Technological Changes, the Labor Market, and Schooling – A General Equilibrium Model with Multidimensional Individual Skills

Zsófia L. Bárány* Moshe Buchinsky† Tamás K. Papp ‡

February 2020
PRELIMINARY AND INCOMPLETE

Abstract

In this paper we develop and estimate a multi-sector general equilibrium growth model of education and sector-occupation choices over the life cycle. The model incorporates the task-based approach by representing jobs in different sector-occupation pairs as different bundles of tasks, requiring specific skill sets of workers. Workers are born with heterogeneous endowments of skills that can be improved over time by schooling and on-the-job accumulation of human capital. They supply their entire bundle of skills to a single job according to a Roy-type sorting model where their selections are along the multi-dimensional vector of skills that they possess. We use data from several U.S. sources that allow us to estimate parameters that correspond to both the decision process of the individuals as well as the parameters that correspond to the production side of the economy. The general equilibrium nature of the model allows us to conduct several policy counterfactuals that take full account of the responses in the economy.

*Sciences Po and CEPR. Email: zsofia.barany@sciencespo.fr
†University of California Los Angeles, Crest-INSEE and NBER
‡Institute for Higher Studies, Vienna

1 Introduction

One of the most widely discussed topics in academic and policy circles is the 'future of work'. The labor markets of most developed countries have undergone large shifts in terms of education, sectors and occupations since the middle of the 20th century. In the US between 1968 and 2014 the share of hours worked by individuals with higher education increased by 28 percentage points, while their skill premium increased from 56% to 80%. The share of hours worked in the goods sector contracted by 21 percentage points, while relative wages between goods and service workers remained quite stable. Finally, labor markets have polarized: the share of hours worked in high-earning (occupation group 1) and low-earning occupations (occupation group 3) increased by 15 and 4 percentage points respectively, and in middle-earning occupations (occupation group 2) it fell by 19 percentage points. Concurrently, the growth of real hourly wages in occupation 2 was the lowest at 41%, followed by occupation 3 at 56%, and it was the highest in occupation group 3 at 61%.[1]

These dramatic changes have had a huge impact on income inequality and on the welfare of many American families. While some workers experienced large increases in the value of their skills, others have suffered from severe reductions, altering the incentives for human capital accumulation. There is a general consensus in the literature that the dominating factor driving these observed changes are the rapidly evolving technologies. Distributing the gains from new technologies more evenly among consumers has become one of the most important issues for policy makers. Altering minimum wages, imposing taxes on the use of robots, new industrial policies, and offering retraining programs for displaced workers are only some of the key policy measures proposed.

One vital issue in assessing systematically the validity and magnitude of the var-

[1]The cutoff for acquiring higher education is at least 15 years of schooling completed. Goods include agriculture, mining, construction and manufacturing, and services comprise of all other industries. Our occupational grouping follows that in the literature (for example Acemoglu and Autor (2011)). Occupation group 1 comprises the non-routine cognitive occupations, managers, professional specialty occupations, technicians and related support occupations; occupation group 2 includes routine (cognitive and non-cognitive) occupations, sales and administrative support occupations, production, construction trades, extractive occupations, precision production occupations, operators, fabricators, laborers, farming occupations; occupation group 3 includes non-routine non-cognitive occupations, protective services, food and cleaning services, personal care occupations.

ious policies is what general framework to adopt. It is clear from the evidence provided below that we must incorporate the determinants of educational attainment, and of the sorting of individuals across sectors and occupations, into a general equilibrium framework. Indeed in this paper we develop such a framework, and we structurally estimate it relying on several data sources. This is the only meaningful way in which one can identify the type of technological changes that gave rise to the observed changes in the labor market. Moreover, it provides the most appropriate tool with which one should evaluate the full impact of the various proposed policies.

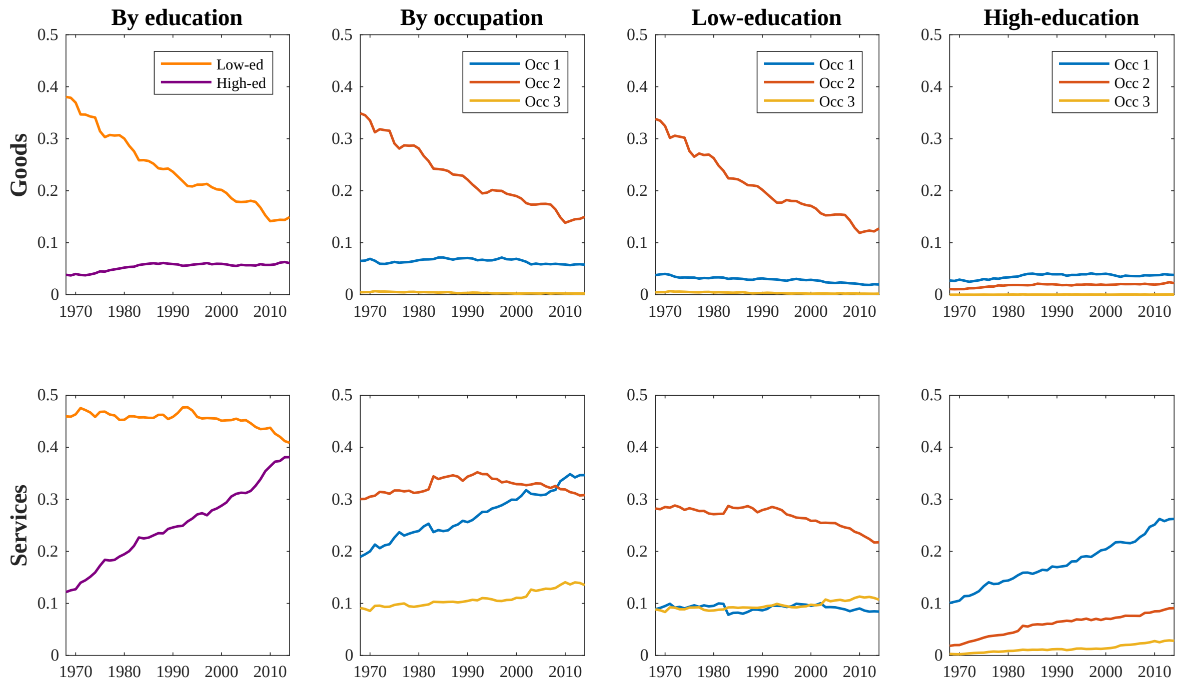

Figure 1: Employment shares by education, sector and occupation, 1968–2014

Notes: Authors' own calculations from the CPS for the years 1968-2014. The curves show the evolution of the shares of hours worked among the 18-65 year old employed population. The top panels show goods, the bottom panel services. The first column shows allocation of hours worked across sector-education pairs, the second column shows allocations across sector-occupation pairs, while the last two columns show the allocation of hours worked across sector-education-occupation triples. For the definition of sectors, education and occupation groups see footnote 1.

Figure 1 shows the allocation of employment in US across the two main sectors – goods and services – and across high- and low-educated individuals, as well as across our three occupation groups. The top row shows employment in goods, the bottom in services. The first column shows employment across sectors-education pairs, and clearly demonstrates that all of the contraction in goods sector employment happened via low-educated individuals, and services expanded via high-educated individuals.

Based on similar observations Buera, Kaboski, Rogerson, and Vizcaino (2018) argue that structural change might be largely driven by skill biased technical change. The second column shows employment across sectors-occupation pairs, and shows that occupation group 2 only shrank in the goods sector, and occupation group 1 and 3 expanded in the services sector. The tight link between structural change and job polarization has been explored in several papers (Bárány and Siegel (2018), Duernecker and Herrendorf (2016), Lee and Shin (2017)). The last two columns show employment allocation across sector-education-occupation triples (i.e. the curves in the four graphs add up to one). All of the changes for the low-educated workers (third column) happened in the goods sector, while all of the changes for the high-educated (fourth column) happened in the services sector. For the low-educated in the services sector, and the high-educated in the goods sector nothing much has changed in the last 50 years or so. This salient pattern – which to our knowledge has been overlooked in the literature – highlights the need to consider educational attainment together with the sorting of workers across sectors and across occupations.

Central in our paper is the modeling of the educational choices and the proceeding sorting of workers into sectors and occupations. We assume that all these choices originate from the heterogeneity in initial *multidimensional* skill vectors with which workers are born. Individuals decide whether or not to acquire higher education before entering the labor market. The educational process increases individuals' various skills differentially, and impacts the arrival rate of jobs. The sorting of workers into sector-occupation cells is based on the task-approach, pioneered by Autor, Levy, and Murnane (2003). Crucially, we assume that jobs across sector-occupation cells are represented by different skill requirements along *all* skill dimensions. As workers supply their entire bundle of skills to a single job, we adopt a multidimensional Roy-type sorting model. Workers in our model conduct directed search, that is, they select the sector-occupation cell in which to search for a job. We further assume that there is on-the-job accumulation of all skills, and that the speed of accumulation depends on the type of job and the type of skill.

We embed this newly developed model of multidimensional skills into a two-sector overlapping generations, general equilibrium growth model, where each sector employs workers of different occupations. The source of growth in the model is sector-occupation specific skill-augmenting technological change. Skill-augmenting technological change, or task-biased technological change, allows us to capture the notion that technologies might be good substitutes for certain skills, while they might complement other skills (as argued for example in Autor et al. (2003), Goos and Manning (2007), Goos, Manning, and Salomons (2014)). Skill-augmenting technologies, as will become clear later, can also capture changing skill requirements. Following Bárány and Siegel (2019a, 2019b), we allow technological change to be sector-occupation specific. This formulation incorporates the leading explanation for both structural change (Baumol (1967), Ngai and Pissarides (2007)) and polarization (Autor and Dorn (2013), Lee and Shin (2017)). Moreover by estimating technological change in such a flexible manner does not hard wire in which way skill-, occupation- and sector -specific components interact.

Estimation of this comprehensive model requires the use of several data sources. We combine four data sets to structurally estimate the model. We use the Current Population Survey (CPS) between 1968 and 2014 to construct economy-wide data on unemployment and on employment and wages across sector-occupation cells, by cohort, age, and education levels. We use data from the National Longitudinal Survey of Youth 1979 (NLSY79) for two purposes. First, we estimate the distribution of initial multidimensional skill endowments (at the age of 16) from this data. Second, we obtain measures of educational attainment, of unemployment, and of employment and wages across sector-occupation cells by initial skill endowment ('type') over time. Using the panel aspect of the NLSY79 we track transitions over time across sector-occupation cells and the associated wage changes. In addition, we use data from the U.S. Bureau of Economic Analysis (BEA) on sectoral value added and on sectoral prices. We use the Occupational Information Network (O*NET) database to construct the skill requirements by fine occupational categories. Combining this with CPS data on employment shares by fine occupational categories and sectors allow us to obtain the skill requirements in our six sector-occupation cells.

The lifetime choices by initial skill endowment together with the estimated skill requirements provide strong support for the multidimensional Roy-type sorting framework that we use. The skill requirements estimated from the O*NET and the CPS shows that occupation 1 requires the highest abstract and interpersonal skills and the lowest manual skills, while occupation 2 requires high manual, and medium abstract and interpersonal skills. Finally occupation 3 has the lowest abstract skill requirements, but it requires high manual and interpersonal skills. Also, the occupations within the goods sector require more abstract and manual skills, while those in the services sector require more interpersonal skills.

The estimates obtained from the NLSY79 for the (discretized) distribution of the multidimensional skill endowments are of particular interests in and by themselves. We find large individual heterogeneity in the combination of abstract, manual and interpersonal skills. Acquisition of higher education is highly correlated with abstract and and to a lesser degree with interpersonal skills. More generally, individuals with different initial skill sets choose very different career paths over their life-cycle. For example, individuals with higher abstract ability are more likely to work in occupation group 1. Those with higher interpersonal ability are more likely to work in the services sector than the goods sector. Individuals with high initial manual skills are less likely to acquire higher education, and more likely to work in occupation 2 over their entire life-cycle.

The rest of the paper is organized as follows. In Section 2 of the paper we provide a brief discussion of the existing literature on the (many) components of our paper. In Section 3 we provide a comprehensive discussion of the multi-dimensional skill requirements and the distribution of skill vectors, and the estimation associated with it. Section 4 is devoted to explaining in detail the model and its various novel features. The estimation and more generally the implementation are discussed in Section 5. The results are provided in Section 6. Section 7 is devoted to the examination of a number of alternative policy measures, while Section 8 provides a brief summary and conclusions [to be added]. Details of the estimation, construction of variables, etc. are provided in an Appendix.

