Financial Integration and Financial Instability*

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Abstract

In the presence of bank funding risk, unregulated issuance of safe short-term liabilities by financial intermediaries leads to excessive reliance on this form of financing, which increases losses associated with financial crises. This paper studies welfare implications of international financial integration in the presence of bank funding risk. First, I show that integration increases the severity of potential financial crises in the countries that receive capital inflows. As a result, integration may reduce welfare for these countries. Second, I show that if macroprudential regulation of banking sector is chosen by each country in an uncoordinated way, the outcome can be Pareto inefficient, so that there is role for global coordination of such policies. This is because the macroprudential regulation that limits overissuance of safe debt liabilities changes the international interest rate creating incentives to manipulate the interest rate. Third, the desire to manipulate the international interest rate when regulating local banking sector creates incentives to use two regulatory tools, macroprudential regulation of banking sector and capital controls.

Keywords: Financial integration, financial stability.

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1 Introduction

Large increase in cross-border banking during the last decade renewed interest in the effects of fluctuations in capital flows. The creation of the Eurozone in 1999 is a case in point. Capital account liberalization as a prerequisite for admission. The cross-border assets of the Eurozone banks in domestic currency increased from 2 trillion dollars in 1999 to 10 trillion dollars in 2008 and the liabilities went up from 2 trillion dollars to 8 trillion dollars. However, these flows were unevenly distributed across Eurozone countries. Slow growing central countries were investing in fast growing peripheral countries. For example, the net foreign asset positions of Spain decreased from -40% as a share of its GDP in 1999 to -80% in 2008 and continued falling after that. More than half of the decline is associated with the banking sector increasing its net foreign liabilities. A large fraction of these Spanish liabilities were held by surplus countries, such as Germany and France. At the same time banking lending to foreign non-banking sector in the Eurozone did not show the same level of integration.

The ongoing global financial crisis, which has had especially serious consequences in the Eurozone periphery, raises the question of whether increased financial integration may have played a role in exacerbating the negative effects of the crisis. Pre-crisis conventional wisdom suggested that financial integration leads to a better risk sharing by smoothing country-specific shocks and to efficient capital reallocation from capital-abundant countries to capital-poor countries. However, in the presence of market imperfections these benefits of financial integration may be mitigated or more than offset by exacerbated financial frictions.

In this paper I ask three questions. First, does the integration of bank short-term funding markets exacerbate financial crises? Second, can this lead to a decrease in social welfare?

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1 The data comes from BIS locational banking statistics, Table 5A. The BIS uses the US dollars as the numeraire in its international banking statistics.
2 The data comes from the International Financial Statistics Database.
3 ECB (2012) presents the data on establishment and activity of foreign branches and subsidiaries across euro area countries. The report concludes that the integration in cross-border retail banking market is limited.
4 The argument that removing a distortion in an environment with other distortions may lead to a reduction in welfare goes back to at least Lipsey and Lancaster (1956). Hart (1975) presents an example in which adding a new market that does not make the market structure complete makes every agent in the economy worth off. Newbery and Stiglitz (1984) show that opening countries to international trade in goods can make agents worth off in participating economies in the absence of insurance markets.
Third, what regulations should be put in place to neutralize the negative consequences that financial frictions have when funding markets are integrated? And in particular, is it necessary for countries to coordinate to achieve optimal regulation?

I present a model of bank funding risk based on Stein (2012). Banks finance themselves by issuing risky and safe debt and invest in long-term risky projects. Entrepreneurs liquidity preferences from holding safe debt. This makes the safe debt a cheaper and therefore a preferable mean of financing for banks relative to risky debt. Because there is more uncertainty in the long run, it is easier for the banks to issue short-term safe debt. For short-term debt to be safe, banks must, however, have enough resources to honor their short-term liabilities in an adverse state. If outside funding is not available in the adverse state banks have to sell their assets at a fire-sale price. Thus banks cannot issue more safe debt than the value of their assets in the adverse state. This implies that banks face an endogenous collateral constraint on the issuance of safe debt. Banks don’t internalize the fact that their choices of safe debt affect the collateral constraints of the other banks. This externality leads banks to issue too much safe debt.

I embed this model of funding risk into a setting with two regions: the center and the periphery. Each country has entrepreneurs and banks. The periphery entrepreneurs have more productive marginal investment opportunities compared to the entrepreneurs in the center. Banks buy risky projects from entrepreneurs and fund themselves by issuing safe and risky debt. The difference in productivities of marginal investment opportunities in the two countries leads to different returns on safe debt before integration. The peripheral entrepreneurs create more risky assets for the peripheral banks to invest in. The banks need more funding to buy these assets which leads to a bigger safe debt issuance. Because entrepreneurs utility from holding safe debt has diminishing returns to scale the interest that banks have to pay to the safe debt holders is higher in the periphery than in the center.

Integration of bank short-term liabilities funding markets leads to equalization of the returns on safe debt across the two countries. As a result rates on safe debt decrease in the periphery which increases bankers incentives to issue safe debt. More safe debt will lead to a larger fire-sale discount in the adverse state of the world in the periphery. The opposite is true for the center.
I show that the center always benefit from integration while the periphery loses under certain conditions. There are two effects of the integration: capital reallocation and a change in the severity of welfare losses due to overissuance of safe debt and the associated exacerbation of fire-sale discount during crises. Consider the periphery. The inflow of resources from the center is a benefit, the banks can issue more safe debt cheaply. However, more safe debt leads to a larger asset fire-sale discounts in the adverse state of the world which exacerbates the negative externality associated with overissuance of safe debt, leading to bigger welfare losses. I show that if the difference in marginal productivities of investment opportunities across the two countries is not too big then the welfare losses associated with fire sales always dominate welfare benefits from having access to cheaper safe debt financing. On the other hand, the banks in the center reduce issuance of safe debt which decreases losses in the adverse state. In addition agents in the center are able to invest their savings at a higher return in the periphery. Thus, both effects increase welfare in the center.

In a closed economy setting, a regulator wants to impose a tax on safe debt issuance, to make banks internalize the social costs of fire-sales. In the two-country model with two local regulators I show that they will choose inefficient tax rates on safe debt issuance. The intuition is as follows. An increase in the tax level decreases the issuance of safe securities which in turn decreases the world equilibrium return on the securities. Because the periphery is a net supplier of safe debt a decrease in the rate of return decreases the amount that bankers have to repay to the agents in the center. Hence, the regulator in this country chooses the level of taxes that is higher than needed to correct the externality in the banking sector. The logic is reversed for the center regulator. The resulting Nash equilibrium can be Pareto improved.

Finally, the desire to manipulate the international interest rates when regulating local banking sector creates incentives to use two regulatory tools, prudential taxes on the banking sector and capital controls, instead of just using prudential taxation in the banking sector.

Related literature. This paper contributes to the recent literature that studies welfare effects of financial integration. Most closely related to this paper is Mendoza et al. (2007). They argue that financial flows that arise from different levels of financial development leads
to an increase in welfare in a more financially developed country and a decrease in welfare in a less financially developed country after integration. Eden (2012) studies the welfare effects of financial integration in the presence of the working capital constraints in a less financially developed country. The author concludes that the more financially developed country that does not face working capital constraints and experiences capital inflows benefits while the less financially developed country loses from financial integration.

My paper is related to the literature that studies models in which the presence of financial frictions give rise to the welfare reducing pecuniary externality. Lorenzoni (2008) builds a model in which financial frictions prevent equalization of the marginal utilities of agents which leads to welfare loses associate with the pecuniary externality. Bianchi and Mendoza (2010) present a quantitative model in which agents face borrowing constraints. In their model agents cannot borrow more than the current value of their collateral. The presence of this constraint leads to a negative pecuniary externality.

My paper is related to the literature that studies the international terms of trade manipulation. Obstfeld and Rogoff (1996) in a two-period and recently Costinot et al. (2011) in a dynamic model study how the incentives to install capital controls may arise because of the desire to manipulate the intertemporal terms of trade. In my paper a regulator that only intends to limit the scope of the negative externality in the banking sector will inevitably affect the international interest rate which will lead to the desire to manipulate the interest rate.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 studies equilibrium properties. Section 4 analyses welfare consequences of integration. Section 5 investigates how incentives to correct the externality changes with the integration. Formal proofs are presented in the Appendix.
2 Model.

In this section I describe a two-country integrated world. I will use superscripts $C$ (the center) and $P$ (the periphery) to distinguish between country-specific variables. Each country is identical except for their marginal productivity of investment opportunities $A^C < A^P$ (see the description below).

The economic environment departs minimally from Stein (2012) original model for a meaningful analysis of a two-country model. First, I assume that the utility from holding safe securities has diminishing returns to scale. This assumption implies non-zero net capital flow after bank funding market integration of two asymmetric countries. Second, I assume that banks don’t invest directly into projects production but instead buy projects from entrepreneurs. This assumption will allow me to consider the effect of the lending market integration in the end of the paper.

I will describe the model in terms of the periphery and then present a two-country equilibrium. The center description is identical. The economy goes on for three dates, $t = 0, 1, 2$, and there is a single consumption good that serves as the numeraire. The economy is populated by three types of agents: entrepreneurs, bankers and outside investors. Each type of agents has measure 1. An entrepreneur has endowment in period 0 and he chooses his consumption plan, portfolio allocation and investments in risky projects that he immediately sells to the bankers in period 0. A banker buys risky projects from the entrepreneurs and finance them by issuing risky and safe debt to the entrepreneurs in period 0. The banker can sell his safe debt to entrepreneurs in both countries. The risky projects will payoff in period 2. The uncertainty structure of the risky projects is presented in figure 1.

In period $t = 1$ news about future payoff of the projects arrives. With probability $p$ there is good news, called the good state and denoted $s^P_1 = G$, where subscript 1 denotes period 1, that ensures that the risky projects will yield positive amount of consumption good in period $t = 2$, the corresponding state in period 2 is denoted by $s^P_2 = G$. With probability $1 - p$ there is bad news, called the bad state and denoted $s^P_1 = B$, informing that the risky projects will yield the same positive amount of consumption good in period 2 with probability $q$, I denote this state by $s^P_2 = Bnc$, and 0 with probability $1 - q$, this state is denoted by $s^P_2 = Bc$. The
risk projects uncertainty is aggregate.

