \in (0,1)$ such that $\Lambda^D > \underline{\lambda}$ if $\underline{\lambda} \in [0, \Lambda_1)$ and $\Lambda^D < \underline{\lambda}$ if $\underline{\lambda} \in (\Lambda_1, 1)$. By definition, Λ_0 is the smallest value of $\underline{\lambda} \geq 0$ for which setting $\underline{\lambda} \geq \Lambda_0$ ensures $\overline{R}^D = \overline{r}^D$. By contrast, Λ_1 is the smallest value of $\underline{\lambda} \geq 0$ for which setting $\underline{\lambda} \geq \Lambda_1$ ensures that no agent borrows to speculate in any market above $\underline{\lambda}$. But in that case, all lending is riskless, and we know that the equilibrium interest rate on loans will equal the return on the asset. Hence, $\Lambda_1 \geq \Lambda_0$. To show that the inequality is strict, recall that when $\underline{\lambda} = 0$, we know that $\gamma^D > 0$ since some agents borrow to buy the asset. But γ^D is continuous in $\underline{\lambda}$, and we know that $\gamma^D = 0$ when $\underline{\lambda} \geq \Lambda_1$. Hence, there must be some value of $\underline{\lambda} \in [0, \Lambda_1)$ for which $\gamma^D < 1$. But $\gamma^D < 1$ iff $\overline{R}^D = \overline{r}^D$. It follows that $\Lambda_1 > \Lambda_0$.

When $\underline{\lambda} > \Lambda_1$ no agent will borrow to buy the asset, so $\gamma^D = 0$. Expected default costs are 0, and so the only effect of increasing $\underline{\lambda}$ is to reduce production. This will leave fewer goods for each cohort to consume.

Finally, we turn to the case where $\Lambda_0 < \underline{\lambda} < \Lambda_1$. We do not analyze this case in general. However, when $\Lambda^D = \underline{\lambda}$, the interest rate in all active markets would equal $\overline{R}^D$, since the only active markets are those with $\lambda \geq \underline{\lambda} = \Lambda^D$. Since $\underline{\lambda} \geq \Lambda_0$, we know that $\overline{R}^D = \overline{r}^D$ and so the interest rate in all active markets is $\overline{r}^D$. The equilibrium condition that determines γ^D is given by

$$\begin{aligned}
\left(1+\overline{r}^D\right) &= (1-(1-\underline{\lambda})\varphi)\left[1+\overline{r}^D - \gamma^D\pi\Phi\right] + 2\varphi\int_0^1 \left[\min\left\{\tfrac{w}{\underline{\lambda}},1\right\} - w\right]\left[1+R^D\left(\max\{w,\underline{\lambda}\}\right)\right]dw \\
&= (1-(1-\underline{\lambda})\varphi)\left[1+\overline{r}^D - \gamma^D\pi\Phi\right] + 2\varphi\left(1+\overline{r}^D\right)\int_0^1 \left[\min\left\{\tfrac{w}{\underline{\lambda}},1\right\} - w\right]dw \\
&= (1-(1-\underline{\lambda})\varphi)\left[1+\overline{r}^D - \gamma^D\pi\Phi\right] + 2\varphi\left(1+\overline{r}^D\right)\left[\underline{\lambda}/2 + (1-\underline{\lambda}) - 1/2\right] \\
&= 1+\overline{r}^D - \gamma^D(1-(1-\underline{\lambda})\varphi)\pi\Phi
\end{aligned}$$

Hence, when $\underline{\lambda} = \Lambda_1$, we have $\gamma^D = 0$. For $\underline{\lambda} < \Lambda_1$, however, $\gamma^D > 0$, since

$$\int_0^1 \left[1 + R^D \left(\max\left\{w, \underline{\lambda}\right\}\right)\right] \left[\min\left\{\frac{w}{\underline{\lambda}}, 1\right\} - w\right] dw$$

will be strictly greater than $\frac{1}{2}\left(1 + \overline{r}^D\right)\left(1 - \underline{\lambda}\right)$. Hence, in the limit as $\underline{\lambda} \uparrow \Lambda_1$, we have $d\gamma^D / d\underline{\lambda} < 0$ expected default costs $\pi\gamma^D \Phi p^D$ must be decreasing in $\underline{\lambda}$ since this expression goes from a positive value to 0.

To show that this can generate a Pareto improvement, observe that increasing $\underline{\lambda}$ while dividends are high will make the initial old at date 0 better off given p_0^D increases. Cohorts born after dividends have fallen will be unaffected if $\underline{\lambda}$ is only increased while dividends are high. Cohorts who are born while dividends are high expect to consume the dividends from the asset net of default costs $E\left[d_{t+1}\right] - \Phi\pi\gamma^D p_t^D$ as well as the output produced by entrepreneurs. If Φ is sufficiently large and φ is small, we can promise these agents a higher expected consumption. ■

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