2 How does our Model Fit into the Literature?

There are a few papers that have estimated dynamic general equilibrium multi-sector models of the labor market with heterogeneous workers. However, none of these consider heterogeneity along multi-dimensional skills. We aim to investigate similar patterns in the data as Lee and Wolpin (2006), namely the shift from the goods to the service sector in the US, while our estimation strategy is closer to the one in Dix-Carneiro (2014). Both of these papers present multi-sector overlapping generations models, with endogenous job-specific human capital accumulation and costly switching of jobs, and study the impact of technology or trade shocks. Both of these papers model the heterogeneity of workers by assuming that there are a few types of workers (3 or 4), and both the characteristics of these types as well as their distribution in the population are part of the structural estimation. This is the first key point where our strategy deviates. We rely on the skill measurements in the National Survey of Youth 1979 (NLSY79) data to identify the types of individuals as well as their distribution in the population. Moreover, the panel aspect of the NLSY79 data allows us to observe the choices made by each type of individual. The second crucial difference is that we aim to model the sorting of workers with multidimensional skills into jobs characterized by multidimensional task requirements, as in Lise and Postel-Vinay (2019). Finally, by assuming human capital accumulation in each of the skill dimensions, our model provides a micro-foundation for the less-than-prefect mobility of human capital across sectors and occupations.

In our modeling of worker productivity across different jobs we build on the task-based approach. Following the seminal paper by Autor et al. (2003), a string of largely empirical work lends support to this approach (e.g. Autor, Katz, and Kearney (2006), Spitz-Oener (2006), Goos and Manning (2007), Dustmann, Ludsteck, and Schönberg (2009), Goos et al. (2014), Autor and Dorn (2013)). There are two important challenges faced by this literature. The first challenge – as pointed out by Autor and Handel (2013) – is that it does not make an explicit link between the *tasks required* in an occupation and the *skills that workers possess*. Instead, most papers assume that the price of a given task (and hence skill) is equated across the economy, thus effectively unbundling the tasks within occupations. This is not consistent with the fact that workers supply their entire bundle of skills to a single occupation. This would suggest that what can be priced are the bundles of skills in the market.

Lise and Postel-Vinay (2019) tackle this issue from the perspective of workers. While we model worker productivity across jobs in a similar way, there are important differences. First, we assume directed search, as opposed to their random search. Second, we allow technologies to change over time. Finally, as they are interested in skill mismatch, they only consider a partial equilibrium framework, taking the demand for workers as given. However, to evaluate the impact of technological change on workers, such a model needs to be embedded in a general equilibrium framework, with well-defined labor demand for each type of job.

3 Sorting along multidimensional skills in the data

In this study we develop and estimate a model where individuals choose in which sector and occupation to work in each period of their life. One key aspect of this choice is that their multidimensional skills should be matched well against the corresponding skill requirements of the job. In this section we describe the construction of the skill requirements of all jobs and the construction of the initial distribution of skills in the population. We also show that the initial individual skill vectors line up well with skill requirements.

3.1 Skill requirements

To construct the skill requirement for every job, and ultimately for the six sector-occupation combinations we consider in this study, we use the O*NET database. This dataset contains hundreds of standardized descriptors for fine occupational categories from the Standard Occupational Classification (SOC). From the O*NET 19.0 release we retain the importance value of all descriptors in Abilities, Knowledge, Skills and Work Activities, and the value of all the descriptors in the Work Context section.[2] This gives

[2] We omit 17 descriptors for which not all occupations have a value.

us 199 descriptors for each of the 954 occupational categories in O*NET. Similarly to Lise and Postel-Vinay (2019), we reduce these descriptors to three dimensions using principal component analysis (PCA) with exclusion restrictions. We order the descriptors such that the first three are: mathematical knowledge, multilimb coordination and social perceptiveness skill.[3] We impose the exclusion restrictions that: (a) mathematical knowledge only reflects abstract requirements; (b) multilimb coordination only captures manual requirements; and (c) social perceptiveness skill only reflects interpersonal requirements. These restrictions make it possible to identify the three components of skills. We then conduct a PCA on all descriptors, and retain only the first three principal components. We use the associated three eigenvectors to construct our abstract, manual and interpersonal skill requirements by a representation of all skill requirements in the space of the first three principal components. We rescale these such that they lie in the $[0, 1]$ interval.

We then use a crosswalk between the O*NET occupations and the 1990 occupational categories (*occ1990*) of the CPS to assign a requirement vector to each *occ1990* category, similarly to Acemoglu and Autor (2011). Finally, using CPS data for 1990, we calculate in each sector-occupation cell the labor supply weighted average of the skill requirements obtained above. We denote these by $\underline{\lambda}_{Jo}$, for the goods and the services sectors, $J = G, S$ and our three occupation groups, $o = 1, 2, 3$.[4] Table 1 provides the resulting estimates for skill requirements in the six cells we consider here. The table indicates that there are substantial differences across occupations and across sectors in skill requirements. For example, occupation 1 has the highest abstract and interpersonal skill requirements, while occupation 2 has high manual requirements, and intermediate abstract and manual requirements. In contrast, occupation 3 has low abstract skill requirements, but relatively high interpersonal and manual skill requirements. Within occupations, the goods sector requires more abstract and manual skills, and the services sector requires more interpersonal skills.

[3] These are '2.C.4.a' from Knowledge, '1.A.2.b.2' from Abilities, and '2.B.1.a' from Skills, for the three descriptors respectively.

[4] Note that since the labor supply weights of each of the *occ1990* category changes within each cell over time, the $\underline{\lambda}_{Jo}$ can potentially change over time. However, in our analysis these are remarkably stable over time.

Table 1: Skill requirement vectors in all sector-occupation pairs

Sector – occupation	Abstract	Manual	Interpersonal
Goods – 1	0.69	0.29	0.53
Services – 1	0.57	0.29	0.66
Goods – 2	0.44	0.65	0.32
Services – 2	0.41	0.43	0.46
Goods – 3	0.15	0.61	0.31
Services – 3	0.23	0.58	0.50

3.2 Skill distribution

We construct the empirical distribution of the initial multi-dimensional skill endowments, denoted by $\underline{a}_0$, for individuals in the original sample of the NLSY79.[5] The procedure we follow is similar to the one described above for extracting the skill requirements of different occupations from the O*NET, and similar to the one in Lise and Postel-Vinay (2019). We first identify all the background variables that are correlated with the individual's innate ability. These variables come from the Armed Services Vocational Aptitude Battery (ASVAB) scores, Rosenberg self-esteem test scores, Rotter locus-of-control scale score, health, family background variables, as well as measures of criminal and anti-social behavior.

We aim to reduce the dimension of the vector representing the individual skill endowment to have only three elements, corresponding to abstract, manual and interpersonal skills. We achieve this using PCA with exclusion restrictions. To identify abstract and interpersonal skills we use the same restrictions as Lise and Postel-Vinay (2019), namely mathematical knowledge (ASVAB section 8) and the Rosenberg self-esteem test score. However, it turns out that the NLSY79 does not have a single measure that credibly *only* impacts individuals' manual skills. To extract such a skill measure, we regress auto and shop information (ASVAB section 7) on mathematical knowledge, and use the residual from this regression.[6] We then order the variables such that the first is mathematical knowledge, the second the residual manual skill, and the third is the Rosenberg self-esteem test score. Following the same procedure as in extract-

[5] To make sure that the sample is representative, in this part we exclude individuals from the supplement and military samples.

[6] The auto and shop information subtest of the ASVAB tests knowledge about automobile systems and about common shop tools and fasteners and their uses.

ing skill requirements, we conduct a PCA on all descriptors, and retain only the first three principal components. We use the associated three eigenvectors to construct abstract, manual and interpersonal skill endowments for each individual in the NLSY79 original sample, which we also rescale to obtain measures in $[0,1]$.

In principle, each person in the population has a different initial endowment $\underline{a}_0$. Estimating the structural model allowing for an almost continuous distribution of individual specific innate ability vectors, or 'types', would be intractable. Therefore, we approximate the continuous distribution of $\underline{a}_0$ by a discrete distribution of individual 'types'. We do so by performing the well known k-medoids algorithm of cluster analysis on the estimated individual ability vectors $\underline{a}_0$ obtained from the PCA to obtain k, in our case twelve, clusters. We then assign each individual in the sample with the appropriate point of support estimated in the cluster analysis. This assigned value for $\underline{a}_0$ is held constant during the estimation of the structural model. We present the estimated $\underline{a}_0$ for the 12 clusters in Table 2. The ordering of types is increasing in abstract ability.

Table 2: Estimated Points of Support for $\underline{a}_0$

Group	a_0			Probability	% with higher education
	Abstract	Manual	Inter-personal		
1	0.17	0.48	0.34	0.07	2.3
2	0.24	0.35	0.70	0.08	6.7
3	0.26	0.42	0.50	0.09	3.3
4	0.44	0.40	0.41	0.09	10.3
5	0.46	0.34	0.63	0.10	12.8
6	0.48	0.25	0.85	0.09	20.9
7	0.62	0.35	0.45	0.09	23.1
8	0.67	0.23	0.68	0.08	38.9
9	0.68	0.42	0.72	0.06	34.9
10	0.82	0.19	0.91	0.08	57.7
11	0.83	0.32	0.50	0.08	57.2
12	0.88	0.23	0.71	0.08	66.9
All	0.54	0.33	0.62	1	27.4

Note that the 12 types have quite different combinations of the three skills. Among the approximately 4600 individual initial endowments the correlation of abstract and manual skills is -0.48, the correlation between abstract and interpersonal skills is 0.31, and the correlation between manual and interpersonal skills is -0.53. These correlations are reflected also in the 12 types identified using the k-medoids procedure, with a slight positive correlation between abstract and interpersonal skills, and a negative association of manual skills with the other two. In the last column of the table we report the fraction of individuals from each type that has acquired higher education, i.e. completed at least 15 years of schooling. The overall fraction is 27%, with large variation across the 12 types. Individuals with high abstract and interpersonal, and low manual skills are more likely to acquire higher education. Take for example types 5 and 6, which have similar abstract skills, but type 6 has lower manual and higher interpersonal skills. 20.9% of type 6 individuals get higher education, while only 12.8% of type 5 do so. A similar pattern arises when comparing type 3 with 2 (with the latter having lower manual and higher interpersonal skills), or type 9 with 8 (where the latte has much lower manual skills). This indicates that the one-factor model, whereby all skills are perfectly correlated, is not likely the correct model.

It is not only that individuals in the groups with higher abstract and interpersonal skills, and lower manual skills are more likely to acquire higher education, but individuals with different initial skill sets are also likely to choose very different career paths and end-up in the long run in different sector-occupation cells. The panel nature of the NLSY79 allows us to follow individuals over time; we tabulate the choices of the twelve individual types from 1979 to 2014.[7] The results are depicted in Figure 2. That is, the figure provides for 4 of the 12 types, the employment share by sector-occupation broken down into those without and with higher education over time. The employment shares sum to 1 by year and type across education and sector-occupation pairs. Goods sector occupation 1 ($G1$) employment is shown in light blue, red shows services sector occupation 1 ($S1$), grey shows goods occupation 2 ($G2$), yellow services occupation 2 ($S2$), dark blue is goods sector occupation 3 ($G3$), and green shows services occupation 3 ($S3$).[8]

Figure 2 indicates that the types with higher abstract ability are more likely to work

[7]Recall that it is annual data until 1994, and only bi-annual thereafter.

[8]As noted above, the occupations in group 1 are non-routine cognitive skills, in group 2 with routine, and in group 3 non-routine manual occupations as defined in Acemoglu and Autor (2011). For more details see below.