The bankers can also sell their risky projects to outside investors in period 1. The outside investors have a fixed endowment of consumption goods in period 1 that they can use to invest in their late arriving technology, invest in the storage technology between period 1 and 2 or to buy bankers assets. Only the outside investors have access to the storage technology.\textsuperscript{5}

2.1 Entrepreneurs

An Entrepreneurs maximizes the following utility function

\[ U = C_0^P + \beta E C_2^P + v(D_d^P) \]  

(1)

where \( C_0^P \) and \( C_2^P \) are consumption levels in period 0 and 2 respectively which have to be non-negative.\textsuperscript{6} \( v(D_d^P) \) represents the additional utility derived from holding safe debt claims on time 2 consumption, \( D_d^P \) is time 0 holdings of safe debt in units of period 2 consumption.\textsuperscript{7,8}

\textsuperscript{5}This assumption can be relaxed in two ways. I can allow all the agents to use the storage technology. In addition I can allow the storage technology to operate between period 0 and 1. By allowing these additional opportunities I need to restrict my analysis to a specific range of parameters under which the bankers issue some amount of private safe debt.

\textsuperscript{6}If I assume that discounting happens between period 0 and period 1 then the absence of consumption in period 1 is without loss of generality.

\textsuperscript{7}Index \( d \) denotes demand. It will be useful later to differentiate it from supply of safe debt.

\textsuperscript{8}The utility from holding safe securities is meant to capture the idea that safe securities provide transaction services. Gorton and Pennacchi (1990) and Dang et al. (2012) argue that safe securities are a preferable mean of transaction because they eliminate the potential for adverse selection between transaction parties. Some examples of private safe securities are repurchase agreements, asset-backed commercial paper, covered bonds with short maturity, etc.
Period 0 entrepreneur budget constraint is given by
\[ C_0^P + D_d^P P_D^P + \sum_{s_2^P} B^P(s_2^P) P_B^P(s_2^P) \leq Y + [P_0^P A^P F(I^P) - I^P], \tag{2} \]
where \( P_D \) is the price of safe debt (the return on the safe debt will be denoted \( R_D = 1/P_D \)). \( \sum_{s_2^P} B^P(s_2^P) P_B^P(s_2^P) \) is the value of the risky portfolio, where \( B^P(s_2^P) \) is the repayment in state \( s_2^P \) and \( P_B^P(s_2^P) \) is the price of a security that pays off one unit of consumption good in period 2 in state \( s_2^P \), i.e., this is an Arrow-Debreu security price. I assume that \( I^P \) units of investment immediately produce \( A^P F(I^P) \) units of risky projects which are sold to the bankers. \( P_0^P A^P F(I^P) - I^P \) is the profit from investing in the risky projects. I assume that \( A^P F(I^P) \) is increasing, strictly concave and twice continuously differentiable in \( I^P \).

The budget constraint of the entrepreneur in period 2 is given by
\[ C_2^P(s_2^P) \leq D_d^P + B^P(s_2^P). \tag{3} \]
The entrepreneur takes prices as given and chooses consumption plan \( C_0^P \), \( C_2^P(s_2^P) \), the amount of safe debt \( D_d^P \), the risky portfolio \( \{ B^P(s_2^P) \} \), investment in production of risky projects \( I^P \). I assume that endowment \( Y \) is large enough so that non-negativity constraints on consumption do not bind.

The optimal interior choice of risky portfolio \( \{ B(s_2^P) \} \) implies
\[ P_B^P(G) = \beta p, \]
\[ P_B^P(Bnc) = \beta(1 - p)q, \]
\[ P_B^P(Bc) = \beta(1 - p)(1 - q). \]
This immediately implies that the return on any risky security is given by
\[ R_B = \frac{1}{\beta}. \tag{4} \]
The optimal interior choice of safe debt by the entrepreneur implies
\[ R_D = \frac{1}{\beta + v'(D_d^P)}. \tag{5} \]
It is immediate that \( R_B^P < R_B \) which represents the liquidity premium from holding safe debt.

The optimal choice of investment in the production risky projects gives
\[ P_0^P A^P F'(I^P) = 1. \tag{6} \]
2.2 Outside Investors

The outside investors have endowment of consumption goods in period 1 that they can use to invest in their late technology and to invest in the storage technology. Because the storage technology is riskless the outside investors can issue safe securities backed by the storage technology output. The outside investors can use their securities to buy bankers assets (risky projects). Each outside investor endowment equals $W$. The price of the bankers assets is $Q$ if bad news arrives. The late technology yields $g(x)$ units of consumption good in case of success in period 2, and 0 in case of failure, if $x$ units of consumption good are invested in period $t = 1$. Success and failure, which happens with probabilities $\delta$ and $1 - \delta$ respectively, are common across the outside investors, i.e., this is aggregate uncertainty.\(^9\) I assume that $g(x)$ is increasing, strictly concave, twice continuously differentiable. I also assume that $\delta g'(W) > 1$. This assumption guarantees that the outside investor trades with the bankers only when bad news arrives. In addition it guarantees that it is more profitable to invest in the late technology than into the storage technology. Imposing this assumption limits the number of uninteresting cases to consider.

In period 1 when bad news arrives the outside investor maximizes his revenue in period 2 from investing his endowment which equals his period 2 consumption. The problem in the bad state is

$$
\max_{K_d^P, D_{OI}} qK_d^P + \delta g(W - D_{OI})
$$

s.t. \[ Q^P K_d^P \leq D_{OI}, \]

where $D_{OI}$ is the amount of the endowment that the outside investor invests into the storage technology. The first term represents the expected payoff of the risky projects that the outside investor buys from the bankers, the second term represents the expected payoff of his investments in the late technology. The optimal choice of the amount of the risky projects implies

$$
q = \delta Q^P g'(W - Q^P K_d^P) \quad (7)
$$

\(^9\)This assumption will prevent the outside investors from issuing safe debt in period 1 backed by the proceeds of the late technology. This assumption is crucial to generate downward sloping demand curve for the bankers assets. Alternatively one can assume that $\delta = 1$, i.e., there is no uncertainty, but the outside investors cannot commit to keep their promises.
Because function $g(\cdot)$ is strictly concave, demand $K_d^P$ decreases with $Q^P$. Intuitively, each additional unit of the bankers assets bought by the outside investor has marginal benefit which is equal to $q$ while the marginal cost, $\delta Q^P g'(W - Q^P K_d^P)$, increases with the price and the amount of the risky projects being purchased. Hence, the optimal level of $K_d^P$ decreases with $Q^P$.

Notice also that the elasticity of the outside investor assets demand with respect to price $Q^P$ is greater than 1. This is because the marginal cost of buying the bankers assets is more sensitive to price change than to a change in the quantity bought. To understand the intuition consider the situation in which the outside investor decreases the risky projects demand by 1% as a response to a 1% increase in price $Q^P$. This does not change the marginal value of additional unit of resource invested in the late technology, $g'(W - Q^P K_d^P)$, however, it increases the marginal cost of investing in the risky projects, $Q^P g'(W - Q^P K_d^P)$, which must be constant according to (7). Thus, the outside investor can gain by decreasing demand $K_d$ by more than 1%.

Finally, it is useful to note for later that (7) together with the assumption that $\delta g'(W) > 1$ implies

$$Q^P < q$$

Intuitively, whenever the outside investor chooses to buy bankers risky projects the price of one unit of the risky projects in the bad state is less than its fundamental value $q$.

### 2.3 Bankers

A banker buys risky projects from the entrepreneurs and raises funding by issuing debt to maximize his period 2 profits which equals his consumption. The banker prefers to issue safe debt because it earns liquidity premium. Because there is positive probability for risky projects to become worthless in period 2, safe debt security cannot be made long-term. However, the banker can issue some amount of safe debt by promising potential holders to repay them early (in period 1) with riskless claims on period 2 consumption in the event of the banker’s assets become risky (bad news arrival).

The banker can issue “risky debt” in addition to the safe debt. Such debt promises
repayment of a fixed amount in period 2, and gives the holders of the debt the following rights: (i) a claim to any resources in the hands of the banker in period 2, after safe debt has been repaid, up to the amount promised to be repaid in period 2 (i.e., the claims of the risky debt holders are junior to those of the holders of the collateralized debt); (ii) a right to prevent the banker from undertaking any transactions in period 1 that would reduce the value of the risky debt, except the early repayment on safe debt.

In the event of bad news arrival the banker has to obtain riskless security to repay his safe debt holders. It is assumed that because of the risky debt financing the severe debt overhang problem prevents the banker to raise outside financing (Myers, 1977).\footnote{Because new funding must be junior to existing bankers liabilities new investors may not be willing to provide resources because the additional revenue that the banker gets because of new funding will be paid of to senior investors.} Hence, the only way the banker can obtain riskless securities to fulfill the promise that he gave his safe debt holders is to sell some of his assets to the outside investors.

The banker’s choice variables in period 0 are the quantity $Z^P$ of risky projects to buy, the quantity of safe debt to issue (measured by the face value $D^P_s$), and the quantity of the risky debt to issue (measured by the amount $B^P$ promised to repay in period 2). These three quantities determine the state-contingent payout to the holders of the risky debt, in each of the three possible states in period 2. There is a well-defined asset-pricing kernel (taken as given by an individual banker, because the financial markets are competitive) that determines the market value in period 0 of any type of risky debt that might be issued; this determines the market value of the risky debt, as a function of the three quantities ($Z^P, D^P_s, B^P$) chosen by the given banker.

Let me now express the banker’s state contingent profit. Denote the banker’s state contingent profit which equals to his period 2 consumption level as $\pi^P_B(s^P_2)$. If there is good news and hence no asset collapse in period 2, i.e., $s^P_2 = G$, the banker collects risky projects payoff $Z$ and he pays out the holders of his safe debt $D^P_s$ and risky bond holders $B^P$. Thus, his profit is $\pi^P_B(G) = Z^P - D^P_s - B^P$. If there is bad news and no asset collapse, i.e., state $s^P_2 = Bnc$ then the banker has to sell part of his risky projects, denoted $K^P_s$, to the outside investors in period 1 to make early repayment $D^P_s$ to safe debt holders, the remainder of the risky projects $Z^P - K^P_s$ pays out at $t = 2$ and the banker repays risky debt holders. In this
state his profit $\pi_B^P(B_{nc}) = Q K_s^P - D_s^P + Z^P - K_s^P - \min \{ B^P, Z^P - K_s^P \}$. The last term takes into account the fact that the banker may end up having less resources than the promised repayment on risky debt. Denote the last term as $B_s(B_{nc})$. If there is bad news and assets collapse the banker, as in the previous case, has to sell $K_s^P$ risky projects in period 1 but then he gets nothing because his risky projects yield zero at $t = 2$. To summarize, the banker expected profits are

$$
E\pi_B^P = p[Z^P - D_s^P - B^P] + (1 - p)q(QK_s^P - D_s^P + Z^P - K_s^P - \min \{ B^P, Z - K_s^P \}) + (1 - p)(1 - q)[Q^P K_s^P - D_s^P].
$$

(9)

The value of bankers safe debt outstanding in period 0 is $D_s^P/R_D$. That is the face value of the safe debt is discounted with riskless discount factor $1/R_D$. The value of the risky debt in period 0 equals $\beta p B^P + \beta (1 - p)q \min \{ B^P, Z^P - D_s^P/Q^P \}$. Hence, the banker period 0 budget constraint is

$$
P_0^P Z^P \leq \frac{D_s^P}{R_D} + \beta p B^P + \beta (1 - p)q \min \{ B^P, Z^P - D_s^P/Q^P \}
$$

(10)

In the bad state in period 1 the banker cannot sell more risky projects to the outside investors than he has on his balance sheet

$$
K_s^P \leq Z^P
$$

(11)

For safe debt to be safe the value of banker assets in the bad state have to be no less than the value of safe debt

$$
D_s^P \leq Q^P K_s^P
$$

(12)

Let’s now characterize the banker’s optimality conditions. First, observe that constraint (12) is always binding. It is not optimal for the banker to sell more then it is required to service the safe debt. Thus, constraints (11) and (12) can be rewritten as a single constraint

$$
D_s^P \leq Q^P Z^P.
$$

(13)

which implies that the value of the safe debt cannot be greater than the value of all the assets on banker’s balance sheet in the bad state. The budget constraint (10) always binds
at the optimum. This observation and the fact that (12) always binds allow me to rewrite the profit function

$$E\pi_B^P = [p + (1 - p)q] Z^P - \left( p + \frac{(1 - p)q}{Q^P} \right) D^P_s - R_B \left( P^P_0 Z^P - \frac{D^P_s}{R_D} \right)$$

(14)

The first term represents expected revenue from buying risky projects from entrepreneurs. The second term is the expected payout to the safe debt holders and the last term the expected payout to the risky bond holders.

