Institut d'Études Politiques de Paris

Master's thesis

Intergenerational transfer without commitment: a macroeconomic framework

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2017

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Abstract

Can a heterogeneous agent model replicate the empirical wealth distribution when individuals are altruistic and strategic? To answer this question, this paper constructs a heterogeneous agent model with a life cycle and overlapping generations. The main unit of interest is the family, constituted of two generations with their own preferences. The older generation values the utility of its descendant. The decision-making process is thought as a two-stage non-cooperative game, and both players do not have access to commitment devices. In this model, intergenerational transfers occur and are driven by two main motives. First, transfers are used by parents to redistribute wealth across the members of the family. Second, they are used as a risk sharing tool to help the young member of the family smooth its consumption when faced by adverse shocks. Both motives eventually push savings upwards. We test the predictions of the model, and find that its qualitative properties are in line with the empirical evidence on life cycle consumption, savings and intergenerational transfer. The model is also able to generate a wealth distribution with a relatively thick right tail.

Keywords: Intergenerational transfer; family insurance; Markov perfect equilibrium; wealth distribution.

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I would like to thank my supervisor, Nicolas Coeurdacier, for his precious comments and advice throughout this project. I would also like to thank Florian Oswald who initiated my love for Julia, Théo Durandard for his mathematical insights, Pia Testa for her wonderful English, and Oliver Cassagneau-Francis and Fabio Palacio for lively discussions. I wish to thank Nadège Capet, who introduced me to economics in the very first place. Finally, I am extremely grateful to my family for their support during these last five years.

1. Introduction

For the last twenty years, quantitative macroeconomics has been relying on heterogeneous agent models to replicate the cross-sectional heterogeneity empirically observed. To understand the determinants of economic inequality, a key element of interest has in particular been the wealth distribution, with its well-known thick upper right tail. However, most of the literature has not succeeded in reproducing the high concentration of wealth at the top of the distribution.[1] For instance, 29% of the total wealth was held by the one percent richest households in 1989 in the United States, 3.2% in Aiyagari (1994).

In heterogeneous agent models, incomplete markets force individuals to save more in order to self-insure against future negative shocks. Yet, as savings are primarily driven by this precautionary motive, once a buffer stock of wealth is constituted, agents stop accumulating wealth. This finally prevents a high concentration of wealth at the top of the distribution. On the contrary, microeconomic and macroeconomic empirical evidence suggest that wealth accumulation is driven by two primary forces. First, wealthy individuals keep saving a large share of their wealth (Lillard and Karoly, 1997; Carroll, 1998; Dynan et al., 2004). Second, intergenerational transmission of wealth accounts for a large part of capital formation, both at the household level (Gale and Scholz, 1994) and at the aggregate (Kotlikoff and Summers, 1981).

In this paper, we ask whether including altruism and strategic interactions can produce more realistic saving decisions, resulting in a wealth distribution that is more skewed. We extend the heterogeneous agent model by adding a deterministic life cycle, overlapping-generations (OLG) and imperfect one-sided altruism. In this economy, the main unit of interest is the family, consisting of two members: the old and the new generation. The former is assumed to value the latter's utility, while the converse is not true. Within the family, the two generations are characterized by individual preferences, and are modeled as separate entities. As such, we depart from the unitary family model, in which a unique utility function is specified for the entire household. Instead, in every period, the decision making process is modeled as a sequential two-stage game. In the first stage, parents decide how much to consume, save and transfer to their descendant. In the second, the child optimally chooses its consumption and savings. The stage game is then repeated throughout the life cycle, until the older generation dies, at which point the young become parents, and the game continues with new players. This no-commitment assumption is in line with the empirical literature on risk sharing at the family level. Hayashi et al. (1996) finds for instance that risk-sharing is imperfect within the family, while Mazzocco (2007) strongly rejects the hypothesis that household members can commit to future allocations of resources.