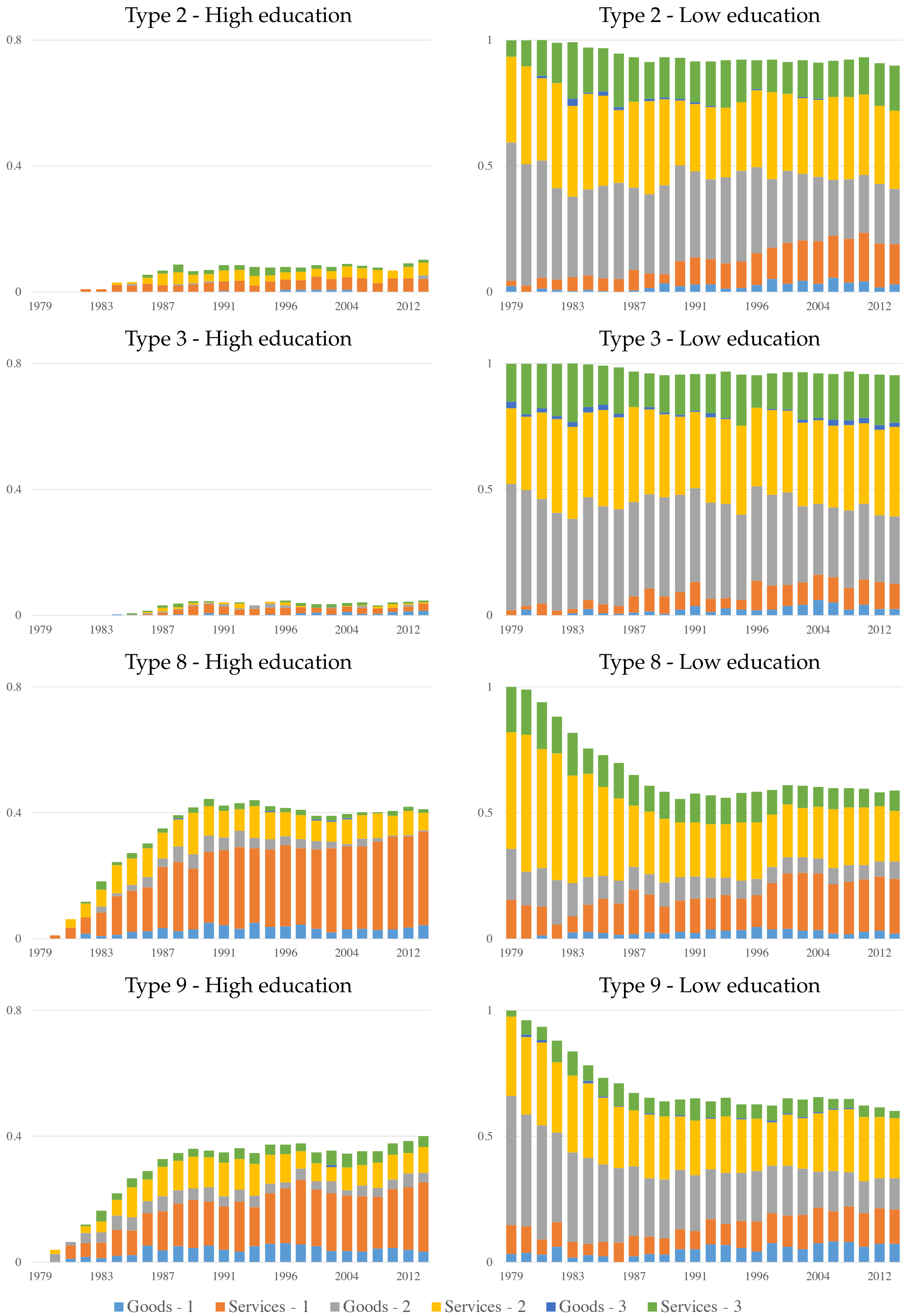

Figure 2: Employment shares by type in sector-occupation cells, 1979–2014

Notes: Authors' own calculations from NLSY79 annual data between 1979-1994, and bi-annual data after that until 2014. Each row of panels shows the share of hours worked over time by an initial ability type. The shares add up to one by year for a given type across high- (left panel) and low-educated individuals (right panel). The sector-occupation groups are distinguished by color, as shown in the legend.

in occupation 1, and less likely to work in occupation 3 and 2, also conditional on the level of education. It is also evident that higher educational attainment is a very strong predictor of employment in occupation group 1. Comparing type 2 to 3, the former with higher interpersonal and lower manual skills initially, we see that type 2 individuals are more likely to work in services, and in occupation group 1. The share of type 2 individuals working in occupation 1 increases more over their life cycle than it does for type 3 individuals. Turning to types 8 and 9, with type 9 having much higher initial manual skills, we see that type 9 individuals are much more likely to work in $G2$, while type 8 individuals are much more likely to work in $S1$ and in $S3$. This figure reinforces the observation that the various dimensions of skills carry meaningful information that would have been lost if one were to adopt a one-skill framework. Moreover, these comparisons show that initial skill endowments are predictive of lifetime choices, and individuals' skills tend to line up rather well with the skill requirements of their employment outcomes, see Table 1.

There are also some type-independent patterns worth noting. First, over time (as individuals age) more people work in the services sector. Second, more people work in occupation group 1 and fewer in occupation group 2 and 3. Since the NSLY79 essentially follows one cohort over time, it does not provide information on the extent to which each of these patterns can be considered as a time effect or an age effect. This is why we turn to the Current Population Survey, where we obtain data on all cohorts of the economy.

3.3 Aggregate employment trends

We use the CPS between 1968 and 2014 to obtain information on all individuals in the economy. In the CPS there are no measures on individual skills from which one can credibly estimate the innate ability vector $\underline{a}_0$. Thus, we can only look at the aggregate economy in terms of cohort, age and education. In Figure 1 in the introduction we already showed employment patterns by education across sectors, as well as across sector-occupation cells.

In Figure 3 we show the employment outcomes broken down by education for

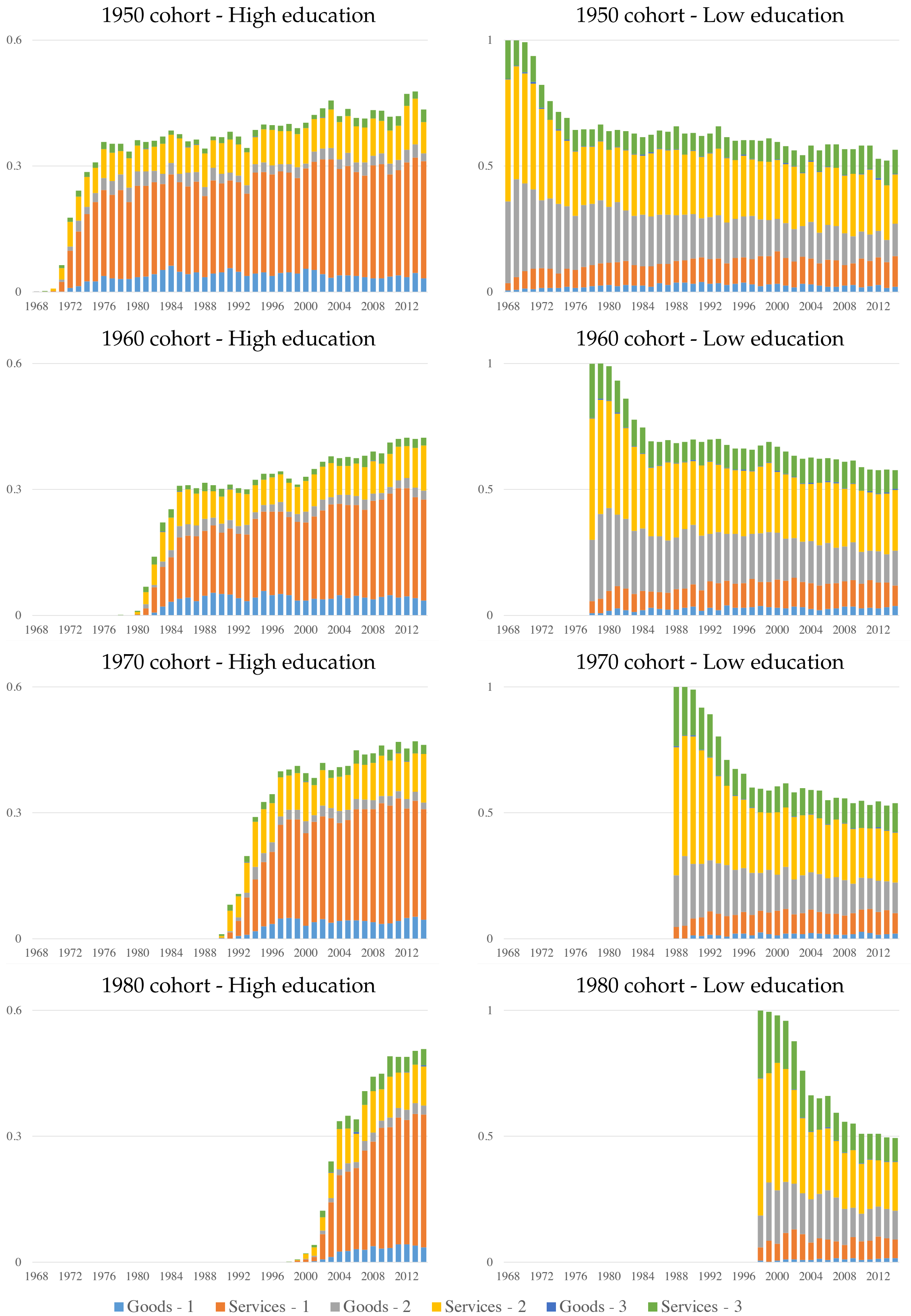

Figure 3: Employment shares by cohort in sector-occupation cells, 1968–2014

Notes: Authors' own calculations from CPS data 1968-2014. Each row of panels shows the share of hours worked over time by a birth cohort. The shares add up to one by year for a given cohort across high- (left panel) and low-educated individuals (right panel). The sector-occupation groups are distinguished by color, as shown in the legend.

four cohorts of the CPS, those born in 1950, 1960, 1970, and 1980. The graphs show employment outcomes from age 18 until our last year of data, namely 2014, within the employed individuals of the cohort. This figure shows clearly for each of the cohorts that as they get older the fraction working in occupation 2 decreases, while the fraction in occupation 1 increases, and the fraction working in occupation 3 is rather stable. For the cohorts born in 1950 and 1960 we also see that the fraction working in the goods sector decreases over time, whereas this is less pronounced for the cohorts born in 1970 and 1980. Finally, comparing these cohorts, we see that a much smaller fraction of the later born cohorts starts working in $G2$, and a larger fraction starts in $S2$ and mainly in $S3$.[9] These patterns suggest that individuals accumulate human capital while working, which allows them to progress across occupations groups. At the same time, there are clear time effects: there is a reduction in the demand for employment in the goods sector, and an increase in the demand for $S2$ and $S3$ employment.

In what follows we formulate a two-sector overlapping generations general equilibrium model with individual education choices and subsequent sector-occupation choices, which allows us first to estimate the type of technological changes which have lead to the discussed patterns, and second to evaluate the impact of various policies.

4 Model

Time is infinite and discrete. The demographic structure is an overlapping generations model. Individuals are heterogeneous in their innate ability, $\underline{a}_0 \in \mathbb{R}^3$, we refer to each dimension as a skill type. Below we first provide details about the structure of the model and its various elements.

Sectoral production and consumption demand There are two sectors in the economy: goods and services. In both sectors, firms employ the available matched labor from each of the O occupations. Firms pay workers their marginal product. The value of the different types of abilities (tasks) is different across the sectors and occupations.

[9] Here, $G1$, $G2$, and $G3$, are occupation 1, 2, and 3, in the goods sector, respectively. Similarly, $S1$, $S2$, and $S3$, are the occupation 1, 2, and 3, in the services sector, respectively.

We assume that technology augments tasks and is sector-occupation specific. Individuals have non-homothetic preferences for consuming services and goods. There is no capital and no saving in the model. Thus, each individual spends his/her current income on consumption in each period.

Education Every individual has to decide whether to acquire education. Education increases the ability of the agent, and increases the arrival rate of jobs for certain occupations. Education has an opportunity cost as it is not possible to work while studying and a *stochastic* utility cost. Furthermore, the expected utility cost is larger for individuals with lower abstract ability.