Given the above analysis the banker maximizes (14) subject to (13) by choosing $Z^P, D^P_s$. The optimal level of the face value of the risky debt can then be determined from (10). Define variable $\theta^P$ such that $\theta^P / Q^P$ is the Lagrange multiplier on constraint (13). The optimal choice of $Z^P$ by the banker leads to

$$[p + (1 - p)q] - R_B P^P_0 + \theta^P = 0.$$

(15)

This condition states that at the optimum the marginal return on additional unit of risky projects equals the marginal cost when it is financed through risky bonds. To see this, consider the following perturbation: the banker increases $Z^P$ by one unit by increasing the issuance of the risky bonds such that period 0 value of the risky debt goes up by 1 unit while keeping $D^P_s$ constant. A unit increase in $Z^P$ delivers additional $[p + (1 - p)q]$ units of period 2 consumption and relaxes the collateral constraint, a unit increase in value of the risky debt increases funding costs by $R_B$. This first-order condition makes clear that when constraint (13) binds, the banker wants to buy more risky projects relative to the case in which the constraint does not bind.

The optimal choice of the face value of the safe debt $D^P_s$ leads to

$$\frac{R_B}{R_D} - \left[ p + \frac{(1 - p)q}{Q^P} \right] - \frac{\theta^P}{Q^P} = 0.$$

(16)

This condition states that at the optimum the banker is indifferent between risky and safe debt financing. To see this, consider the following perturbation: the banker increases the face value of the safe debt by one unit but decreases the period 0 value of the risky debt by $1/R_D$. This variation does not change the size of banker’s balance sheet (it does not change

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11In equilibrium the banker will always choose positive amount of $Z^P$
the banker’s budget constraint in period 0). However, it affects the future repayments. First, it decreases the expected risky debt payments (the first term) which is a benefit for the banker. Second, it increases the expected payments on the safe debt (the second term) which is an additional cost to the banker. Third, it tightens the collateral constraint (13) (the third term) which is a loss to the banker if the constraint is binding. At the optimum, this variation has no affect on the banker’s profits.

After solving for $D^P_s$, $Z^P$ one can determine the optimal face value of the risky debt using period 0 budget constraint (10) as follows

**Lemma 1.** The optimal face value of the risky debt

$$
B^P = \begin{cases} 
\frac{1}{p+(1-p)q} R_B \left( P^0_P Z^P - \frac{D^P}{R_D} \right) 
\quad \text{if } R_B \left( P^0_P Z^P - \frac{D^P}{R_D} \right) < \left( Z^P - \frac{D^P}{Q^P} \right) [p + (1-p)q], \\
\frac{1}{p} \left[ R_B \left( P^0_P Z^P - \frac{D^P}{R_D} \right) - (1-p)q \left( Z^P - \frac{D^P}{Q^P} \right) \right] 
\quad \text{if } R_B \left( P^0_P Z^P - \frac{D^P}{R_D} \right) \geq \left( Z^P - \frac{D^P}{Q^P} \right) [p + (1-p)q]. 
\end{cases}
$$

The first situation corresponds to the case when the banker only defaults on the risky debt if there is a collapse of his assets in period 2. The state contingent payoff of the risky debt is $B^P_s = B^P_s(Bnc) = B^P$ and $B^P_s(Bc) = 0$. The second situation corresponds to the case when the banker defaults on the risky debt in states $s_2 = \{Bnc\}$ and $s_2 = \{Bc\}$. The state contingent payoff of the risky debt is $B^P_s = \bar{B}^P$, $B^P_s(Bnc) = Z^P - D^P_s/Q^P$, $B^P_s(Bc) = 0$.

**Equilibrium.** An equilibrium in a two-country model is a collection of plans \( \{C^P_0, C^P_2(s_2), D^P_d, I^P, D^P_s, B^P(s_2), B^P_s(s_2), Z^P, K^P_s, K^P_d\} \) in the periphery and a collection of plans \( \{C^C_0, C^C_2(s_2), D^C_d, I^C, D^C_s, B^C(s_2), B^C_s(s_2), Z^C, K^C_s, K^C_d\} \) in the center and prices \( \{P^P_0, P^C_0, R_D, P^P_B(s_2), P^C_B(s_2)Q^P, Q^C\} \) such that all the agents solve their problems taking the prices as given and all the markets clear, i.e.,

1. markets for risky projects in period 0 in both countries:
   \[ Z^P = A^P F(I^P) \text{ and } Z^C = A^C F(I^C), \]

2. markets for risky projects in period 1 in both countries:
   \[ K^P_d = K^P_s \text{ and } K^C_d = K^C_s, \]
3. risky debt markets in both countries:

\[ B^P(s_2) = B^P_s(s_2) \] \[ ^{12} \]

4. integrated safe debt market:

\[ D^P_d + D^C_d = D^P_s + D^C_s \] \[ (17) \]

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\(^{12}\text{In general, it would not be enough for the risky debt markets clearing to require that period 0 value of risky bonds supplied to be equal to the amount of resources that the entrepreneurs pay for this value, because this would allow the entrepreneurs to demand a portfolio with different state-contingent payoffs than the supply by the bankers.}\)
3 Equilibrium

In this section I describe the properties of model equilibria. I start by considering an equilibrium in the periphery in which I set \( D_s^C - D_d^C = 0 \), i.e., effectively making the economy closed. This will allow me to consider comparative statics which will be useful when I consider an integrated equilibrium. Then I characterize an integrated equilibrium.

3.1 Closed Economy

All possible equilibria can be divided into those for which: (i) collateral constraint (13) does not bind, i.e., \( \theta^P = 0 \), (ii) collateral constraint (13) binds, i.e., \( \theta^P > 0 \).

If the collateral constraint does not bind optimality condition (16) pins down price \( Q^P \) as a function of the endogenous return on safe debt \( R_D^Q \)

\[
Q^P = \frac{1 - p}{R_B^R_D}q^P.
\]

The banker’s optimality condition (15) with respect to purchases of the risky projects and the entrepreneur optimality condition with respect to his investment in the risky project production determine the level of investments \( I^P \)

\[
[p + (1 - p)q] A^P F'(I^P) = R_B.
\]

These two equations in addition to the outside investor optimality condition and the entrepreneur demand for safe debt (5) fully characterize the solution of the model. It is clear that the solution is unique.

When the collateral constraint binds I can combine both optimality conditions of the bankers with optimal investment condition of the entrepreneurs to get

\[
\left[ \left( \frac{R_B}{R_D^P} - p \right) Q^P + p \right] A^P F'(I^P) = R_B.
\]

The equilibrium level of banker’s risky project purchases \( Z^P \) depends on price \( Q^P \) and the return on safe debt \( R_D \). To understand how \( Q^P \) affects the bankers consider the following intuition: an increase in \( Q^P \) raises the collateral value of \( Z^P \), it becomes more profitable to buy risky projects \( Z^P \). This increases price \( P^P_0 \) which in turn increases the entrepreneur
investment $I^P$ raising equilibrium level of $Z^P$. To understand how $R_D$ affects the banker, consider the following intuition: an increase in $R_D$ makes safe debt a less attractive mean of financing which increases banker’s financing costs decreasing the desire to buy risky projects $Z^P$. As a result price $P_0^P$ falls and the consumer-entrepreneur invests less in the risky technology which makes $Z^P$ fall in equilibrium. Note also that the left-hand side of the above equation is an increasing function of $A^P$.\textsuperscript{13}

Outside investor’s optimality condition (7) together with market clearing condition $K_s^P = K_d^P$ and the fact that the collateral constraint binds ($K_s^P = Z^P$) imply

$$g'(W - Q^PZ^P) = \frac{q}{Q^P},$$

We can solve the last two equations for $Q^P = Q^P(R_D)$ and $Z^P = Z^P(R_D)$ for a given $R_D$. The solution can be graphically represented as in the left panel of Figure 2. The line labeled as $B$ corresponds to equilibrium condition (20). The line labeled as $OI$ corresponds to equilibrium condition (21). The solution determines the supply of safe debt in the economy $D_s^P(R_D) = Q^P(R_D)Z^P(R_D)$.

If I were to plot $B$- and $OI$- curves for a higher value of $R_D$ on the same left panel of Figure 2 this would correspond to the leftward shift in $B$-curve only (see the intuition after equation (20)). As a result $Q^P$ would increase and $Z^P$ would decrease. Although, $Q^P$ and $Z^P$ change in the opposite directions we can still unambiguously determine the affect on

\textsuperscript{13}This is because $\frac{d}{dA^P} F'(I^P) = F'(I^P) - \frac{F''(I^P)}{F'(I^P)} > 0.$
Because the elasticity of the outside investor demand for bankers assets with respect to price $Q^P$ is greater than one, product $Q^P Z^P$ goes down. Thus the supply of safe debt decrease with $R_D$ which is represented on the right panel of Figure 2 with downward sloping $D_s$-curve. The upward sloping $D_d$-curve represents the entrepreneur optimality condition (5). The intersection of these two curves determine the equilibrium level of $D^P$ and $R_D^P$. The equilibrium level of $R_D^P$ determines the position of $B$-curve on the left panel of the figure which in turn determines equilibrium $Q^P$ and $Z^P$.