In this setup, one-sided altruism gives rise to intergenerational transfer for redistributive and risk-sharing motives. As an illustration of the former, we

[1] See for instance De Nardi (2015) for an excellent summary of this literature.

study a simplified, two-period version of the general model, in which income is deterministic. There, intervivos transfers are solely used to achieve a more equal allocation of resources across the members of the family. Heterogeneity within the family is thus key to explaining these transfers. Transferring a positive amount is however costly for the parents, and requires a higher initial wealth, pushing their savings upwards. Moreover, the young, helped by their parent, uses a fraction of the transfer to increase their savings as well. This is the redistributive effect of transfer on savings.

To add the risk-sharing motive into the picture, we then compute the steady-state recursive partial equilibrium of the dynamic game. Compared to the usual heterogeneous agent models, the parent's utility is subject to the fluctuation of its income and that of its child. Being risk averse, the parent saves a higher proportion of its wealth to self-insure against adverse shocks. This helps maintaining high saving rates throughout the life cycle, even when the parent's uncertainty on its own income is reduced. This effect percolates backward, and raises the young's savings in apprehension of this higher uncertainty. Additionally, it diminishes the impact of strategic interactions on the young's choices. While the latter could indeed decide to over-consume to extract more weatlh from its parent, it does not in anticipation of its future needs.

The predictions of the model are eventually tested against the relevant empirical literature. We first confirm that the generated consumption and savings follow hump-shaped profiles over the life cycle. Moreover, the predicted timing of intervivos transfers is empirically relevant. Specifically, in the middle of the life cycle, intergenerational transfers are more likely when the young is facing adverse shocks, while they are less responsive to the child's resources at the end of the parent's life. Besides, comparing the outcomes of the model with and without altruism, we find that intervivos transfers are an effective tool of risk-sharing as they help to reduce consumption fluctuation. Finally, the simulated wealth distribution features a relatively thick-right tail and could eventually match the empirical skewness with an accurate calibration of the model.

Related literature The existing literature on intergenerational transfer is already vast and contains several attempts at constructing heterogeneous agent models with intervivos transfers. Altig and Davis (1989, 1991, 1993) were among the first to study intergenerational transfers in an overlapping-generations environment. Assuming full-commitment within the family, they show how intergenerational transfers tend to mitigate the utility losses incurred from the presence of borrowing constraints. Meanwhile, numerous papers worked on more quantitative models, either assuming full-commitment, or two-sided perfect altruism. This is for instance the case in Laitner (1992) and Fuster et al. (2003, 2007). However, these models predicted too many intervivos transfers and a relatively low fraction of individuals at the credit constraint. Finally, De Nardi (2004) considered the impact of the intergenerational transmission of wealth and human capital on the wealth accumulation process, and finds that these elements help to obtain a more skewed distribution. However, she assumes a warm-glow motive for transfer to prevent strategic parent-child interaction.

Lindbeck and Weibull (1988) were the first to study intergenerational transfers when parents cannot commit to future transfers. In a two-period environment, they find that the receivers would free-ride on the other's concern by under-saving in the first period to receive a higher transfer in the second. At the same time, Laitner (1988) built one of the first quantitative macroeconomic model with imperfect altruism and no-commitment. Yet, in his model, generations overlapped for only one period, thus limiting the scope of transfer behavior. Nishiyama (2002) adopts a four-period OLG model in which households in the same family behave strategically in a simultaneous-move game. While he concludes that his model better explains the observed wealth distribution, it is not certain that the game played by the family members is simultaneous rather than sequential. Additionally, the simultaneity assumption, added to a discrete time model, prevents the young from using the transfer for their current savings, considerably reducing strategic interactions.

In a non-OLG framework, Kaplan (2012) studies the strategic interaction between a young worker who has the option to move in and out of the parental home, and their parent. If his paper assumes that parents cannot commit to transfer, he also restricts them to not saving either. Finally, Boar (2016) is the closest to this paper. Building up on Barczyk and Kredler (2014), she studies the same dynamic game as ours. However, she finds that intervivos transfer flows from the parent to the young only when the latter is credit constrained, a statement that we proved to be wrong. Moreover, she solves the model using value function iteration, whereas we show how noisy this method can be in the presence of strategic interactions. Instead, we use policy function iteration.

