

# Cyclical ascent: a fast algorithm for computing Wasserstein barycenters

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## Abstract

Jacobs and Léger recently proposed a back-and-forth gradient ascent algorithm for numerical optimal transport. The algorithm is surprisingly effective; moreover, in a fixed dimension with quadratic costs it is scalable, in the sense that for a grid with  $N$  points, each step of the ascent can be computed in  $O(N \log N)$  operations. Inspired by their approach, we propose an analogous cyclical gradient ascent algorithm for interpolating between  $m$  distributions by computing a weighted Wasserstein barycenter. Each step of our cyclical gradient ascent takes either  $O(mN \log N)$  or  $O(mN + N^2)$  operations, depending on the geometry and/or choice of cost.

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