What happens to the equilibrium when the marginal productivity of investment opportunities, i.e., $A^P$ goes up? Given the price $P_0^P$ the entrepreneurs want to invest more $I^P$ and sell more risky projects $Z^P$ to the bankers. Price $P_0^P$ will fall in equilibrium. The behavior of the rest of the equilibrium variables depends on whether the bankers collateral constraints bind or not. The following lemma summarizes the comparative statics

**Lemma 2.** There always exists $\overline{A}$ such that for any $A^P < \overline{A}$ the collateral constraint binds, $\theta^P > 0$, and for any $A^P \geq \overline{A}$ the collateral constraint does not bind.

- If $A^P < \overline{A}$ then investment in risky projects $I^P$, amount of risky projects $Z^P$, safe debt $D^P$ and return on safe debt $R_D^P$ go up while price of loans $P_0^P$ in period 0 and price $Q^P$ in period 1 go down after an increase in $A^P$. In addition the shadow value of risky projects $\theta$ strictly decreases.

- If $A^P \geq \overline{A}$ then investment in risky projects $I^P$, amount of risky projects $Z^P$ go up, price $P_0^P$ goes down and all the other variables: $D^P$, $Q^P$, $R_D^P$, $\theta$ stay the same after an increase in $A^P$.

The intuition behind this lemma is as follows. When $A^P$ is sufficiently small the amount of risky projects $Z^P$ produced is small in equilibrium. Price $Q^P$ is bounded by $q$ from above. If the collateral constraint does not bind then amount of deposits $D^P$ is smaller than $Z^P Q^P$. When the level of the safe debt is small the return on it $R_D^P$ is small creating strong incentives for the bankers to issue more of safe debt which eventually leads to the collateral constraint being binding. When $A^P$ is sufficiently high this logic is reversed which implies that for high $A^P$ the collateral constraint does not bind. Next, when the collateral constraint binds
an increase in marginal productivity $A^P$ allows the entrepreneur to produce more and the bankers to buy more of the risky projects. To do that the bankers increase the safe debt issuance which leads to a smaller price $Q^P$ in the bad state and higher return on safe debt that consumers have to be compensated with. When the collateral constraint does not bind the amount of the risky projects bought by the bankers is decoupled from the safe debt issuance decision of the bankers because marginal projects don’t serve as collateral in this case.

### 3.2 Integrated Equilibrium

Now I remove the assumption that $D_s^P - D_u^P = 0$ and study the properties of the integrated economy. Specifically, I compare how the equilibrium allocations and prices under integration differ from those under autarky.

The effects of integration will depend on the type of equilibrium in each country before the integration. As I discussed in the previous subsection each country can have one of the two possible types of equilibrium before the integration. Thus there are four possibilities to consider when integrating two countries. However, lemma 2 allows me to remove one possibility immediately. It is not possible for the collateral constraint to be binding in peripheral economy with higher $A^P$ while the collateral constraint is not binding in the center with smaller $A^C$. It would contradict the fact that the shadow value of risky projects decreases with an increase in $A$. This leaves three possibility to consider.

**Case 1** The first case is the situation in which the collateral constraints do not bind in both countries. According to lemma 2 the interest rate on safe debt is the same in both countries before the integration. This implies that opening up the two countries to trade in safe debt does not lead to changes in prices. Hence none of the equilibrium variables change in both countries.

**Case 2** Consider next the case in which the collateral constraints are binding in both countries before the financial integration. This case is graphically illustrated on figure 3 which is an extension of figure 2 to the multicountry case. The left column of plots represents
the determination of equilibrium in the center while the right column presents the equilibrium in the periphery.

Let’s first focus on autarky equilibrium. Plot (c) of figure 3 presents the equilibrium on the market for risky projects in period 1 in the center. The green solid line $OI$-line is the outside investors demand for the risky projects. The red dashed $B(R_D^C(Aut), A^C)$-line is the combination of the bankers and the entrepreneurs optimality conditions and can be thought of as the supply of the risky projects in period 1. The last curve represents the supply given the equilibrium level of the interest rate on safe debt $R_C^D(Aut)$ in case of autarky. Plot (d) similarly presents the equilibrium on the risky projects market in period 1 in the periphery. $B(R_D^P(Aut), A^P)$-line is shifted to the right on plot (d) compared to respective line in plot (c). This is because of the difference in productivity of the risky projects production $A^P > A^C$ which makes the supply of the risky projects higher in the periphery (conditional on the same interest rate). Note that there is an opposing force: the interest rate on safe debt is higher in equilibrium in the periphery which dampens the effect of difference in productivity on the supply of the risky projects. However, the interest rate effect is always smaller (lemma 2). Because the supply of the risky projects in period 1 in the bad state is higher in the periphery for a given value of safe return $R_D$ the supply of safe debt in period 0 by bankers is higher in the periphery compared to the center. This fact is represented on plots (a) and (b) of figure 3: $D_s^P$-curve is shifted relative to $D_s^C$-curve. However, the demand for safe debt is the same in both countries. As a result the interest rate in the periphery is higher than in the center before the integration.

Let’s now consider the effects of integration. Arbitrage forces equalize the returns on safe debt in both countries $R_D^C = R_D^P$ as a result the return in the center rises compared to the autarky case while the return in the periphery falls. This leads to flow of resources from the center to the periphery. One can see on plots (a) and (b) that at interest rate $R_D$ the periphery is a net supplier of safe debt while the center is a net buyer of safe debt. A decline in safe interest rate in the periphery increases the supply of the risky projects in bad state in period 1 as is indicated by the shift of the supply curve from $B(R_D^P(Aut), A^P)$ to $B(R_D, A^P)$ on plot (d). This leads to a decline in price $Q^P$ of the risky projects and a rise in equilibrium amount of the risky projects $Z^P$. The center experiences the reverse effect. An
increase in safe interest rate decreases the supply of the risky projects in period 1 which is indicated by the shift from $B(R^C_{Aut}, A^C)$ to $B(R_D, A^C)$ on plot (c). As a result the price $Q^C$ increases and the amount of the risky projects produced $Z^C$ falls.

**Case 3** The third and final case is a situation in which the banker collateral constraints don’t bind in the periphery ($\theta^P(Aut) = 0$) while they are binding in the center ($\theta^C(Aut) > 0$). According to lemma 2 the safe interest rate is higher in the periphery. Thus financial integration leads to flows of resources from the center to the periphery. As a result of integration the equilibrium may have one of the following three types: (i) the center collateral constraints bind ($\theta^C > 0$) while the periphery collateral constraints don’t bind ($\theta^P = 0$); (ii) collateral constraints are binding in both countries ($\theta^P > 0$, $\theta^C > 0$); (iii) collateral constraints don’t bind ($\theta^P = 0$, $\theta^C = 0$). However, independent of the type of equilibrium of
integrated economy the effect of integration on equilibrium variables is qualitatively similar
to the previous case and can be summarized by the following lemma

**Lemma 3.** The financial integration of two countries with the periphery having higher pro-
ductivity of investment opportunities than the center \((A^P > A^C)\) and collateral constraints
being binding in the center but not in the periphery (before the integration) results in the
following changes

1. in the periphery: increase in investment investment in the risky projects \(I^P\), amount of
   risky projects \(Z^P\), supply of safe debt \(D^P_s\) and decrease in safe interest rate \(R^P_D\), risky
   projects price in period 1 in bad state \(Q^P\) and demand for safe debt \(D^P_d\).

2. in the center: decrease in investment in the risky projects \(I^C\), production of risky
   projects \(Z^C\), supply of safe debt \(D^C_s\) and increase in safe interest rate \(R^C_D\), risky projects
   price in period 1 in bad state \(Q^C\) and demand for safe debt \(D^C_d\).
4 Welfare

In this section, I study the welfare effects of short-term liabilities funding markets integration between the two countries. All the agents are risk-neutral with respect to their consumption, I will evaluate the level of social welfare in each country by adding consumption levels of all the agents in each period.\(^{14}\) The following lemma expresses the social welfare in the periphery.

**Lemma 4.** The expectation in period 0 of the social welfare in the periphery is

\[
\mathbb{E}U^P = Y - I^P + \frac{D_s^P - D_d^P}{R_D} \\
+ v(D_d^P) \\
+ \beta [p + (1 - p)q] A^P F(I^P) \\
+ \beta \left\{ p \left[ g(W) - (D_s^P - D_d^P) \right] + (1 - p) \left[ g(W - D_s^P) + D_s^P - (D_s^P - D_d^P) \right] \right\} \tag{22}
\]

The first line represents the amount of consumption goods available for consumption in period 0: \(Y\) is the initial endowment of the entrepreneurs, \((D_s^P - D_d^P)/R_D\) is the amount of consumption goods that the entrepreneurs in the center pay to peripheral bankers to obtain \(D_s^P - D_d^P\) units of safe debt. The second line represents non-pecuniary utility from holding \(D_d^P\) units of safe debt. The third line represents the expected discounted output of the risky projects. The last line is the expected discounted revenue of the outside investors net of the repayments to the entrepreneurs in the center. Term \([g(W - D_s^P) + D_s^P - (D_s^P - D_d^P)]\) in the last line takes into account that in the bad state \(D_s^P\) units of the outside investors endowment have to be invested in the storage technology rather then in the risky technology.

The welfare in the center has the same form.

As was shown before there are three possible outcomes of the integration. The first case, the collateral constraints do not bind before the integration, is trivial in the sense that the integration has no effect on welfare. This is because, as I showed in the previous section, the returns on safe debt are the same in both countries before the integration. The integration does not lead to a change in safe debt return and thus the equilibrium allocations do not...

\(^{14}\)Alternatively I can assume that all three types of agents belong to the same large household with utility function similar to the entrepreneur utility function.
change. This immediately implies that the social welfare is the same in both countries.
The third case, the collateral constraints bind in the periphery but not in center (before
integration), features similar effects that will be analyzed in the second case. This is why I
think the second case, the collateral constraints bind in both countries before the integration,
is the most interesting to analyze.

The next proposition is summarizes the welfare effects of the integration.

**Proposition 1.** The financial integration of the two countries in which the collateral con-
straints bind before and after the integration has the following effects on welfare:

1. The center always benefits from integration.

2. There always exists $\bar{A}$ such that if $A^P \in (A^C, \bar{A})$ then the periphery loses from the
integration.

Proof. Proof will be added to appendix A.

There are two welfare effects of the financial integration. The first effect is an efficient
capital reallocation. Both countries benefit. The entrepreneurs in the center invest their
resources in debt of the peripheral bankers and receive higher interest rate. Although this
means that they invest less in their local banks which in turn means that bankers buy smaller
amount of risky projects which reduces home entrepreneurs profits. The net effect is positive.
The bankers in the periphery can fund themselves cheaper after the integration. Although
this effect is dampened by smaller holding of safe debt by the entrepreneurs, the net effect
is positive.