The remainder of the paper is organized as follows. Section 2 defines the model and the equilibria considered. Section 3 studies the simplified, two-period version of the model. Section 4 then computes the steady-state recursive partial equilibrium of the general model, and tests whether its predictions match the empirical data. Section 5 concludes.

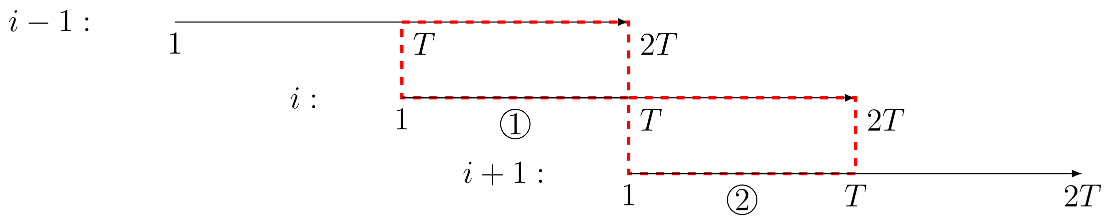
2. Model: intergenerational transfer without commitment

The model's foundations are drawn from the heterogeneous agent literature. All individuals are *ex-ante* similar. However, in every period, they face idiosyncratic income shocks, as well as a borrowing constraint that prevents them from perfectly smoothing their consumption. Both features lead to *ex-post* heterogeneity in consumption and asset holding. Our model adds to this framework a life cycle and overlapping generations.

Life cycle The life cycle is entirely deterministic, and is decomposed into two life stages, themselves subdivided into several periods (Figure 1). In the first one, individuals are called "young", while they are "old" in the second.

Newborns are *ex-ante* identical, do not yield additional costs for their parent and do not undertake any decisions until they exit their parental household in period 1. At this point, they become "young", starting off their life with no asset. From period 1 to T, they provide labor inelastically, and their parent is still alive, representing a potential source of (intervivos) transfers – ①. At time $T+1$, their parent, then aged $2T+1$, dies and their child, aged 1, forms a household on their own. From $T+1$ to $2T$, they are thus "old", and value the utility of their direct descendant. They then continue to work and can decide to transfer a fraction of their wealth to the next generation in every period – ②.

Figure 1: Overlapping generations



At any point in time, a household is made of a single agent, that has a unique ancestor and a unique descendant. However, rather than the individual, the main unit of interest is the family, defined as a pair (y, o), with y the young and o the old. A dynasty is a sequence of family.

Stage game To model the decision making process at the family level, we depart from the unitary model, and suppose that each member is a separate entity, with its own preferences. In every period, the choices of each entity are determined within a non-cooperative two-stage game, with the following structure.

1. The stochastic incomes are determined, and are publicly observed by the family members.

2. The old decides how much to transfer to the young and how much to save for the next period. If the old is about to die, these savings will be transfered as bequest to its child in the next period. It consumes the remaining of its wealth.

3. The young observes the savings and the transfer of the old, and decides in turn how much to save and consume.

One can think of this setting as a Stackelberg game, where the old is the leader and the young the follower.[2] If the former can thus influence the latter's choices through its own decisions, it cannot however impose to its child specific consumption and savings patterns. This game is then repeated throughout the life cycle, and players do not have access to committing devices. In particular, the parent's threat to not increase the transfer if the child is poor in the next

[2] Alternatively, one can interpret it as a dynamic principal agent model. The principal, here the old, maximizes its utility, subject to an incentive constraint that represents the young's optimal behaviors.

period is not credible. This setup thus brings parent-child strategic interactions into the model.