Structure of the labor market All workers start out as unemployed and have to decide in which of the sector-occupation cells to search for a job; that is, we assume *directed search*. There is on-the-job search, so if a worker wants to switch sectors and/or occupations, he/she endures a non-pecuniary search cost, and if he/she gets a job offer from the other sector/occupation, he/she has to accept it. Jobs get destroyed at an exogenous rate, that is, potentially, sector-occupation specific. Jobs arrive to unemployed and to searching employed workers at exogenous rates that are sector-occupation specific and are different for unemployed and employed workers. Workers accept any job offer that arrives. Each worker in each period draws a mean zero i.i.d. preference shock for searching for a job in each sector-occupation pair (cite papers that have this feature).[10]

Human capital accumulation Workers acquire skills on the job in all three dimensions of ability. The speed of skill accumulation depends on the sector-occupation of the worker and on the skill type. When choosing which sector-occupation to search in the worker has to consider both the current wage difference and the difference in the continuation value due to the differential impact on his/her human capital. We provide a more detailed explanation below.

[10]This feature is introduced largely for computational reasons. The i.i.d. continuous preference shocks make the solution of the model sufficiently smooth. Thus it reduces significantly the computational time and guarantee convergence to a solution.

Equilibrium The economy is in a decentralized equilibrium at all times: individuals make educational decisions and sector-occupation choices to maximize their lifetime utility; this entails allocating all their income in each period optimally between services and goods. Production is perfectly competitive, wages are equal to the marginal product of an efficiency unit of labor in each sector-occupation cell, prices are such that the goods markets clear.

4.1 Sectors and production

There are two sectors in the model: goods (G) and services (S). Both sectors hire workers from all types of occupations O, and produce in perfect competition according to the following production function:

$$Y^J = \left(\sum_{o=1}^{O} \alpha^{Jo} (L^{Jo})^{\frac{\rho^J - 1}{\rho^J}} \right)^{\frac{\rho^J}{\rho^J - 1}} \qquad \text{for } J = G, S, \tag{1}$$

where α^{Jo}, for $o = 1, ..., O$, are distribution parameters, with $\sum_{o=1}^{O} \alpha^{Jo} = 1$, and where L^{Jo} for $o = 1, ..., O$, are the total amounts of efficiency units of labor in the sector-occupation cell (J, o), and $\rho^J \in [0, \infty)$ is the elasticity of substitution between workers in different occupations in sector J. Since the job destruction and arrival rates are exogenous, firms do not make hiring and firing decisions. Consequently, firms always use the labor that is matched with them.[11]

Given sector-occupation specific wages per efficiency units of labor, w^{Jo}, the relative occupational labor demand (of occupation o and o') in sector J can be expressed as:

$$\frac{L^{Jo}}{L^{Jo'}} = \left(\frac{\alpha^{Jo}}{\alpha^{Jo'}} \frac{w^{Jo'}}{w^{Jo}} \right)^{\rho^J}. \tag{2}$$

Using the optimal relative labor use in the production function, we can express the price of sector J output as:

$$p^J = \left(\sum_{o=1}^{O} (\alpha^{Jo})^{\rho^J} (w^{Jo})^{(1-\rho^J)} \right)^{\frac{1}{1-\rho^J}}. \tag{3}$$

[11] For $\rho^J < 1$ it is always optimal to employ workers in each occupation.

4.2 Efficiency labor – Task based approach

The amount of efficiency units of labor a worker has depends on the value of his/her ability in the different skills/tasks, and on the sector-occupation cell that he/she is in. The effective ability vector of worker i with ability vector $\underline{a}_i \in \mathbb{R}^s$ in occupation o when working in sector J is given by

$$\underline{h}_i^{Jo} = \underline{\lambda}^{Jo} \circ \underline{a}_i \quad \in \mathbb{R}^s,$$

where $\underline{\lambda}^{Jo}$ is an $s \times 1$ vector with the skill requirements in sector J occupation o, and $\circ$ denotes the Hadamard product.[12] Let $\underline{Z}^{Jo}$ denote the sector-occupation specific task augmenting technology in sector-occupation cell (J, o). Then, the efficiency units of labor of individual i with skill vector $\underline{a}_i$ in (J, o) is

$$l_i^{Jo} = (\underline{Z}^{Jo})' \underline{h}_i^{Jo} = (\underline{Z}^{Jo})'(\underline{\lambda}^{Jo} \circ \underline{a}_i) = \sum_{s=1}^{S} \underline{Z}^{Jo}(s) \underline{\lambda}^{Jo}(s) \underline{a}_i(s),$$

with earnings $w^{Jo} l_i^{Jo}$. The above specification implies that the total efficiency units of labor in sector-occupation (J, o) is given by

$$L^{Jo} = \sum_{i \in (J,o)} l_i^{Jo}.$$

4.3 The individual's problem

Time is infinite and discreet, and in each period a new cohort is born, which lives for T periods. As a convention we denote the birth period (cohort) of individuals by b, the age of the individual by e, and the period of the model by t.

[12] In practice we use $s = 3$ skill dimensions corresponding to abstract, manual, and interpersonal skills.

4.3.1 Demand for consumption goods

The consumer cannot borrow or lend, and maximizes in each period the following utility by spending all of his/her current period disposable income:

$$\begin{aligned}
&\max_{c^G, c^S} \left(\theta_G^{\frac{1}{\rho}} (c^G)^{\frac{\rho-1}{\rho}} + \theta_S^{\frac{1}{\rho}} (c^S + \bar{c}^S)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \\
&\text{s.t.} \quad p^G c^G + p^S c^S \leq m \\
&\qquad c^G \geq 0, \ c^S \geq 0,
\end{aligned}$$

where $\theta_G + \theta_S = 1$, $\rho \in [0, \infty)$, $\bar{c}^S > 0$.[13] Given these non-homothetic preferences, if individuals have sufficiently low income, i.e. $\frac{m}{p^G} < \frac{\theta_G}{\theta_S} \left(\frac{p^S}{p^G}\right)^{\rho} \bar{c}^S$, then they spend all their income on goods, namely $c^G = m/p^G$ and $c^S = 0$.

Otherwise, these preferences lead to the following demands for goods and services, respectively:

$$c^G = \frac{m + p^S \bar{c}^S}{p^G + p^S \frac{\theta_S}{\theta_G} \left(\frac{p^G}{p^S}\right)^{\rho}}, \tag{4}$$

$$c^S = \frac{\frac{\theta_S}{\theta_G} \left(\frac{p^G}{p^S}\right)^{\rho} m - p^G \bar{c}^S}{p^G + p^S \frac{\theta_S}{\theta_G} \left(\frac{p^G}{p^S}\right)^{\rho}}. \tag{5}$$

Given these demands we can define the consumer's indirect utility as a function of income, m, and prices, p^G, p^S in the given period:

$$v(m, p^G, p^S) \equiv \begin{cases} \left(\theta_G^{\frac{1}{\rho}} \left(\frac{m}{p^G}\right)^{\frac{\rho-1}{\rho}} + \theta_S^{\frac{1}{\rho}} (\bar{c}^S)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} & \text{if } m < \frac{\theta_G}{\theta_S} \left(\frac{p^S}{p^G}\right)^{\rho} \bar{c}^S p^G \\ \frac{m + p^S \bar{c}^S}{p^G + p^S \frac{\theta_S}{\theta_G} \left(\frac{p^G}{p^S}\right)^{\rho}} \left[\theta_G^{\frac{1}{\rho}} + \theta_S^{\frac{1}{\rho}} \left(\frac{\theta_S}{\theta_G}\right)^{\frac{\rho-1}{\rho}} \left(\frac{p^G}{p^S}\right)^{\rho-1} \right]^{\frac{\rho}{\rho-1}} & \text{otherwise} \end{cases} \tag{6}$$

The per period utility from consumption of an individual with ability vector $\underline{a}_i$ working in sector-occupation (J, o) in period t is:

$$u_t^{Jo}(\underline{a}_i) \equiv v(p_t^G, p_t^S, w_t^{Jo} \left(\underline{Z}_t^{Jo}\right)' \left(\underline{\lambda}^{Jo} \circ \underline{a}_i\right)). \tag{7}$$

[13] The data seems to suggest that with a homothetic utility we will not be able to match relative sectoral prices and value added shares (see appendix A.5).

Note that $v(0, p^G, p^S) = 0$, and hence we assume that the unemployed have no income and thus get zero utility.

4.3.2 Education and work

Each worker is born with an innate ability $\underline{a}_0 \in \mathbb{R}^s$, drawn from the same, time invariant, discrete distribution of N types.[14] Individuals have the option to acquire higher education before entering the labor market. The costs of education are twofold. For $e_H > 0$ periods (4 years in our case) the individual, while at school, cannot work and thus has zero income. In addition he/she also suffers a *stochastic* utility cost, $\varepsilon^\kappa \cdot \mathcal{U}_t$, in each of these periods. The stochastic component ε^κ is individual specific and is drawn when the agent is born from a distribution $F(\cdot)$ with mean $\mu^\kappa(\underline{a}_0)$ and standard deviation σ^κ. We allow the mean of the cost distribution, μ^κ, to depend on the individual's initial ability, $\underline{a}_0$. We do so, to be able to capture the possibility that costs of education may be lower for individuals with higher abstract ability. We denote by $F(\cdot | \, \underline{a}_0)$ the cost distribution for individuals with initial ability $\underline{a}_0$. Note that this non-pecuniary cost (like all other non-pecuniary costs in the model) is scaled by $\mathcal{U}_t$. We discuss the criterion for, and choice of, $\mathcal{U}_t$ below. Acquiring education increases the worker's ability to $\underline{a}_1 = f_H(\underline{a}_0) > \underline{a}_0$, while no education leaves the worker's ability unchanged: $\underline{a}_1 = \underline{a}_0$. Moreover, we allow the arrival rate of jobs for certain occupations to be higher for those with higher education. Workers acquire education if the expected present value of their lifetime utility is higher with education than without.

Every individual starts out as unemployed. Unemployed individuals in period $t-1$ choose in which sector-occupation (J, o) to search in for potential employment in period t (recall that we assume directed search). The arrival rate of jobs in sector-occupation (J, o) to an unemployed with education level $k \in \{NE, ED\}$ is η_k^{Jo}.

Employed individuals in period $t-1$ can search on-the-job for period t, which entails a non-pecuniary cost of $\chi \cdot \mathcal{U}_t$. Jobs arrive at rate $\zeta \eta_k^{Jo}$ in (J, o), where $\zeta \leq 1$ captures that the employed may be less efficient in searching for jobs. In case no job offer arrives to the worker who is searching on-the-job, he/she remains in the same

[14] As discussed earlier we discretize the almost continuous initial skill distribution obtained from the NLSY79, and represent it by $N = 12$ types.

job in period t.

Jobs in (J, o) are destroyed at an exogenous rate δ_{Jo}: if this hits a worker in period $t-1$, he/she will be unemployed in period t, and cannot start searching for another job in period $t-1$. We assume that individuals draw *i.i.d.* preference shocks ε_t^{Jo} for searching in each sector-occupation (J, o) for each period t. Then $\varepsilon_t^{Jo} \cdot \mathcal{U}_t$ can be seen as the utility/disutility of searching in a given sector-occupation, or if the worker is employed in sector-occupation (J, o), then this is the utility from not searching. The preference shock for period t is drawn in period $t-1$, when individuals have to decide in which sector-occupation, if any, to search in for period t. Individuals have to make decisions at all ages from 0 for those not acquiring higher education, and from e_H for those getting higher education until period $T-1$.

An unemployed worker needs to decide where to search.Employed workers, who do not get hit by a job destruction shock, need to decide whether to search somewhere else; and if so, where. An employed worker who is hit by a job destruction does not make any decision, as he/she will spend the next period unemployed.

The noise in the cost of education (ε^{κ}) and in the search process (ε_t^{Jo}) makes the solution of the model much smoother, thus making it easier to find market clearing wages. Furthermore it also allows the model to fit the data better, since in the data people with the same initial skills make different education decisions, and people with similar initial skills and employment paths find jobs in different sector-occupations. We assume that these patterns arise as a result of the random shock ε^{κ} and ε_t^{Jo}, respectively.

We want the model to be homothetic in all $\{w^{Jo}\}$ jointly, as well as in $\bar{c}^S$ and in all $\{\underline{Z}^{Jo}\}$ jointly. For this, we need to scale all non-pecuniary costs in the model with something that is homogeneous of degree 0 in all $\{w^{Jo}\}$ jointly, and homogeneous of degree 1 in $\bar{c}^S$ and in all $\{\underline{Z}^{Jo}\}$ jointly. The period t utility factor defined as

$$\mathcal{U}_t \equiv v(p_t^G, p_t^S, (\underline{Z}_t^{G1})' \underline{\lambda}^{G1}),$$

satisfies both of these criteria. In other words we scale the non-pecuniary costs of the model with a utility factor defined as the indirect utility from working in sector G

occupation group 1 with unit ability in each skill (i.e. $\underline{a} = [1, 1, 1]$).