The second effect has to do with the change in welfare losses due to the negative pecuniary
externality. The periphery bankers start to issue more safe debt after the integration which
leads to larger losses in the bad state. On the other hand, because the center bankers issue
smaller amount of safe debt after the integration the welfare losses gets smaller in the center.

To formally see the influence of these two effects on, for example, the level of the social
welfare in the periphery I can express the change in the welfare due to the integration as
follows

\[ X^P = U^P(A^P, A^C; \text{integration}) - U^P(A^P, A^C; \text{autarky}) \]

\[ = U^P(A^P, A^C; \text{integration}) - U^P(A^P, A^P; \text{integration}) \]

\[ = - \int_{A^C}^{A^P} \frac{dU^P(A^P, \tilde{A}; \text{integration})}{d\tilde{A}} d\tilde{A}, \]  \hspace{1cm} (23)

\(U^P(A^P, A^C; \text{integration})\) is a social welfare function where the first argument is the marginal productivity of investment opportunities in country \(P\), the second argument is the marginal productivity of investment opportunities in country \(C\), the third argument is a dummy variable that indicates if the two countries are integrated. In the proof of proposition 1 I show that

\[ \frac{dU^P(A^P, \tilde{A}; \text{integration})}{d\tilde{A}} = \beta \theta^P A^P F(I^P) \frac{dQ^P}{d\tilde{A}} - \frac{D^P_s - D^P_d}{R^2_D} \frac{dR_D}{d\tilde{A}} \]

In the above formula the first term is positive because \(dQ^P/d\tilde{A} > 0\): an increase in \(\tilde{A}\) in the center leads to a larger supply of safe debt \(D^C_s\) which decreases the equilibrium issuance of safe debt in the periphery and an increase in price \(Q^P\). When \(\tilde{A} < A^P\) there is an inflow of resources to the periphery, hence, \(D^P_s - D^P_d > 0\). Derivative \(dR_D/d\tilde{A}\) is positive because the increase in the issuance of safe debt in the center leads to higher holdings of safe debt in both countries which increases safe debt return \(R_D\). Thus, the second term of the above formula is positive. We can now see that

\[ X^P = \int_{A^C}^{A^P} \left( -\beta \frac{\theta^P A^P F(I^P)}{Q^P} \frac{dQ^P}{d\tilde{A}} \right) d\tilde{A} + \int_{A^C}^{A^P} \left( -\frac{D^P_s - D^P_d}{R^2_D} \frac{dR_D}{d\tilde{A}} \right) d\tilde{A}, \]

where the first term represents increased losses due to the negative pecuniary externality while the second term represents the efficient capital reallocation effect. In the proof of the proposition I show that \(dQ^P/d\tilde{A}\) and \(dR_D/d\tilde{A}\) are bounded from zero for all \(\tilde{A}\), however, because the second term features the difference \(D^P_s - D^P_d\) the value of this term can be arbitrarily close to zero. This observation implies that the efficient capital reallocation benefits from the integration are smaller than the welfare loss coming from exacerbated pecuniary externality when the difference \(A^P - A^C\) is small.

The same reasoning may be applied the center to show that both effects go in positive direction.
5 Regulation

In this section I study policies that can be used to prevent negative effects due to the negative externality. I show how the role of the policies change with the integration.

A number of recent papers suggested that a system of Pigouvian taxes can be used to bring financial sector incentives closer to social interests.\textsuperscript{15} Stein (2012) and Woodford (2011) argued that interest rate paid on reserves may be used as a macroprudential tool.\textsuperscript{16} I start this section by studying the optimal policy in the presence of just one tool, safe debt taxes. Later I investigate whether additional tools can help improve welfare.

5.1 Safe Debt Taxation

In this section I consider the problem faced by the regulator in country \( P \) who maximizes the social welfare function in his country by choosing safe debt taxes in his country given all the equilibrium conditions and fixed behavior of the regulator in the other country. The regulator rebates the proceeds of taxes to the entrepreneurs in non-distortionary way.

I formally introduce safe debt taxes into the banker period 0 budget constraint as follows

\[
P_0^P Z^P \leq V^P_B + \frac{D^P_s}{R^P_D} (1 - \tau^P).
\]

where \( V^P_B \) is period 0 value of the risky debt. The optimal choice of safe debt funding \( D^P_s \) leads to

\[
\frac{R^P_B}{R^P_D} (1 - \tau^P) - \left( p + \frac{(1-p)q}{Q^P} \right) - \frac{\theta^P}{Q^P} = 0.
\]

\( \tau^P \) reduces the benefit from using cheaper safe debt financing. The optimal choice of the risky projects purchases \( Z^P \) is given as before by (15) because proportional taxes on safe debt do not distort this choice.

The regulator maximizes the peripheral social welfare given by (22), i.e., the sum of all agent types consumption (after I took into account the budget constraints of all the agents) by choosing \( \tau^P \) subject to seventeen equilibrium conditions: bankers optimality condition in

\textsuperscript{15}See, for example, Jeanne and Korinek (2010), Perotti and Suarez (2010).

\textsuperscript{16}The effectiveness of this tool depends on the ability of the government to impose reserve requirements on the issuance of assets that create systematic risk to the stability of financial system. For example, if the government only imposes reserve requirements on traditional banking sector deposits this may not be welfare increasing if deposits are already appropriately insured by the government.
the periphery (24) and (15), constraint on the issuance of safe debt (13), non-negativity of the Lagrange multiplier \( \theta^P \), the complementarity slackness condition, outside investor optimality condition (7), entrepreneurs optimality condition with respect to safe debt holdings (5), entrepreneurs optimality condition with respect to investments into the risky projects, eight similar equations for the center and the safe debt market clearing condition. The proof of lemma 5 states this problem explicitly and derives the first order necessary condition.

**Lemma 5.** At an optimum of the regulator problem the following condition must hold

\[
\frac{dU^P}{d\tau^P} = \beta A^P F(I^P) \frac{dQ^P}{Q^P} \left( \theta^P - \tau^P R_B Q^P \right) \frac{D_s^P - D_d^P}{\bar{R}_D^2} \frac{dR_D}{d\tau^P} = 0
\]  

(25)

**Proof.** Proof will be added to appendix A.

This lemma states that if the regulator chooses to impose taxes \( \tau^P \in (0, 1) \) on its bankers then the above condition should hold for optimal choice of \( \tau^P \).\(^{17}\) The first term of this optimality condition represents two effects. On the one hand, an increase in \( \tau^P \) has a positive effect because it mitigates the negative externality. Observe that this effect is only present when \( \theta^P > 0 \). On the other hand, an increase in \( \tau^P \) makes it more expensive for the bankers to fund themselves which leads to a lower production of the risky projects that yield less consumption in period 2. The second term \( \frac{D_H - D_d}{\bar{R}_D} \frac{dR_D}{d\tau^P} \) reflects the policy maker understanding that his decision changes the international interest rate and thus he can benefit his economy by changing the interest rate. Because the periphery experiences inflow of resources directed to investment in safe debt, a decrease in the interest rate will benefit the bankers in the periphery because they will have to repay less in period 2 to entrepreneurs in the center. The policy maker decreases the interest rate by taxing its bankers more than he would otherwise do without interest rate manipulation motive. The same intuition applies to the policy maker in the center. She wants to regulate his banking sector less relative to no manipulation case to increase the interest rate.

\(^{17}\)Note that this condition is trivially satisfied if the regulator taxed out the safe debt in the periphery from existence. That is, when the tax rate is high enough so that the bankers in the periphery do not issue the safe debt at all. As a result further changes in the tax rate can not alter the equilibrium variables, i.e., \( dQ^P/d\tau^P = 0 \) and \( dR_D/d\tau^P = 0 \).
Example. Figure 4 presents a numerical example which shows how the peripheral social welfare function depends on safe debt taxes. The parameters are chosen such that the periphery is a net issuer of safe debt when there is no regulation in this country which corresponds to the assumption that $A^P > A^C$ and the collateral constraints bind at least in the center. Several observations can be made looking at this example. First, a small positive debt taxes level benefits the periphery. This is a combination of the interest rate manipulation benefit and the externality correction benefit. At $\tau^P \approx 0.25$ the social welfare in the periphery attains its maximum. This point corresponds to the condition in lemma 5. A further increase in the level of taxes makes losses from higher funding costs for banks dominate the benefits from the taxes. At $\tau^P \approx 0.57$ there is the first kink, the collateral constraints stop being binding in the periphery. It becomes costly enough for banks to issue safe debt that they decide to issue less safe debt than the value of their risky projects in the bad state. At $\tau^P \approx 0.64$ there is the second kink. It becomes extremely costly for banks to issue safe debt that they decide not to issue it at all.

What happens when two local regulators determine their policies independently at the
same time? Formally, I solve for a Nash equilibrium of the regulation game. A regulator in each country chooses optimal level of taxes taking the behavior of the other regulator as given. I will only focus on the case in which regulators optimal choices can be described using the first order necessary conditions from lemma 5.

The next proposition shows that the outcomes of optimal choices by regulators can be improved.

**Proposition 2.** A Nash equilibrium can be locally Pareto improved if the regulator in country $P$ decreases taxes and the regulator in country $C$ increases taxes.

**Proof.** Proof will be added to appendix A. □

Note that I did not mention whether the collateral constraints bind or not in this proposition. The proposition holds in both cases.

To explain the effects at work behind the result in the proposition consider the effect of a marginal increase in taxes in the periphery on the center social welfare

$$\frac{dU^C}{d\tau^P} = \beta \frac{F(I^C)}{Q^C} \frac{dQ^C}{d\tau^P} \theta^C - \beta \frac{F(I^C)}{Q^C} \frac{dQ^C}{d\tau^P} \tau^C R_B Q^C R_D \tilde{r}^D C \tilde{g}^C \tilde{r}^C R_D \tilde{D} < 0$$

The first term is a loss due to the exacerbated negative externality after a marginal increase in $\tau^P$. An increase in $\tau^P$ decreases the supply of the safe debt in the periphery which makes the world supply of safe debt smaller leading to an increase in the price (and decrease in returns) of safe debt $1/R_D$. As a result the center bankers incentives to issue safe debt go up. However, this leads to a more severe decline in bankers assets price in the bad state, i.e., $dQ^C/d\tau^P < 0$, which has negative consequences for welfare. The second term shows that the losses associated with distortional taxes in the periphery become smaller. The last term represents the loss for the entrepreneurs in the center who now receive smaller return on their purchases of safe debt from the periphery.

