

On the Monopolist's Problem Facing Consumers with Nonlinear Price Preferences

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Abstract

The principal-agent problem is an important paradigm in economic theory for studying the value of private information; the nonlinear pricing problem faced by a monopolist is a particular example. In this lecture, we identify structural conditions on the consumers' preferences and the monopolist's profit functions which guarantee either concavity or convexity of the monopolist's profit maximization. Uniqueness and stability of the solution are particular consequences of this concavity. Our conditions are similar to (but simpler than) criteria given by Trudinger and others for prescribed Jacobian equations to have smooth solutions. By allowing for different dimensions of agents and contracts, nonlinear dependence of agent preferences on prices, and of the monopolist's profits on agent identities, we improve on the literature in a number of ways. The same mathematics can also be adapted to the maximization of societal welfare by a regulated monopoly. In the classical case of bilinear preferences, we introduce a new duality for certifying solutions, which leads to a free boundary formulation for the missing region in the square example of Rochet and Chone.

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