4.3.3 Human capital accumulation

Each year of work experience increases the agent's abilities. One year of experience in sector-occupation (J, o) increases the agent's ability vector by

$$\Delta \underline{a}^{Jo} = \underline{\varphi} \circ \underline{\lambda}^{Jo}. \tag{8}$$

This formulation captures two ideas in a parsimonious way. First, that the accumulation of different types of skills depends on the type of job that the worker performs. More precisely, $\underline{\lambda}^{Jo}$ captures the idea that skills which are used more intensively are accumulated faster. Second, the vector $\underline{\varphi}$ allows for a different accumulation speed across skills. This captures the idea that certain skills are easier to learn than others.[15] Note that human capital accumulation is assumed to be linear in the years of experience, and that the change is independent from the level of starting skills. These assumptions simplify the quantitative implementation.

4.3.4 Timing of Decisions

In Figure 4 we depict the specific decisions that an individual has to make in each time period and age. The figure specifically shows the two routes that an individual can take based on his/her decision whether or not to acquire education. The education decision is made at the very first period after high school, the period in which, from the perspective of the model, the individual is born. The decisions after graduation is similar for the educated individuals as for the non-educated, only that the labor market environment for the two groups can, and generally is, very different.

4.3.5 Optimal search

As Figure 4 shows the decision that individuals have to make in each period, depending of whether they are employed or unemployed. In addition, individuals have to

[15] Lise and Postel-Vinay (2019) find for example that the accumulation speed is the fastest in manual, and is the slowest in interpersonal ability, in this notation: $\varphi_2 > \varphi_1 > \varphi_3 \geq 0$.

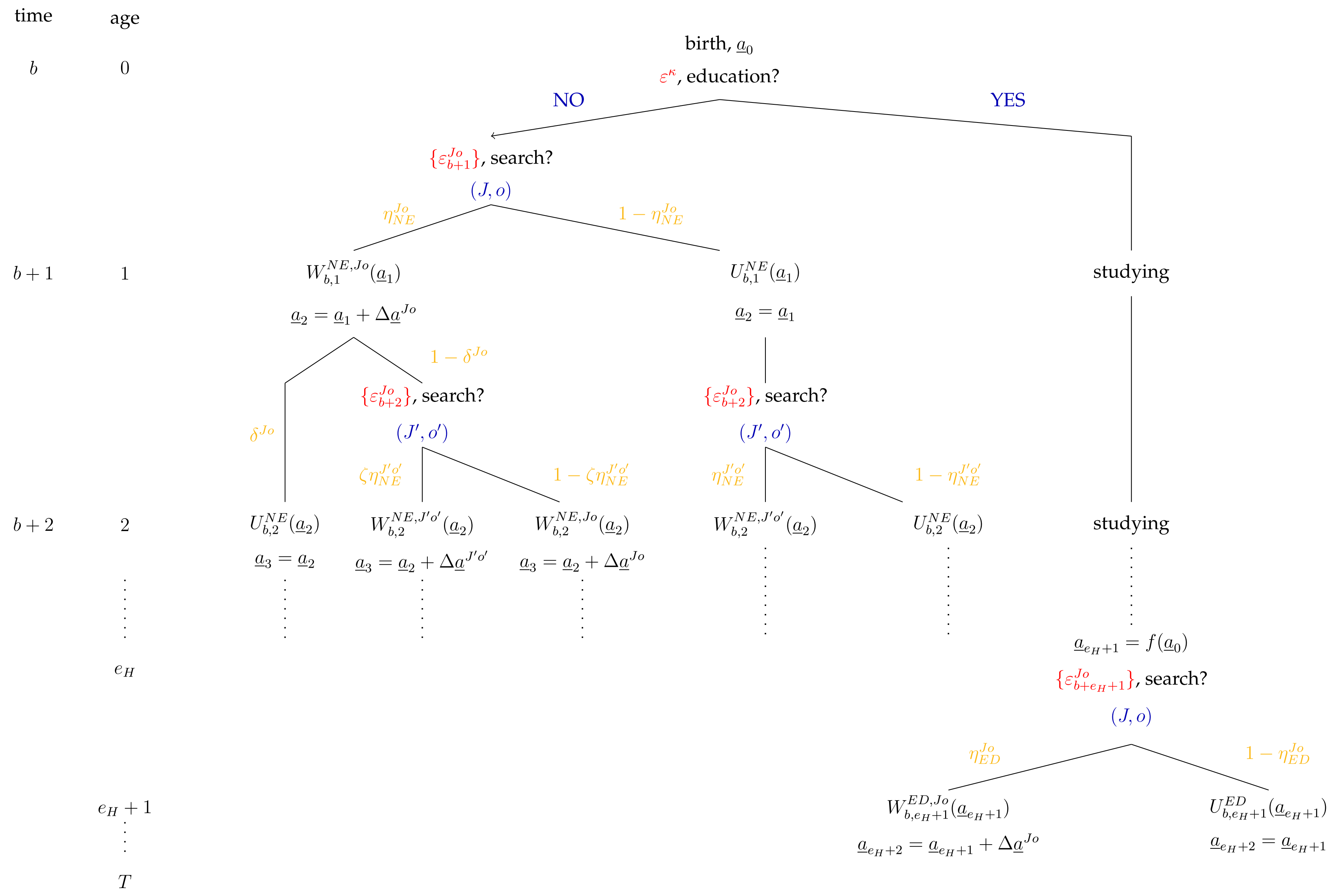

Figure 4: Individual's problem: timing and decisions

decide whether to acquire higher education in the first period, and in that period only. Below we discuss the nature and implication of the decisions individuals face.

Decisions for the unemployed. Consider an $e-1$ year old unemployed worker from the cohort born in period b, with current ability $\underline{a}_{e-1}$, and education level k. The worker has to choose which sector-occupation (J, o) to search in for the next period. The optimal decision depends on the expected value of holding a job in that sector-occupation from next period onward, denoted $W_{b,e}^{k,Jo}(\underline{a}_e)$, on the value of unemployment from next period on $U_{b,e}^{k}(\underline{a}_e)$, as well as the worker's idiosyncratic search preference shock in this period, ε_t^{Jo}, where $t = b + e$. Note that these expressions depend on the age of the individual, e, as that determines the number of periods the individual will live, as well as on the birth cohort of the individual, b, since these are the elements that determine the period in the model. We need to keep track of both, as the expected value of unemployment and employment depend on all the (current and future) wage rates that an e year old individual from cohort b might face ($\{w_{t'}^{Jo}\}$ for all $t' \in \{b+e, ..., b+T\}$). These wages determine the earnings in each sector-occupation in each period, and thus they also determine the prices of goods and services in each period. To save on notation, we do not explicitly include these (common) wage rates in our definition of the various expected wage quantities. The value of employment and unemployment also depend on the level of education of the individual, $k \in \{NE, ED\}$, because the job arrival rates depend on the individual level of education, k. Note that if the worker is unemployed at age $e-1$ then his ability does not change between $e-1$ and e, i.e. $\underline{a}_e = \underline{a}_{e-1}$.

The worker will choose to search in the sector-occupation $(J, o) \in \{\{G, S\} \times \{1, ..., O\}\}$ which maximizes the following:

$$\max_{(J,o)} \left\{ (1 - \eta_k^{Jo}) U_{b,e}^{k}(\underline{a}_e) + \eta_k^{Jo} W_{b,e}^{k,Jo}(\underline{a}_e) + \varepsilon_{b+e}^{Jo} \mathcal{U}_{b+e} \right\} =$$

$$U_{b,e}^{k}(\underline{a}_e) + \max_{(J,o)} \left\{ \underbrace{\eta_k^{Jo} \left[W_{b,e}^{k,Jo}(\underline{a}_e) - U_{b,e}^{k}(\underline{a}_e) \right]}_{\equiv V_{b,e}^{k,Jo}(U,\underline{a}_e)} + \varepsilon_{b+e}^{Jo} \mathcal{U}_{b+e} \right\}, \quad (9)$$

where $V_{b,e}^{k,Jo}(U, \underline{a}_e)$ denotes the expected gain of an individual with eduction level k

at age e, born in period b, with age e ability $\underline{a}_e$ from searching in sector-occupation (J, o) relative to continuing in unemployment (the agent's current state). Thus workers choose to search in the sector-occupation (J, o) for which $V_{b,e}^{k,Jo}(U, \underline{a}_e) + \varepsilon_{b+e}^{Jo}\mathcal{U}_{b+e}$ is the largest. We assume that there are i.i.d. search preference shocks, ε_t^{Jo}, drawn from a mean zero Gumbel distribution with scale parameter σ. Under this assumption we can obtain a closed-form expression for the probability that sector-occupation (J, o) is the best option for a worker. This probability is given by:

$$\pi_{b,e}^{k,U}((J,o), \underline{a}_e) \equiv Prob((J,o) \text{ is better than } (J', o')) = \frac{\exp\left(\frac{V_{b,e}^{k,Jo}(U,\underline{a}_e)}{\sigma\mathcal{U}_{b+e}}\right)}{\sum_{J'o'} \exp\left(\frac{V_{b,e}^{k,J'o'}(U,\underline{a}_e)}{\sigma\mathcal{U}_{b+e}}\right)}. \tag{10}$$

We denote by $\pi_{b,e}^{k,U}((J,o), \underline{a}_e)$ for all (J, o) pairs the probability that an $e-1$ year old unemployed worker from the cohort born in period b with ability $\underline{a}_e$ and education level k at age e searches in sector-occupation (J, o) for age e (period $b+e$).

Using these probabilities we can can now express the expected value of being unemployed for an age $e-1$ individual whose ability is $\underline{a}_{e-1}$. This is given by:

$$\begin{aligned}
U_{b,e-1}^{k}(\underline{a}_{e-1}) &= \underline{u} + \beta E\left[U_{b,e}^{k}(\underline{a}_e) + \max_{(J,o)}\left(\eta_k^{Jo}\left(W_{b,e}^{k,Jo}(\underline{a}_e) - U_{b,e}^{k}(\underline{a}_e)\right) + \varepsilon_{b+e}^{Jo}\mathcal{U}_{b+e}\right)\right] \\
&= \underline{u} + \beta U_{b,e}^{k}(\underline{a}_e) + \beta E\left[\mathcal{U}_{b+e}\max_{(J,o)}\left(\frac{V_{b,e}^{k,Jo}(U, \underline{a}_e)}{\mathcal{U}_{b+e}} + \varepsilon_{b+e}^{Jo}\right)\right] \\
&= \underline{u} + \beta U_{b,e}^{k}(\underline{a}_e) + \beta\mathcal{U}_{b+e}\sigma\log\sum_{Jo}\exp\left(\frac{V_{b,e}^{k,Jo}(U, \underline{a}_e)}{\sigma\mathcal{U}_{b+e}}\right),
\end{aligned} \tag{11}$$

where $\underline{u} = 0$ is the indirect utility from consumption when unemployed. Thus we expressed $U_{b,e-1}^{k}(\underline{a}_{e-1})$ as a function of $U_{b,e}^{k}(\underline{a}_e)$ and $W_{b,e}^{k,Jo}(\underline{a}_e)$ for all $(J, o) \in \{\{G, S\} \times \{1, ..., O\}\}$ and the model's parameters.