When choosing its optimal level of taxes the regulator in country $P$ does not internalize that he has the above three effects on the center economy. The proposition states that in a Nash equilibrium the net effect of the three effects is negative. In addition, the net effect of marginal increase in taxes in the center on the welfare in the periphery is positive.
Next I ask whether the welfare effects of the integration that were obtained in the previous section survive when the regulators choose safe debt taxation optimally. To derive a sharp result, I will assume that there is a unique Nash equilibrium of the regulation game and there are capital flows from the center to the periphery when the regulation is chosen optimality.

**Proposition 3.** If the regulators choose the level of safe debt taxation optimally before and after integration, there is a unique Nash equilibrium of the regulation game, the collateral constraints bind in both countries before and after integration and the periphery is a net supplier of safe debt then both countries benefit from the integration.

**Proof.** Proof will be added to appendix A.

### 5.2 Safe Debt Taxation and Capital Controls

Because the regulators have incentives to use macroprudential safe debt taxation to manipulate the interest rate it is logical to add another tool to their policy choice sets. One such tool can be capital controls. By capital controls I will mean a proportional tax or subsidy on capital flows. Consider the periphery. If the local interest rate on the safe debt equals $R_{PD}$ then the agents in the center who invest in safe debt in the periphery will receive $(1 - \tau_P^f)R_{PD}$ units of consumption good in period 2 for each unit invested in period 0. Symmetric definition is applied to the center.

In the next lemma I solve for the first order necessary conditions of the peripheral regulator problem assuming that the other regulator is passive. The full problem that the regulator solves defined in the proof of this lemma.

**Lemma 6.** At an optimum of the regulator problem the following condition must hold

$$\tau_P^f = \theta_P^f \overline{\varepsilon}_g \frac{R_P^D}{Q_P^P \bar{R}_B},$$  

$$\tau_P^f = \frac{-\frac{R_P^D}{R_D^P} \frac{D_P^f - D_P^f}{D_P^f - D_P^f}}{\frac{R_P^D}{R_D^P} \frac{D_P^f - D_P^f}{R_D^P}} \frac{dR_P^D}{dD_P^f} (D_P^f - D_P^f) \frac{dD_P^f}{dD_P^f},$$  

$$\tau_P^f = \frac{\frac{R_P^D}{R_D^P} \frac{D_P^f - D_P^f}{D_P^f - D_P^f}}{1 - \frac{\frac{R_P^D}{R_D^P} \frac{D_P^f - D_P^f}{D_P^f - D_P^f}}{\frac{R_P^D}{R_D^P} \frac{D_P^f - D_P^f}{D_P^f - D_P^f}}} \frac{dR_P^D}{dD_P^f} (D_P^f - D_P^f).$$

\(^{18}\)Subscript $f$ distinguishes this tool from the tax on safe debt.
Proof. Proof will be added to appendix A.

It is easy to see that $\tau^P, \tau_f^P \in [0, 1]$. The first condition states that with two tools the regulator does not use safe debt tax to manipulate the interest rate. The second condition states that capital control tax is proportional to the regulators effect on the interest rate in the center, i.e., $dR_C^D(D_s^P - D_d^P)/dD_d^P$ and to the level of cross-border net safe debt $D_s^P - D_d^P$.

Jeanne and Korinek (2010) and Bianchi (2011) argued that negative externality associated with borrowing from abroad give rise to prudential capital controls. In my paper, borrowing from abroad per se does not create inefficiencies. However, the desire to manipulate the international interest rates when regulating local banking sector will lead to the desire to operate two tools, prudential taxes on banking sector and capital controls, instead of just using prudential taxation in the banking sector.
References


Woodford, Michael, “Monetary Policy and Financial Stability,” presentation at NBER Summer Institute, 2011.
A Appendix: Proofs

A.1 Proof of Proposition 1.

I first prove that welfare in the center unambiguously goes up.

Step 1. Let’s denote the social welfare in country $C$ by $U^C = U^C (A^C, A^P; \cdot)$, where the first argument is the marginal productivity of investment opportunities in country $C$, the second argument is the marginal productivity of investment opportunities in country $P$, the third argument is a dummy variable that indicates if the two countries are integrated. We are interested in computing the following difference

$$X^C = U^C (A^C, A^P; \text{integration}) - U^C (A^C, A^P; \text{autarky})$$

I can express the social welfare in country $C$ as follows

$$U^C (A^C, A^P; \text{integration}) = \int_{A^C}^{A^P} \frac{dU^C (A^C, \tilde{A}; \text{integration})}{d\tilde{A}} d\tilde{A} + U^C (A^C, A^P; \text{integration})$$

Observe that if the two countries have the same level of $A$ then there is no gains from integration. Formally, $U^C (A^C, A^P; \text{integration}) = U^C (A^C, A^P; \text{autarky})$. Thus, the variable of interest $X^C$ can be expressed as follows

$$X^C = \int_{A^C}^{A^P} \frac{dU^C (A^C, \tilde{A}; \text{integration})}{d\tilde{A}} d\tilde{A} \tag{A.1}$$

Step 2. I now show that $dU^C (A^C, \tilde{A}; \text{integration}) / d\tilde{A} > 0$. Thus, from (A.1) I will get that $X^C > 0$. In words, country $P$ unambiguously benefits from integration when $A^P > A^C$.

The social welfare function in country $C$ is

$$U^C (A^C, \tilde{A}; \text{integration}) = Y - I^C + \frac{D^C_s - D^C_d}{R_D} + \beta \left\{ [p + (1 - p)q] A^C F(I^C) + p [g(W) - (D^C_s - D^C_d)] ight\} + (1 - p) \left\{ g(W - D^C_s) + D^C_s - (D^C_s - D^C_d) \right\} + v(D^C_d).$$

Rearranging last equation I get

$$U^C (A^C, \tilde{A}; \text{integration}) = Y - I^C + (D^C_s - D^C_d) \left( \frac{1}{R_D} - \beta \right) + \beta \left\{ [p + (1 - p)q] A^C F(I^C) + pg(W) ight\} + (1 - p) \left\{ g(W - D^C_s) + D^C_s \right\} + v(D^C_d)$$
Now, I take the full derivative of the above expression with respect to \( \tilde{A} \)

\[
\frac{dU^C(A^C, \tilde{A}; \text{integration})}{d\tilde{A}} = -\frac{dI^C}{d\tilde{A}} + \left( \frac{dD^C_s}{d\tilde{A}} - \frac{dD^C_d}{d\tilde{A}} \right) \left( \frac{1}{R_D} - \beta \right) - \frac{D^C_s - D^C_d}{R_D^2} \frac{dR_D}{d\tilde{A}} \\
+ \beta \left\{ [p + (1 - p)q]A^C F'(I^C) \frac{dI^C}{d\tilde{A}} \\
+ (1 - p) \left[ 1 - g'(W - D^C_d) \right] \frac{dD^C_d}{d\tilde{A}} \right\} + v'(D^C_d) \frac{dD^C_d}{d\tilde{A}}
\]

Rearranging I get

\[
\frac{dU^C(A^C, \tilde{A}; \text{integration})}{d\tilde{A}} = \left\{ \beta [p + (1 - p)q] A^C F'(I^C) - 1 \right\} \frac{dI^C}{d\tilde{A}} \\
- \frac{D^C_s - D^C_d}{R_D^2} \frac{dR_D}{d\tilde{A}} \\
+ \left[ -\frac{1}{R_D} + \beta + v'(D^C_d) \right] \frac{dD^C_d}{d\tilde{A}} \\
+ \left\{ \frac{1}{R_D} - \beta + \beta (1 - p) \left[ 1 - g'(W - D^C_d) \right] \right\} \frac{dD^C_d}{d\tilde{A}} \tag{A.2}
\]

If \( \tilde{A} > A^C \) which is the case of interest then \( D^C_s < D^C_d \). It is also true that \( dQ^C/d\tilde{A} > 0 \), \( dI^C/d\tilde{A} < 0 \), \( dD^C_s/d\tilde{A} > 0 \) and \( dD^C_d/d\tilde{A} < 0 \). Before I simplify the above formula it useful to interpret all the terms to understand the effects of the marginal increase in \( \tilde{A} \). Consider the first line of (A.2). An increase in \( \tilde{A} \) leads to decrease in investment in country \( C \). This has two effects. First, the expected revenue of the bankers projects goes down which is represented by the first term in curly brackets. Second, the entrepreneurs in country \( C \) have now more endowment in period \( t = 0 \) to consume which is represented by the second term. When the collateral constraint binds the net effect of these two effects is positive. This is because in equilibrium the marginal product of investment is smaller than the marginal financing cost of investment. This is because a unit of risky projects has additional benefit of increasing the amount of collateral for the bankers. See the first line of (A.3). Consider the second line. Because \( D^C_s < D^C_d \) country \( C \) is net lender of resources to country \( P \) in period \( t = 0 \). An increase in \( \tilde{A} \) leads to an increase in \( R_D \) which means that the entrepreneurs in country \( C \) have to lend less to banks in country \( H \) to get 1 unit return in the future. This is a benefit. Consider the third line. An increase in \( \tilde{A} \) increases demand for riskless securities in country \( C \). This has a cost \(-1/R_D \) because the entrepreneurs give part of their endowment to buy the securities. It has two benefits: (i) the entrepreneurs get a unit of consumption at period \( t = 2 \) but discount this at rate \( \beta \); (ii) the entrepreneurs benefit from using more riskless securities in their transactions which is captured by \( v'(D^C_d) \). Observe that in equilibrium these two benefits exactly equal to the cost. This follows from the entrepreneurs optimality condition (5). Thus, the third line equals zero. Consider the

\[19\]I don’t show this formally here but it can be simply obtained by differentiating the integrated market equilibrium conditions with respect to \( A^P \).
fourth line. An increase in $\tilde{A}$ leads to a decrease in riskless securities issuance $D^C_s$. A unit decrease in $D^C_s$ have several effects on the welfare in country $C$. First, it decreases the amount of resources that the bankers in country $C$ use to invest by $1/R_D$. Second, it decreases the amount of consumption goods that has to be paid out by the bankers in period $t = 2$ which adds $\beta$ to the welfare. Third, it decreases the reallocation of resources from the outside investors to the bankers in the bad state which has the following effect on welfare $-\beta(1 - p) [1 - g'(W - D^C_s)]$. That’s, the output of projects that are run by the outside investors increases by $g'(W - D^C_s)$ in the bad state while the bankers get 1 unit less of consumption goods. The bankers optimality condition with respect to riskless securities (16) can be used to simplify the fourth line. See the third line of (A.3).

$$\frac{dU^C}{d\tilde{A}}\left(A^C, \tilde{A}; \text{integration}\right) = -\beta \theta^C A^C F'(I^C) \frac{dI^C}{d\tilde{A}} - \frac{D^C_s - D^C_d}{R^2_D} \frac{dR_D}{d\tilde{A}} + \frac{\beta \theta^C dD^C_s}{Q^C} \frac{d\tilde{A}}{d\tilde{A}} \tag{A.3}$$

Next, I use that $\theta^C[D^C_s - Q^C A^C F(I^C)] = 0$ to combine the first and third line of the equation above to get

$$\frac{dU^C}{d\tilde{A}}\left(A^C, \tilde{A}; \text{integration}\right) = \beta \theta^C A^C F(I^C) \frac{dQ^C}{d\tilde{A}} - \frac{D^C_s - D^C_d}{R^2_D} \frac{dR_D}{d\tilde{A}}$$

The first term in the above formula is positive while the second one is negative which makes the overall expression positive. This completes the proof that the center benefits from the integration.