Decisions for the employed. Consider an $e-1$ years old worker, with education level k, from cohort b, who is currently employed in sector-occupation (J, o). The individual has to decide in which sector-occupation (if any) to search in for potential employment in the next period, conditional on not becoming unemployed at $e-1$. The decision of such a worker consists of choosing the $(J', o') \in \{\{G, S\} \times \{1, ..., O\}\}$ which maximizes:

$$\max_{(J',o')} \left\{(1-\zeta\eta_k^{J'o'})W_{b,e}^{k,Jo}(\underline{a}_e) + \zeta\eta_k^{J'o'}W_{b,e}^{k,J'o'}(\underline{a}_e) + \varepsilon_{b+e}^{J'o'}\mathcal{U}_{b+e} - \chi\mathcal{U}_{b+e}\mathbf{I}_{(J',o')\neq(J,o)}\right\} =$$
$$W_{b,e}^{k,Jo}(\underline{a}_e) + \max_{(J',o')}\left\{\underbrace{\zeta\eta_k^{J'o'}\left(W_{b,e}^{k,J'o'}(\underline{a}_e) - W_{b,e}^{k,Jo}(\underline{a}_e)\right) - \chi\mathcal{U}_{b+e}\mathbf{I}_{(J',o')\neq(J,o)}}_{\equiv V_{b,e}^{k,J'o'}((J,o),\underline{a}_e)} + \varepsilon_{b+e}^{J'o'}\mathcal{U}_{b+e}\right\}. \tag{12}$$

Here $V_{b,e}^{k,J'o'}((J,o),\underline{a}_e)$ denotes the expected gain for an individual with education level k at age e of cohort b with ability $\underline{a}_e$ from searching in sector-occupation (J', o') relative to continuing in sector-occupation (J, o) (the agent's current state). Note that for $(J', o') = (J, o)$, this is zero by construction. Thus, an employed worker will choose the sector-occupation (J', o') to search in (or choose to not search if $(J', o') = (J, o)$) for which $V_{b,e}^{k,J'o'}((J,o),\underline{a}_e) + \varepsilon_{b+e}^{J'o'}\mathcal{U}_{b+e}$ is the largest. Again, given that the preference shocks are drawn from a Gumbel distribution, the probability that sector-occupation (J', o') is the best option for a worker is given by:

$$\pi_{b,e}^{k,Jo}((J',o'),\underline{a}_e) = \frac{\exp\left(\frac{V_{b,e}^{k,J'o'}((J,o),\underline{a}_e)}{\sigma\mathcal{U}_{b+e}}\right)}{\sum_{(\widehat{J},\widehat{o})}\exp\left(\frac{V_{b,e}^{k,\widehat{J}\widehat{o}}((J,o),\underline{a}_e)}{\sigma\mathcal{U}_{b+e}}\right)}. \tag{13}$$

The expected value of working in sector-occupation (J, o) at age $e-1$ from cohort b with ability $\underline{a}_{e-1}$ can be expressed as:

$$W_{b,e-1}^{k,Jo}(\underline{a}_{e-1}) = u_{b+e-1}^{Jo}(\underline{a}_{e-1}) + \beta\delta^{Jo}U_{b,e}^{k}(\underline{a}_e) +$$
$$+\beta(1-\delta^{Jo})E\left[W_{b,e}^{k,Jo}(\underline{a}_e) + \max_{(J',o')}\left(\zeta\eta_k^{J'o'}\left(W_{b,e}^{k,J'o'}(\underline{a}_e) - W_{b,e}^{k,Jo}(\underline{a}_e)\right) - \chi\mathcal{U}_{b+e}\mathbf{I}_{(J',o')\neq(J,o)} + \varepsilon_{b+e}^{J'o'}\mathcal{U}_{b+e}\right)\right]$$
$$= u_{b+e-1}^{Jo}(\underline{a}_{e-1}) + \beta\delta^{Jo}U_{b,e}^{k}(\underline{a}_e) + \beta(1-\delta^{Jo})\left[W_{b,e}^{k,Jo}(\underline{a}_e) + \mathcal{U}_{b+e}\sigma\log\sum_{(J',o')}\exp\left(\frac{V_{b,e}^{k,J'o'}((J,o),\underline{a}_e)}{\sigma\mathcal{U}_{b+e}}\right)\right], \tag{14}$$

where $u^{Jo}_{b+e-1}(\underline{a}_{e-1})$ is the indirect utility from consumption when working in sector-occupation (J,o) in period $b+e-1$ with ability $\underline{a}_{e-1}$ as defined in (7), and $\underline{a}_e = \underline{a}_{e-1} + \Delta\underline{a}^{Jo}$, as the individual works in sector-occupation (J,o) at age $e-1$. With this we expressed for all (J,o) pairs $W^{k,Jo}_{b,e-1}(\underline{a}_{e-1})$ as a function of $U^k_{b,e}(\underline{a}_e)$ and $W^{k,J'o'}_{b,e}(\underline{a}_e)$ for all $(J',o') \in \{\{G,S\} \times \{1,...,O\}\}$ and parameters of the model.

As each individual in the model only lives for T years, the continuation values after T are zero, i.e. $U^k_{b,T+1}(\underline{a}) = W^{k,Jo}_{b,T+1}(\underline{a}) = 0$, for all education levels, all (J,o) pairs, and all abilities, $\underline{a}$. Thus, we can calculate the optimal search probabilities and implied value functions backwards, starting from the final year, age T to age 1.

The education decision. Finally, consider the education decision of the cohort born in period b. Individuals have to decide (i) whether to acquire education or not (after their disutility cost ε^κ from studying is drawn, but prior to the realization of their initial search preference shocks $\{\varepsilon^{Jo}_{b+1}\}$ or $\{\varepsilon^{Jo}_{b+e_H+1}\}$ for all (J,o) pairs), and (ii) which sector-occupation to search in for their first period on the labor market (once the search preference shocks are realized). The latter problem of a new entrant is identical to the problem of an unemployed worker, given in (9), with $e=1$ and $\underline{a}_e = \underline{a}_0$ in case of no education, and $e = e_H + 1$ and $\underline{a}_e = f_H(\underline{a}_0)$ in case of education. The search probabilities $\pi^{NE}_b((J,o),\underline{a}_0)$ and $\pi^{ED}_b((J,o), f_H(\underline{a}_0))$ are given by the equivalent of (10), and the lifetime value without and with education can be expressed in a similar fashion to (11):

$$V^{NE}_b(\underline{a}_0) = U^{NE}_{b,1}(\underline{a}_0) + \sigma \log \sum_{(J,o)} \exp\left(\frac{V^{NE,Jo}_{b,1}(U,\underline{a}_0)}{\sigma}\right), \tag{15}$$

$$V^{ED}_b(\underline{a}_0) = -\varepsilon^\kappa \sum_{e=1}^{e_H} \beta^{e-1}\mathcal{U}_{b+e} + U^{ED}_{b,e_H+1}(f(\underline{a}_0)) + \sigma \log \sum_{(J,o)} \exp\left(\frac{V^{ED,Jo}_{b,e_H+1}(U, f_H(\underline{a}_0))}{\sigma}\right), \tag{16}$$

where V^{ED}_b reflects that education has a time cost and a non-pecuniary cost: A worker who decides to get education cannot work in the first e_H periods of his/her life. In addition, in each of these periods he/she has a disutility of $\varepsilon^\kappa \mathcal{U}_{b+e}$. Since the disutility of studying is stochastic, and since there is a continuum of individuals with innate ability $\underline{a}_0$, in general, only a given fraction of the individuals with ability $\underline{a}_0$ will choose to obtain more schooling. The cutoff stochastic disutility multiplier for type $\underline{a}_0$ of cohort b is given by:

$$\bar{\varepsilon}_b^{\kappa}(\underline{a}_0) = \max \left\{ \frac{U_{b,e_H+1}^{ED}(f(\underline{a}_0)) + \sigma \log \sum_{(J,o)} \exp\left(\frac{V_{b,e_H+1}^{ED,Jo}(U, f_H(\underline{a}_0))}{\sigma}\right) - V_b^{NE}(\underline{a}_0)}{\sum_{e=1}^{e_H} \beta^{e-1} \mathcal{U}_{b+e}}, 0 \right\}, \tag{17}$$

and the implied fraction of workers who get education is then $F\left(\bar{\varepsilon}_b^{\kappa}(\underline{a}_0) \middle| \, \underline{a}_0\right)$.

Aggregating individual decisions. Given these education and search target decisions we go through the life of each cohort b period by period (for each age $e = 1, ..., T$) for each initial $\underline{a}_0$, where in each period we apply the previously found optimal search probabilities for the next period, and the implied evolution of abilities.

Out of all cohort b initial type $\underline{a}_0$ individuals fraction $F\left(\bar{\varepsilon}_b^{\kappa}(\underline{a}_0) \middle| \, \underline{a}_0\right)$ acquire higher education, and $1 - F_{\underline{a}_0}(\bar{\varepsilon}_b^{\kappa}(\underline{a}_0))$ do not. Those who do not acquire education enter the labor force at age $e = 1$ with ability $\underline{a}_1 = \underline{a}_0$, while those who acquire education only enter the labor force at age $e = e_H + 1$, with ability $\underline{a}_{e_H+1} = f(\underline{a}_0)$.

For the first period in the labor market of cohort b with education level k:

$$\pi_b^k((J, o), \underline{a}_e)\eta_k^{Jo}$$

fraction of the individuals will be employed in cell (J, o), for $J = G, S$, and $o = 1, 2, 3$. Consequently the fraction of individuals who will be unemployed consist of the remaining individuals in the population, that is

$$1 - \sum_{(J,o)} \pi_b^k((J, o), \underline{a}_e)\eta_{NE}^{Jo}$$

will be unemployed.

The age $e + 1$ ability for those who are employed in cell (J, o) at age e is given by $\underline{a}_{e+1} = \underline{a}_e + \Delta\underline{a}^{Jo}$, while for those who are unemployed $\underline{a}_{e+1} = \underline{a}_e$.

We can calculate fractions and abilities for all future ages similarly. For an individual who is unemployed at age $e-1$ the following fractions arise at age e, where the probabilities $\pi_{b,e}^{k,U}((J,o),\underline{a}_e)$ are as in (10)

$$\pi_{b,e}^{k,U}((J,o),\underline{a}_e)\eta_k^{Jo}$$

will be employed in cell (J,o), while fraction

$$1-\sum_{(J,o)}\pi_{b,e}^{k,U}((J,o),\underline{a}_e)\eta_k^{Jo}$$

will remain unemployed.

Among those employed in (J,o) a fraction δ^{Jo} become unemployed for the next period, while the remaining $1-\delta^{Jo}$ fraction of individuals will search with probabilities $\pi_{b,e}^{k,Jo}((J',o'),\underline{a}_e)$ as in (13). Consequently, fraction

$$(1-\delta^{Jo})\pi_{b,e}^{k,Jo}((J',o'),\underline{a}_e)\zeta\eta_k^{J'o'}$$

will be working in (J',o'), where in $(J',o')\neq(J,o)$, a fraction

$$(1-\delta^{Jo})\left[\pi_{b,e}^{k,Jo}((J,o),\underline{a}_e)+\sum_{(J',o')\neq(J,o)}\pi_{b,e}^{k,Jo}((J',o'),\underline{a}_e)(1-\zeta\eta_k^{J'o'})\right]$$

will remain in (J,o), while the remainder, δ^{Jo}, will become unemployed.

The age $e+1$ ability is determined the same way as for the new entrants. The total amount of efficiency units of labor available in sector-occupation (J,o) in period t is then given by

$$(\underline{Z}_t^{Jo})'(\underline{\lambda}^{Jo}\circ\underline{A}_t^{Jo}), \tag{18}$$

where $\underline{A}_t^{Jo}$ is the sum of ability vectors of cohorts born in period $b=t-T,...,t-1$ (currently at corresponding ages $e=T,...,1$) who have a job in sector-occupation (J,o) in period t.

4.4 Competitive Equilibrium

A competitive equilibrium is a sequence of cutoff education costs $\{\bar{\varepsilon}_b^{\kappa}\}_{b=1}^{\infty}$, search probabilities $\{\pi_b^{NE}, \pi_b^{ED}, \{\pi_{b,e}^{k,U}, \pi_{b,e}^{k,Jo}\}_{e=1}^{T}$ for $k = NE, ED\}_{b=1}^{\infty}$ for each possible ability vector, wages $\{w_t^{Jo}\}_{t=1}^{\infty}$, prices $\{p_t^G, p_t^S\}_{t=1}^{\infty}$, given the path of productivities $\{\underline{Z}_t^{Jo}\}_{t=1}^{\infty}$ and initial education cutoffs $\bar{\varepsilon}_0^{\kappa}$ (by type) such that:

1. cutoff education costs are optimal, as in (17);
2. search probabilities arise as the result of maximizing expected lifetime utility from the current period onwards, given by (10) and (13);
3. the unit wage rates are such that the labor market in all sector-occupation (J, o) cells clears, i.e. given employment in each sector-occupation cell (18)) wages are equal to marginal products (2);
4. p^G and p^S are such that the market for G (from (1) and (4)) and S (from (1) and (5)) clears.

The economy is always in a competitive equilibrium, where newborns choose their education optimally, all cohorts' search probabilities are in line with optimal decisions, firms maximize their profits and markets clear.

5 Estimation

We want to estimate the technological changes which gave rise to the observed educational decisions, labor market outcomes, VA shares and prices. To do so we simulate the model's transition from an initial to a final steady state, where technology exogenously changes according to:

$$\underline{Z}_t^{Jo} = \underline{Z}_0^{Jo} \circ (1 + \underline{g}^{Jo})^t.$$

This implies that we allow for the skill-augmenting technological change to be sector and occupation specific, and of course to differ by skill type. We estimate the model in the transition using indirect inference and relying on four data sources. The targets

that we calculate from the data are listed below.
From NLSY79 we compute

- educational attainment by type,
- unemployment by education and type by year between 1979 and 2014,
- employment and wages by sector-occupation by education and type by year between 1979 and 2014,
- transition matrices across employment states by type, education, and year,
- average experience in all sector-occupation cell by type, education and year.

From the CPS between 1968 and 2014 we calculate

- education by cohort
- unemployment by education, age and year
- employment and wage by sector-occupation cells by education, age and year.

We use the BEA to calculate sectoral

- nominal value added shares,
- price indices,
- quantity indices for each year between 1968 and 2014.

5.1 Implementation in code

Numerically, the mapping from model parameters to simulated moments consists of *three* nested loops: one for calculating individual policies and then their allocations, one for finding the market clearing prices (and wages), and then finally an outer optimization for indirect inference.

The dimensionality of the model, in the context of indirect inference, requires that we solve for the market clearing allocation very quickly, which in turn necessitates that we can solve a numerical approximation of the individual decision problems for the life cycle very fast, *and* that we obtain a derivative of the excess demand in the labor and the goods market at the same time. In order to do this, we combine Smolyak sparse grid methods (Judd, Maliar, Maliar, and Valero (2014), Maliar and Maliar (2014)) with algorithmic differentiation (Griewank and Walther (2008), as implemented in Revels, Lubin, and Papamarkou (2016)). For calculating labor supply aggregates by sector-occupation from optimal policies, we use a method inspired by Reiter (2010), calculating transition matrices for bins of uniform approximations. This is much faster than simulation and provides exact results without stochastic jitter.

We illustrate the conceptual framework in an abstract manner here. First, consider the life-cycle optimization problem of the individuals. Given prices p (which, for the purposes of this discussion, also contain wages), we derive values and policies using *backward induction*. This requires an accurate representation of value functions over $\mathbb{R}^3_+$ (the 3 skill dimensions), for each of the two sectors and three occupations. For a given set of parameters θ, we can derive theoretical bounds for the ability in each coordinate for each time period, making sure that abilities are in a hypercube $A(t, \theta) \subset \mathbb{R}^3_+$. We then perform our approximation step very efficiently by calculating the Smolyak grid points and the corresponding basis matrix B for Chebyshev polynomials on $[-1, 1]^3$. Mapping this to $A(t)$ for each t is a *linear* transformation, which is very cheap and also differentiable, and also allows us to reuse the LU decomposition of B, which we just calculate once.

Having calculated the policies $G(p, \theta)$ over the life cycle, we need to obtain market clearing prices that satisfy

$$F(G(p, \theta), \theta) = 0 \tag{19}$$

for some market clearing condition F. In programming G, we make sure that it works well with automatic differentiation, and thus

$$dF = F_1 G_1 dp \tag{20}$$

is readily available. This allows us to use a straightforward trust-region nonlinear solver to obtain the market clearing prices $p(\theta)$ implicitly defined by

$$F(G(p(\theta),\theta),\theta) = 0 \tag{21}$$

For indirect inference (Smith (2008)), either by using a quasi-maximum-likelihood or Wald-type estimator, derivatives of various moments $H(G(p(\theta)),\theta)$ with respect to θ are also useful since they allow first-order quasi-Newton or trust region methods (Nocedal and Wright (2006)) which are much faster than first-order ones like Nelder-Mead or stochastic methods like SPSA (Spall (1998)) or CMA-ES. From (19),

$$F_1\left(G_1 dp + G_2 d\theta\right) + F_2 d\theta = 0 \tag{22}$$

which allows us to obtain

$$\frac{\partial p}{\partial \theta} = \frac{F_2 - F_1 G_2}{F_1 G_1} \tag{23}$$

where, again, all the derivatives on the right hand side can be obtained using algorithmic differentiation.

Then

$$dH = \left(H_1 G_1 \frac{\partial p}{\partial \theta} + H_2\right) d\theta \tag{24}$$

We implement the algorithm in Julia (Bezanson, Edelman, Karpinski, and Shah (2017)), in a package complete with documentation and unit tests, which we intend to make available under an open-source license when the paper is submitted to journals.

6 Results

TO BE ADDED.

7 Policy analysis

TO BE ADDED.

8 Summary

TO BE ADDED.

References

Daron Acemoglu and David Autor. Chapter 12 - Skills, Tasks and Technologies: Implications for Employment and Earnings. In Orley Ashenfelter and David Card, editors, *Handbook of Labor Economics*, volume 4, Part B, pages 1043 – 1171. Elsevier, 2011.

David H. Autor and David Dorn. The Growth of Low-Skill Service Jobs and the Polarization of the US Labor Market. *American Economic Review*, 103(5):1553–97, 2013.

David H. Autor and Michael J. Handel. Putting tasks to the test: Human capital, job tasks, and wages. *Journal of Labor Economics*, 31(2):S59–S96, 2013.

David H. Autor, Frank Levy, and Richard J. Murnane. The Skill Content of Recent Technological Change: An Empirical Exploration. *The Quarterly Journal of Economics*, 118(4):1279–1333, 2003.

David H. Autor, Lawrence F. Katz, and Melissa S. Kearney. The Polarization of the U.S. Labor Market. *The American Economic Review*, 96(2):189–194, 2006.

Zsófia L. Bárány and Christian Siegel. Job Polarization and Structural Change. *American Economic Journal: Macroeconomics*, 10(1):57–89, January 2018.

Zsófia L. Bárány and Christian Siegel. Engines of Sectoral Labor Productivity Growth. Working paper, July 2019a.

Zsófia L. Bárány and Christian Siegel. Biased Technological Change and Employment Reallocation. Working paper, Ovtober 2019b.

William J. Baumol. Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis. *The American Economic Review*, 57(3):415–426, 1967.

Jeff Bezanson, Alan Edelman, Stefan Karpinski, and Viral B. Shah. Julia: A fresh approach to numerical computing. 59(1):65–98, 2017. doi: 10.1137/141000671. URL `https://doi.org/10.1137/141000671`.

Francisco J. Buera, Joseph P. Kaboski, Richard Rogerson, and Juan I. Vizcaino. Skill Biased Structural Change. Working paper, October 2018.

Rafael Dix-Carneiro. Trade liberalization and labor market dynamics. *Econometrica*, 82 (3):825–885, 2014.

Georg Duernecker and Berthold Herrendorf. Structural Transformation of Occupation Employment. Working paper, February 2016.

Christian Dustmann, Johannes Ludsteck, and Uta Schönberg. Revisiting the German Wage Structure. *The Quarterly Journal of Economics*, 124(2):843–881, 2009.

Maarten Goos and Alan Manning. Lousy and Lovely Jobs: The Rising Polarization of Work in Britain. *The Review of Economics and Statistics*, 89(1):118–133, February 2007.

Maarten Goos, Alan Manning, and Anna Salomons. Explaining Job Polarization: Routine-Biased Technological Change and Offshoring. *American Economic Review*, 104(8):2509–26, 2014.

Andreas Griewank and Andrea Walther. *Evaluating derivatives: principles and techniques of algorithmic differentiation*, volume 105. Siam, 2008.

Kenneth L Judd, Lilia Maliar, Serguei Maliar, and Rafael Valero. Smolyak method for solving dynamic economic models: Lagrange interpolation, anisotropic grid and adaptive domain. 44:92–123, 2014.

Donghoon Lee and Kenneth I. Wolpin. Intersectoral labor mobility and the growth of the service sector. *Econometrica*, 74(1):1–46, 2006. ISSN 00129682, 14680262.

Sang Yoon (Tim) Lee and Yongseok Shin. Horizontal and Vertical Polarization: Task-Specific Technological Change in a Multi-Sector Economy. Working Paper 23283, National Bureau of Economic Research, March 2017.

Jeremy Lise and Fabien Postel-Vinay. Multidimensional Skills, Sorting, and Human Capital Accumulation. Working paper, 2019.

Lilia Maliar and Serguei Maliar. Numerical methods for large-scale dynamic economic models. In *Handbook of computational economics*, volume 3, pages 325–477. Elsevier, 2014.

L. Rachel Ngai and Christopher A. Pissarides. Structural Change in a Multisector Model of Growth. *The American Economic Review*, 97(1):429–443, 2007.

Jorge Nocedal and Stephen J Wright. Numerical optimization 2nd, 2006.

Michael Reiter. Approximate and almost-exact aggregation in dynamic stochastic heterogeneous-agent models, 2010.

J. Revels, M. Lubin, and T. Papamarkou. Forward-mode automatic differentiation in julia. 2016. URL `https://arxiv.org/abs/1607.07892`.

A Smith. Indirect inference. 2008.

James C Spall. Implementation of the simultaneous perturbation algorithm for stochastic optimization. 34(3):817–823, 1998.

Alexandra Spitz-Oener. Technical Change, Job Tasks, and Rising Educational Demands: Looking outside the Wage Structure. *Journal of Labor Economics*, 24(2):235–270, 2006.

A Appendix

A.1 Data

A.1.1 Sector and occupation classification

Occupations are classified in the following way:

Table 3: Classification of occupations into groups

Category	occ1990	Group		
		1	2	3
A. MANAGERIAL & PROFESSIONAL SPECIALTY OCCUPATIONS				
A.1 Executive, Administrative, and Managerial Occupations	3 - 22	X		
A.2 Management Related Occupations	23 - 37	X		
A.3 Professional Specialty Occupations	43 - 200	X		
B. TECHNICAL, SALES & ADMIN. SUPPORT OCCUPATIONS				
B.1 Technicians and Related Support Occupations	203-235	X		
B.2 Sales Occupations	243 - 290		X	
B.3 Administrative Support Occupations, Including Clerical	303 - 390		X	
C. SERVICE OCCUPATIONS	405 - 469			X
D. FARMING, FORESTRY & FISHING OCCUPATIONS	473 - 498		X	
E. PRECISION PRODUCTION, CRAFT & REPAIR OCCUPATIONS	503 - 699		X	
F. OPERATORS, FABRICATORS, AND LABORERS	703 - 890		X	

A.1.2 Principal component analysis for the O*NET and the NLSY79 data

Denote the data matrix by D, which contains in each row the descriptors for a given occupation or the characteristics of each individual. The first three columns correspond to the descriptors or characteristics which are assumed to only impact abstract, manual and interpersonal skill requirements or endowments respectively. We run the following principal component analysis:

$$D = F\Gamma,$$

where each row in Γ contains the coefficients for one principal components, and F contains the representation of data D in the principal component space. Denote the first three columns of F by F_3, and the first 3-by-3 matrix of Γ by Γ_{33}. We obtain a representation of all skill requirements or skill endowments in the space of the first three principal components as:

$$D \approx X_0 = F_3 \Gamma_{33}.$$

We normalize all elements (i.e. each column of X_0) such that all values lie between 0 and 1, denote this normalized matrix by X. The rows of X give the abstract, manual and interpersonal skill requirement of each occupation from the O*NET or the initial skill endowment of each individual in the NLSY79.

A.1.3 NLSY79 descriptors

We retain the variables listed below which describe individual characteristics. Family background variables that we include are the highest grade completed for the father and the mother of the respondent. We include a set of dummy variables for the race of the individual (white, Hispanic, African American, and others), dummy variable for the gender of the individual, height, weight and Body-Mass-Index (BMI) of the respondent. We also include a number of measures of criminal and anti-social behavior.[16] From the Attitudes section we retain the Rotter locus-of-control scale score and the Rosenberg self-esteem test score. To directly account for measure of ability we include also a number of variables from the Armed Services Vocational Aptitude Battery (ASVAB), as follows: General knowledge (section 1), arithmetic reasoning (section 2), word knowledge (section 3), paragraph comprehension (section 4), numerical operations (section 5), coding speed (section 6), auto and shop information (section 7), mathematics knowledge (section 8), mechanical comprehension (section 9), and electronics information (section 10).