I consider the periphery next. The proof of this result uses the same idea as the proof of the previous result.

Step 1. Let’s denote the social welfare in country $P$ by $U^P = U^p (A^P, A^C, \cdot)$, where the first argument is the marginal productivity of investment opportunities in country $P$, the second argument is the marginal productivity of investment opportunities country $C$, the third argument is a dummy variable that indicates if the two countries are integrated. We are interested in computing the following difference

$$X^P = U^P (A^P, A^C, \cdot; \text{integration}) - U^P (A^P, A^C, \cdot; \text{autarky}) = - \int_{A^C}^{A^P} dU^P \left(A^P, \tilde{A}; \text{integration}\right) d\tilde{A}. \tag{A.4}$$

Step 2. Repeating calculations in Step 2 of the previous proof I get

$$\frac{dU^P}{d\tilde{A}}\left(A^P, \tilde{A}; \text{integration}\right) = \beta \theta^P A^P F(I^P) \frac{dQ^P}{d\tilde{A}} - \frac{D^P_s - D^P_d}{R^2_D} \frac{dR_D}{d\tilde{A}}$$
Because $dQ^P/d\tilde{A} > 0$ the first term of this expression is positive. Because for $\tilde{A} < A^P$ it is true that $D_s^P > D_d^P$ and $dR_P/d\tilde{A} > 0$ the second term is negative (taking into account the sign in front of this term). If I plug the above expression into (A.4) and take into account the negative sign in front of the integral the effects of the two terms in the above formula reverses. However, because the two terms have the opposite effects the net effect can be either negative or positive.

**Step 3.** Consider a case in which the marginal productivity of investment opportunities in the two countries are as follows $(A^P, A^C) = (A + \epsilon, A)$ where $A$ is some positive number and $\epsilon$ is small and positive number. In this case I can write

$$X^P \approx -\frac{dU^P (A, A; \text{integration})}{d\tilde{A}} \cdot \epsilon$$

$$= -\frac{\theta^P A^P F(I^P) dQ^P}{Q^P} \frac{dQ^P}{d\tilde{A}} \bigg|_{(A^P, A^C) = (W, W)} \cdot \epsilon < 0$$

By continuity there exists $\tilde{A} > A^C$ such that for all $A^P \in (A^C, \tilde{A})$ it is true that $X^P < 0$. QED.

**A.2 Proof of lemma 5.**

$$\max_{x^P} Y - I^P + \frac{D_s^P - D_d^P}{R_d} + \nu(D_d^P) + \beta[p + (1 - p)q]A^P F(I^P)$$

$$+ \beta \left\{ p \left[ g(W) - (D_s^P - D_d^P) \right] + (1 - p) \left[ g(W - D_s^P) + D_s^P - (D_s^P - D_d^P) \right] \right\},$$
subject to the following system of equilibrium conditions

\[
\frac{R_B}{R_D} (1 - \tau^P) - \left( p + \frac{(1 - p)q}{Q^P} \right) - \frac{\theta^P}{Q^P} = 0,
\]

\[
[p + (1 - p)q] A^P F'(I^P) - R_B + \theta^P A^P F'(I^P) = 0,
\]

\[
g'(W - D_s^P) = \frac{q}{Q^P},
\]

\[
D_s^P \leq Q^P A^P F(I^P), \theta^P \geq 0,
\]

\[
\theta^P (D_s^P - Q^P A^P F(I^P)) = 0,
\]

\[
R_D = \frac{1}{\beta + v'(D_d^C)},
\]

\[
\frac{R_B}{R_D} (1 - \tau^C) - \left( p + \frac{(1 - p)q}{Q^C} \right) - \frac{\theta^C}{Q^C} = 0,
\]

\[
[p + (1 - p)q] A^C F'(I^C) - R_B + \theta^C A^C F'(I^C) = 0,
\]

\[
g'(W - D_C^C) = \frac{q}{Q^C},
\]

\[
D_s^C \leq Q^C A^C F(I^C), \theta^C \geq 0,
\]

\[
\theta^C (D_s^C - Q^C A^C F(I^C)) = 0,
\]

\[
R_D = \frac{1}{\beta + v'(D_d^C)},
\]

\[
D_d^P + D_d^C = D_s^P + D_s^C.
\]

This system of eleven equations and four constraints uniquely defines a mapping from \( \tau^P \) to eleven variables \( I^P(\tau^P), I^C(\tau^P), Q^P(\tau^P), Q^C(\tau^P), D_s^P(\tau^P), D_s^C(\tau^P), D_d^P(\tau^P), D_d^C(\tau^P), R_D, \theta^P(\tau^P), \theta^C(\tau^P) \). The uniqueness comes from the analysis similar to the one presented in section 3. The mapping is differentiable for any \( \tau^P \in (0, 1) \) except for those values of \( \tau^P \) for which the collateral constraints change from being binding to not being binding. Given an implicit mapping of \( \tau^P \) to all the equilibrium variables I can write the first order necessary condition by differentiating the welfare function with respect to \( \tau^P \)

\[
\frac{dU^P}{d\tau^P} = -\frac{dI^P}{d\tau^P} + \left( \frac{dD_s^P}{d\tau^P} - \frac{dD_d^P}{d\tau^P} \right) \left( \frac{1}{R_D} - \beta \right) - \frac{D_s^P - D_d^C}{R_D} \frac{dR_D}{d\tau^P} + v'(D_d^C) \frac{dD_d^P}{d\tau^P}
\]

\[
+ \beta \left\{ [p + (1 - p)q] A^P F'(I^P) \frac{dI^P}{d\tau^P} + (1 - p) \left[ 1 - g'(W - D_s^P) \right] \frac{dD_s^P}{d\tau^P} \right\} = 0
\]
Rearranging I get

\[
\frac{dU^P}{d\tau^P} = \{ \beta[p + (1-p)q]A^P F'(I^P) - 1 \} \frac{dI^P}{d\tau^P} - \frac{D^P_s - D^P_d}{R^2_D} dR_D \\
+ \left[ -\frac{1}{R_D} + \beta + v'(D^P_d) \right] \frac{dD^P_d}{d\tau^P} \\
+ \left\{ \frac{1}{R_D} - \beta + \beta(1-p) \left[ 1 - g'(W - D^P_s) \right] \right\} \frac{dD^P_s}{d\tau^P} = 0
\]

After plugging in the bankers and the entrepreneurs optimality conditions the regulator first condition can be written as follows

\[
\frac{dU^P}{d\tau^P} = -\beta \theta^P \left( A^P F'(I^P) \frac{dI^P}{d\tau^P} - \frac{1}{Q^P} \frac{dQ^P}{d\tau^P} \right) - \frac{D^P_s - D^P_d}{R^2_D} dR_D + \frac{R_B}{R_D} \tau^P \frac{dD^P_s}{d\tau^P} \\
= \beta \frac{A^P F'(I^P)}{Q^P} \frac{dQ^P}{d\tau^P} - \frac{D^P_s - D^P_d}{R^2_D} dR_D + \frac{R_B}{R_D} \tau^P \frac{dD^P_s}{d\tau^P} \\
= \beta \frac{A^P F'(I^P)}{Q^P} \frac{dQ^P}{d\tau^P} \left( \theta^P - \tau^P \frac{R_B Q^P}{R_D \hat{\epsilon}_g} \right) - \frac{D^P_s - D^P_d}{R^2_D} dR_D = 0,
\]

where the second line uses the observation that the derivative of \( \theta^P[D^P_s - Q^P A^P F(I^P)] = 0 \) with respect to \( \tau^P \) equals

\[
\theta^P \left[ \frac{dD^P_s}{d\tau^P} - \frac{dQ^P}{d\tau^P} A^P F(I^P) - Q^P A^P F'(I^P) \frac{dI^P}{d\tau^P} \right] = 0,
\]

and the third line uses

\[
\frac{1}{D^P_s} \frac{dD^P_s}{d\tau^P} = -\frac{1}{\hat{\epsilon}_g Q^P} \frac{dQ^P}{d\tau^P}.
\]

**A.3 Proof of Proposition 2.**

From lemma 5 the optimal level of taxes in country \( P \) satisfies

\[
\beta \frac{A^P F(I^P)}{Q^P} \frac{dQ^P}{d\tau^P} \left( \theta^P - \tau^P \frac{R_B Q^P}{R_D \hat{\epsilon}_g} \right) - \frac{D^P_s - D^P_d}{R^2_D} dR_D = 0. \tag{A.5}
\]

Similar equation holds for country \( C \)

\[
\beta \frac{A^C F(I^C)}{Q^C} \frac{dQ^C}{d\tau^C} \left( \theta^C - \tau^C \frac{R_B Q^C}{R_D \hat{\epsilon}_g} \right) - \frac{D^C_s - D^C_d}{R^2_D} dR_D = 0. \tag{A.6}
\]

Let’s denote a solution to these equations, a Nash equilibrium, as \((\hat{\tau}^C, \hat{\tau}^P)\).
Next I consider the effect of the marginal change in $\tau^P$ on the social welfare function in country $C$ evaluated at a Nash equilibrium $(\hat{\tau}^C, \hat{\tau}^P)$. Repeating the algebra from lemma 5 I obtain

$$\frac{dU^C}{d\tau^P} = \beta \frac{F(I^C)}{Q^C} \frac{dQ^C}{d\tau^P} \left( \theta^C - \tau^C \frac{R_B Q^C}{R_D g} \right) - \frac{D^C_s - D^C_d}{R^2_D} \frac{dR_D}{d\tau^P}$$  \tag{A.7}$$

This formula is key to understanding the coordination failure result. A marginal increase in the taxes in the periphery has three effects: (i) it makes the welfare losses from the externality bigger (first term in the brackets); (ii) it decreases country $C$ tax-induced bank funding costs; (iii) it decreases interest rate which makes entrepreneurs gain from investing in peripheral safe debt smaller.