[16]These measures include the number of times used drugs or other chemical in the past 12 moth to get "high", the number of times charged with illegal activity, age at which first charged with illegal activity, etc.

A.2 The price of sector J output

Using the optimal relative labor use within a sector (2), we can express the amount of L^{J1} labor needed to produce one unit of sector J output as:

$$1=\left[\sum_{o=1}^{O}\alpha^{Jo}(L^{Jo})^{\frac{\rho^J-1}{\rho^J}}\right]^{\frac{\rho^J}{\rho^J-1}}=L^{J1}\left[\sum_{o=1}^{O}\alpha^{Jo}\left(\frac{L^{Jo}}{L^{J1}}\right)^{\frac{\rho^J-1}{\rho^J}}\right]^{\frac{\rho^J}{\rho^J-1}}=L^{J1}\left[\alpha^{J1}+\sum_{o=2}^{O}\alpha^{Jo}\left(\frac{\alpha^{Jo}}{\alpha^{J1}}\frac{w^{J1}}{w^{Jo}}\right)^{\rho^J-1}\right]^{\frac{\rho^J}{\rho^J-1}}$$

$$\Leftrightarrow L^{J1}=\left[\alpha^{J1}+\sum_{o=2}^{O}\alpha^{Jo}\left(\frac{\alpha^{Jo}}{\alpha^{J1}}\frac{w^{J1}}{w^{Jo}}\right)^{\rho^J-1}\right]^{\frac{-\rho^J}{\rho^J-1}}$$

Using this in the production function, we can express the price of sector J output as:

$$\begin{aligned}
p^J&=\sum_{o=1}^{O}w^{Jo}L^{Jo}=w^{J1}L^{J1}\left[1+\sum_{o=2}^{O}\left(\frac{\alpha^{Jo}}{\alpha^{J1}}\right)^{\rho^J}\left(\frac{w^{J1}}{w^{Jo}}\right]^{\rho^J-1}\right)\\
&=w^{J1}\left(\alpha^{J1}+\sum_{o=2}^{O}\alpha^{Jo}\left(\frac{\alpha^{Jo}}{\alpha^{J1}}\frac{w^{J1}}{w^{Jo}}\right)^{\rho^J-1}\right)^{\frac{-\rho^J}{\rho^J-1}}\left(1+\sum_{o=2}^{O}\left(\frac{\alpha^{Jo}}{\alpha^{J1}}\right)^{\rho^J}\left(\frac{w^{J1}}{w^{Jo}}\right)^{\rho^J-1}\right)\\
&=\left[\sum_{o=1}^{O}(\alpha^{Jo})^{\rho^J}(w^{Jo})^{1-\rho^J}\right]^{\frac{1}{1-\rho^J}}
\end{aligned}$$

A.3 Labor demands

To produce Y^J units of output the labor demand for each occupation o in sector J, using optimal relative demands (2) and sectoral price can be expressed as:

$$\begin{aligned}
Y^J&=\left[\sum_{o=1}^{O}\alpha^{Jo}(L^{Jo})^{\frac{\rho^J-1}{\rho^J}}\right]^{\frac{\rho^J}{\rho^J-1}}=L^{J1}\left(\alpha^{J1}+\sum_{o=2}^{O}\alpha^{Jo}\left(\frac{\alpha^{Jo}}{\alpha^{J1}}\frac{w^{J1}}{w^{Jo}}\right)^{\rho^J-1}\right)^{\frac{\rho^J}{\rho^J-1}}\\
&=L_{J1}\left(\alpha^{J1}+\sum_{o=2}^{O}(\alpha^{Jo})^{\rho^J}(\alpha^{J1})^{1-\rho^J}\left(\frac{w^{J1}}{w^{Jo}}\right)^{\rho^J-1}\right)^{\frac{\rho^J}{\rho^J-1}}\\
&=L^{J1}\left((\alpha^{J1})^{1-\rho^J}(w^{J1})^{\rho^J-1}\left((\alpha^{J1})^{\rho^J}(w^{J1})^{1-\rho^J}+\sum_{o=2}^{O}(\alpha^{Jo})^{\rho^J}(w^{Jo})^{1-\rho^J}\right)\right)^{\frac{\rho^J}{\rho^J-1}}\\
&=L^{J1}(\alpha^{J1})^{-\rho^J}(w^{J1})^{\rho^J}\left(\sum_{o=1}^{O}(\alpha^{Jo})^{\rho^J}(w^{Jo})^{1-\rho^J}\right)^{\frac{\rho^J}{\rho^J-1}}=L^{J1}(\alpha^{J1})^{-\rho^J}(w^{J1})^{\rho^J}(p^J)^{-\rho^J}
\end{aligned}$$

Re-arranging we get

$$L^{J1} = \left(\frac{p^J \alpha^{J1}}{w^{J1}}\right)^{\rho^J} Y^J$$

And similarly for the other occupations:

$$L^{Jo} = \left(\frac{\alpha^{Jo}}{\alpha^{J1}}\frac{w^{J1}}{w^{Jo}}\right)^{\rho^J} L^{J1} = \left(\frac{\alpha^{Jo}}{\alpha^{J1}}\frac{w^{J1}}{w^{Jo}}\right)^{\rho^J} \left(\frac{p^J \alpha^{J1}}{w^{J1}}\right)^{\rho^J} Y^J = \left(\frac{p^J \alpha^{Jo}}{w^{Jo}}\right)^{\rho^J} Y^J$$

A.4 Preference shocks

An individual i is faced with a choice between N options, each $n \in N$ providing utility according to:

$$\begin{aligned} U_{i,n} &= V_n + \varepsilon_{i,n} \cdot \mathcal{U} \Leftrightarrow \\ \widetilde{U}_{i,n} &\equiv \frac{U_{i,n}}{\mathcal{U}} = \frac{V_n}{\mathcal{U}} + \varepsilon_{i,n}, \end{aligned}$$

where V_n is known, and $\varepsilon_{i,n}$ is drawn i.i.d. from a Gumbel distribution with location parameter μ and scale parameter $\sigma > 0$, and $\mathcal{U}$ is a utility factor which scales the preference shock. To ensure that the mean of these shocks is zero, we impose that $\mu = -\sigma\gamma$, where γ is the Euler–Mascheroni constant. The probability that an individual chooses option n is given by

$$\begin{aligned} \mathbf{P}(n) &\equiv Prob(U_{i,n} > U_{i,j} \text{ for all } j \neq n) = Prob(\widetilde{U}_{i,n} > \widetilde{U}_{i,j} \text{ for all } j \neq n) \\ &= \frac{\exp\left(\frac{\mu + \frac{V_n}{\mathcal{U}}}{\sigma}\right)}{\sum_j \exp\left(\frac{\mu + \frac{V_j}{\mathcal{U}}}{\sigma}\right)} = \frac{\exp\left(\frac{V_n}{\sigma\mathcal{U}}\right)}{\sum_j \exp\left(\frac{V_j}{\sigma\mathcal{U}}\right)} \end{aligned}$$

The vector $\mathbf{P}$ contains the **policy** that we need to save for all individuals (in each age, in each state, at each ability): it contains the probability that this individual searches in each of the N sector-occupation pairs.

We also need to save for each individual (in each age, in each state, at each ability) **the expected lifetime value** of being in that state from the current period onwards.

To calculate this we need to express the expected present value from the next period onwards *before* drawing the preference shocks. Denote by $y \equiv \max_n \left(\frac{V_n}{\mathcal{U}} + \varepsilon_n\right)$, we show that this random variable also follows a Gumbel distribution.[17]

For this express the following:

$$\begin{aligned}
\log P(y < z) &= \sum_{j=1}^{N} \log P\left(\varepsilon_j + \frac{V_j}{\mathcal{U}} < z\right) = \sum_{j=1}^{N} \log P\left(\varepsilon_j < z - \frac{V_j}{\mathcal{U}}\right) \\
&= \sum_{j=1}^{N} -e^{-\frac{z-\frac{V_j}{\mathcal{U}}-\mu}{\sigma}} = -e^{-\frac{z-\mu}{\sigma}} \sum_{j=1}^{N} e^{\frac{V_j}{\sigma\mathcal{U}}} = -e^{-\frac{z-\mu-\sigma\log\sum_{j=1}^{N} e^{\frac{V_j}{\sigma\mathcal{U}}}}{\sigma}}
\end{aligned}$$

This shows that y is a random variable with a Gumble distribution with location parameter $\mu + \sigma \log \sum_{j=1}^{N} e^{\frac{V_j}{\sigma\mathcal{U}}}$ and scale parameter σ. This immediately implies that its mean is given by:

$$E[y] = E\left[\max_n \left(\frac{V_n}{\mathcal{U}} + \varepsilon_n\right)\right] = \mu + \sigma \log \sum_{j=1}^{N} e^{\frac{V_j}{\sigma\mathcal{U}}} + \sigma\gamma.$$

A.5 Initial parameter guess for preference parameters

To get a good initial guess for the parameters of the utility function – θ_G, θ_S, ρ and $\bar{c}^S$ – we can use aggregate data on value added per hour worked by industry as well as price indexes by industry, and estimate under what parameters would a representative household choose the observed expenditure shares. Looking at per capita value added shares by sector is equivalent to assuming that the demand aggregates, i.e. to assuming a representative household. This aggregation only works if the utility function is homothetic ($\bar{c}^S = 0$), otherwise the average expenditure share is not equal to the expenditure share resulting from the aggregation of individual demands, i.e. this approach is just an approximation.

Thus we have two options.

First, we can assume that $\bar{c}^S = 0$, in which case the expenditure share of every single

[17] The cdf of a Gumbel distribution with location parameter μ and scale parameter $\sigma > 0$ is:

$$F(x; \mu, \sigma) = e^{-e^{-\frac{x-\mu}{\sigma}}}.$$

household (and also in the whole economy) is given by:

$$\left.\frac{p^S C^S}{CE}\right|_{\bar{c}^S=0} = \frac{1}{\frac{\theta_G}{\theta_S}\left(\frac{p^G}{p^S}\right)^{1-\rho} + 1}. \tag{25}$$

Manipulating the above equation we get:

$$\begin{aligned}
\frac{CE}{p^S C^S} &= \frac{\theta_G}{\theta_S}\left(\frac{p^G}{p^S}\right)^{1-\rho} + 1 \\
\frac{CE}{p^S C^S} - 1 &= \frac{\theta_G}{\theta_S}\left(\frac{p^G}{p^S}\right)^{1-\rho} \\
\log\left(\frac{CE}{p^S C^S} - 1\right) &= \log\left(\frac{\theta_G}{\theta_S}\right) + (1-\rho)\log\left(\frac{p^G}{p^S}\right),
\end{aligned}$$

where $CE = VA_S + VA_G$ is total value added per capita, $VA_S = p^S C^S$ is value added per capita in services, and p^G/p^S is the relative price of goods to services. We can estimate the above equation with a simple OLS on the time series of US data, available from the BEA. The estimates imply a negative ρ, which is not compatible with the utility function.

Second, allowing $\bar{c}^S > 0$ we can estimate the parameters under which a representative household's expenditure share would be in line with the data. Note that in general the expenditure share in the economy is not equal to the expenditure share of the representative household, so this is just an approximation. In the general case the expenditure share on services can be expressed as:

$$\frac{p^S C^S}{CE} = \frac{\frac{\theta_S}{\theta_G}\left(\frac{p^G}{p^S}\right)^{\rho-1} - \frac{p^S \bar{c}^S}{CE}}{1 + \frac{\theta_S}{\theta_G}\left(\frac{p^G}{p^S}\right)^{\rho-1}} \tag{26}$$

The above can be estimated using non-linear least squares. The results are summarized in the table below.

Table 4: Estimated utility parameters for representative household

	1947-2017	1968-2017	1968-2008	1980-2008
ρ	0.2100	0.4164	0.4305	0.1660
θ_G	0.1434	0.0925	0.0866	0.1669
$\bar{c}^S$	24.45	69.00	77.11	12.43