Taking into account the optimality condition (A.6) I can rewrite the previous equation as follows

$$\left. \frac{dU^C}{d\tau^P} \right|_{(\hat{\tau}^C, \hat{\tau}^P)} = \frac{D^C_s - D^C_d}{R^2_D} \left[ \frac{dR_D}{d\tau^C} \frac{dQ^C}{d\tau^P} - \frac{dR_D}{d\tau^P} \right] < 0 \quad \tag{A.8}$$

This expression is negative because (i) by the assumption of the proposition the center is a net buyer of safe debt $D^C_s - D^C_d < 0$, (ii) an increase in the tax level in the periphery decreases the level of safe debt in the world making it more expensive which implies $dR_D/d\tau^P < 0$, (iii) analogously $dR_D/d\tau^C < 0$, (iv) an increase in taxes $\tau^P$ increases the issuance of safe debt in the center (because the return on safe debt falls) which implies a more severe fire-sale price decline (relative to fundamental value of the risky projects $q$) $dQ^C/d\tau^P < 0$, however, at the same time the fire-sale price in the periphery rises $dQ^F/d\tau^F > 0$. Negative sign in (A.8) implies that there is a gain for agents in country $C$ from a marginal decrease in taxes in country $P$.

I can analogously compute the marginal effect of change in $\tau^C$ on $U^P$.

$$\left. \frac{dU^P}{d\tau^C} \right|_{(\hat{\tau}^C, \hat{\tau}^P)} = \frac{D^P_s - D^P_d}{R^2_D} \left[ \frac{dR_D}{d\tau^C} \frac{dQ^C}{d\tau^P} - \frac{dR_D}{d\tau^C} \right] > 0$$

This expression is positive because $D^P_s - D^P_d > 0$, $dR_D/d\tau^C < 0$, $dR_D/d\tau^P < 0$, $dQ^P/d\tau^C < 0$ and $dQ^P/d\tau^C > 0$. Positive sign of this expression implies that there is gain for agents in country $P$ from a marginal increase in taxes in country $C$.

Thus, the following perturbation $d(\tau^C, \tau^P) = (\Delta^C, \Delta^P)$, where $\Delta^C$ and $\Delta^P$ are small and positive numbers, increases the social welfare functions in both countries. Hence, if the policy makers could coordinate on their decisions they could achieve higher welfare than in a Nash equilibrium by decreasing taxes in country $P$ and increasing taxes in country $C$. QED.
A.4 Proof of Proposition 3.

Consider the periphery. The social welfare function change after integration equals

\[ X^P = U^P (A^P, A^C; \text{integration}) - U^P (A^P, A^C; \text{autarky}) \]
\[ = U^P (A^P, A^C; \text{integration}) - U^P (A^P, A^P; \text{autarky}) \]
\[ = - \int_{AC}^{A^P} \frac{dU^P (A^P, \tilde{A}; \text{integration})}{d\tilde{A}} d\tilde{A} \]

The derivative of the social welfare function with respect to the marginal productivity of investment opportunities \( \tilde{A} \) in the center is

\[ \frac{dU^P (A^P, \tilde{A}; \text{integration})}{d\tilde{A}} = \beta \frac{F(I^P)}{Q^P} \frac{dQ^P}{d\tilde{A}} \left( \theta^P - \tau^P \frac{R^P}{\tau^P} \right) - \frac{D^P_s - D^P_d}{R^2_D} \frac{dR^P_D}{d\tilde{A}} \]

Using optimality condition of the regulator in country \( P \) from lemma 5 I obtain

\[ \frac{dU^P (A^P, \tilde{A}; \text{integration})}{d\tilde{A}} = \frac{D^P_s - D^P_d}{R^2_D} \left[ \frac{dR^P_D}{d\tau^P} \frac{dQ^P}{d\tau^P} - \frac{dR^P_D}{d\tilde{A}} \right] \]

where \( D^P_s - D^P_d > 0, dR^P_D/d\tau^P < 0, dQ^P/d\tilde{A} > 0, dQ^P/d\tau^P > 0, dR^P_D/d\tilde{A} > 0. \) This implies that \( dU^P (A^P, \tilde{A}; \text{integration})/d\tilde{A} < 0. \) Thus, \( X^P > 0. \)

Consider the center. The social welfare function change equals

\[ X^C = \int_{AC}^{A^P} \frac{dU^C (A^C, \tilde{A}; \text{integration})}{d\tilde{A}} d\tilde{A} \]

where

\[ \frac{dU^C (A^C, \tilde{A}; \text{integration})}{d\tilde{A}} = \frac{D^C_s - D^C_d}{R^2_D} \left[ \frac{dR^C_D}{d\tau^C} \frac{dQ^C}{d\tau^C} - \frac{dR^C_D}{d\tilde{A}} \right] > 0 \]

Hence, \( X^C > 0. \) QED.

A.5 Proof of lemma 6.

I start by defining the problem of the regulator in the periphery

\[ \max_{\tau^P, \tau^C} Y - I^P + \frac{D^P_s - D^P_d}{R^P_D} + v(D^P_d) + \beta[p + (1-p)q]A^P F(I^P) \]
\[ + \beta \{ p [g(W) - (D^P_s - D^P_d)] + (1-p) [g(W - D^P_s) + D^P_s - (D^P_s - D^P_d)] \} , \]
subject to the following system of equilibrium conditions

\[
\frac{R_B}{R_D} (1 - \tau^P) - \left( p + \frac{(1-p)q}{Q^P} \right) - \frac{\theta^P}{Q^P} = 0, \quad (A.9)
\]

\[
[p + (1-p)q] A^P F'(I^P) - R_B + \theta^P A^P F'(I^P) = 0, \quad (A.10)
\]

\[
g'(W - D^P) = \frac{q}{Q^P},
\]

\[
D_s^P \leq Q^P A^P F(I^P), \theta^P \geq 0,
\]

\[
\theta^P (D_s^P - Q^P A^P F(I^P)) = 0,
\]

\[
R_D^P = \frac{1}{\beta + \nu'(D^P_d)},
\]

\[
\frac{R_B}{R_D} (1 - \tau^C) - \left( p + \frac{(1-p)q}{Q^C} \right) - \frac{\theta^C}{Q^C} = 0, \quad (A.11)
\]

\[
[p + (1-p)q] A^C F'(I^C) - R_B + \theta^C A^C F'(I^C) = 0,
\]

\[
g'(W - D^C) = \frac{q}{Q^C},
\]

\[
D_s^C \leq Q^C A^C F(I^C), \theta^C \geq 0,
\]

\[
\theta^C (D_s^C - Q^C A^C F(I^C)) = 0,
\]

\[
R_D^C = \frac{1}{\beta + \nu'(D^C_d)},
\]

\[
D_d^P + D_d^C = D_s^P + D_s^C,
\]

\[
R_D = (1 - \tau_{fD}^P) R_D^P.
\]

Instead of solving this problem I propose to solve less constrained problem and then show that the solution satisfies omitted constraints. The less constrained problem looks as follows

\[
\max_{D^P_d, D^P_s, I^P} Y - I^P + \frac{D^P_s - D^P_d}{R_D^P} + \nu(D^P_d) + \beta [p + (1 - p)q] A^P F(I^P) + \beta \left\{ p \left[ g(W) - (D^P_s - D^P_d) \right] + (1 - p) \left[ g(W - D^P_s) + D^P_s - (D^P_s - D^P_d) \right] \right\},
\]

subject to the following subset of the equilibrium conditions

\[
g'(W - D^P) = \frac{q}{Q^P},
\]

\[
D_s^P \leq Q^P A^P F(I^P),
\]

\[
\frac{R_B}{R_D^C} (1 - \tau^C) - \left( p + \frac{(1-p)q}{Q^C} \right) - \frac{\theta^C}{Q^C} = 0,
\]

\[
[p + (1-p)q] A^C F'(I^C) - R_B + \theta^C A^C F'(I^C) = 0,
\]

\[
g'(W - D^C) = \frac{q}{Q^C},
\]
Observe that the regulator can directly affect the first two conditions. All the remaining conditions are affected through changes in $D^s - D^d$ (because of the safe debt market clearing condition). These remaining conditions determine the equilibrium in the center conditional on $D^s - D^d$. Because only one variable from the center the peripheral welfare function and the first to constraints the only thing we need to know about the remaining conditions is how $R^C_R$ depends on $D^s - D^d$. Hence, the problem can be written as follows

$$\max_{D^s, D^d, I^P} Y - I^P + \frac{D^P_s - D^P_d}{R^P_R(D^s - D^d)} + v(D^P_s) + \beta[p + (1 - p)q]A^P F(I^P)$$

$$+ \beta [g(W - (D^P_s - D^P_d)) + (1 - p) [g(W - D^P_s) + D^P_s - (D^P_s - D^d)]] ,$$

subject to

$$g'(W - D^P_s) = \frac{q}{Q^P},$$

$$D^P_s \leq Q^PA^P F(I^P).$$

The optimal choice of $I^P$ leads to

$$[p + (1 - p)q] A^P F'(I^P) - R_B + \theta^P A^P F'(I^P) = 0 \quad (A.12)$$

The optimal choice of $D^P_s$ leads to

$$\frac{R_B}{R^P_R} - \left( \frac{p}{Q^P} \right) - \frac{\theta^P}{Q^P} = -\theta^P D^P_s \frac{g''(W - D^P_s)}{q} + \frac{D^P_s - D^P_d}{R^P_P} \frac{R_B dR^C_R(D^P_s - D^P_d)}{dD^P_s} . \quad (A.13)$$

The optimal choice of $D^P_d$ leads to

$$\frac{1}{R^C_R} = \beta + v'(D^P_d) - \frac{D^P_s - D^P_d}{R^P_P} \frac{R_B dR^C_R(D^P_s - D^P_d)}{dD^P_s} . \quad (A.14)$$

Note that

$$\frac{dR^C_R(D^P_s - D^P_d)}{dD^P_s} + \frac{dR^C_R(D^P_s - D^P_d)}{dD^P_d} = 0.$$

Finally, the complementarity slackness conditions should be satisfied

$$\theta^P [D^P_s - Q^PA^P F(I^P)] = 0.$$

I now show that the optimality conditions of the less constrained problem satisfy the condition omitted from the more constrained problem. Pick $\tau^P_f$ such that

$$\tau^P_f = \frac{-\frac{R^P_P D^P_f D^P_d}{R^P_R} dR^C_R(D^P_f - D^P_d)}{1 - \frac{R^P_P D^P_f - D^P_d}{R^P_R} dR^C_R(D^P_f - D^P_d)} . \quad (A.15)$$
This \( \tau_f^P \) together with (A.14) implies (A.11). Next, (A.15) together with (A.13) and the following choice of \( \tau^P \)

\[
\tau^P = \theta^p \tilde{\epsilon}_g \frac{R^p_D}{Q^P R_B},
\]

(A.16)
imply (A.9). Next, (A.12) implies (A.10). Thus, I showed that the less constrained problem optimum is feasible under the more constrained problem optimum.