Assumption 1. *We restrict our attention to Markov perfect equilibria.*

Assumption 1 restricts drastically the set of equilibria considered. Markov perfect equilibrium forms a subset of subgame perfect equilibrium, in which the best responses – alternatively, the Markov strategies – depend only on the current values of the state variables. The equilibrium remains constructed by backward induction, in that the Markov strategies are defined for all possible states, including those that will never be visited along the equilibrium path. Moreover, as usual in dynamic games, agents form their expectations by assuming that the other players will revert to their best responses in all the subsequent periods.

State spaces In a Markov world, the best response functions are ultimately defined by the state variables, and deciding what states are relevant is a crucial step towards the definition of the model. The choice of the state space is dictated by the idea of payoff-relevant history, as stated in Definition 1 and 2.

Definition 1 (Sufficient partition by Fudenberg and Tirole (1991))**.** *Let h^t denote the entire history of the game, from period 1 to t, for all players. A partition of this history set, $\{H^t(\bullet)\}_{t=0,\dots,T}$, is sufficient if, for all t, h^t and $\tilde{h}^t$ such that $H^t(h^t) = H^t(\tilde{h}^t)$, the subgames starting at date t after histories h^t and $\tilde{h}^t$ are strategically equivalent. That is, (i) the action spaces are identical, (ii) the players' utility functions conditional on h^t and $\tilde{h}^t$ are representations of the same preferences.*

Definition 2 (Payoff-relevant history by Fudenberg and Tirole (1991))**.** *The payoff-relevant history is the minimal (i.e. coarsest) sufficient partition.*

Using Definition 2, the state spaces are

- For the young, their beginning-of-period wealth, their parent's optimal choices, i.e. their savings and their transfer, and both their income and that of their parent,
- For the old, both their beginning-of-period wealth and that of their child, as well as both incomes.

The current wealth of the old could for instance be included in the child's state vector. However, thanks to the sequential structure of the game, this information is not payoff-relevant as it does not influence today's consumption, nor does it affect the child's future payoffs. On the contrary, the parent's savings are crucial to form correct expectations on the future transfer, and are therefore included in the state space.

Notation Throughout the paper, $\tilde{\bullet}$ refers to the child's variables, cursive capital letters denote Markov strategies, and bold letters represent vectors. Thanks to Assumption 1, time subscripts are removed, and $\bullet'$ denotes next period's variables.

Recursive formulation Let t denote the age of the old. For all $t \in \{T+1, \ldots, 2T-1\}$, they decide how much to consume, save and transfer to their descendant, by maximizing their utility. Being altruistic, the latter includes the child's utility, discounted by an altruism parameter, α. Recursively, the problem writes

$$\begin{aligned} V_t(\boldsymbol{s}) = \sup_{a',\, b}\, & u(c) + \alpha u(\tilde{c}^\star) + \beta \mathbb{E}\left[V_{t+1}(\boldsymbol{s}'^\star) \mid \tilde{y},\, y\right], \\ \text{s.t.} \quad \boldsymbol{s} &= \left\{a,\, \tilde{a},\, y,\, \tilde{y}\right\}, \\ \boldsymbol{s}'^\star &= \left\{a',\, \tilde{a}'^\star,\, y',\, \tilde{y}'\right\}, \\ c &= Ra + y - b - a', \\ \tilde{c}^\star &= R\tilde{a} + \tilde{y} + b - \tilde{a}'^\star, \\ \tilde{a}'^\star &\in \tilde{\mathcal{A}}_{t-T}(\tilde{a},\, a',\, b,\, \tilde{y},\, y), \\ a',\, b &\in [0,\, Ra + y]. \end{aligned} \tag{1}$$

In (1), a refers to the beginning-of-period wealth, b to the transfer made to the young, and y to the income. Regarding the parameters, R is the interest rate – constant over time due to the absence of aggregate shock, and β is the discount factor. The expectation in the objective function is over the next period's incomes, and $u(\bullet)$ is the utility function defined in Assumption 2.

Assumption 2. *Let* $u : \mathbb{R}_{++} \mapsto \mathbb{R}$, $u(\bullet) \in \mathcal{C}^\infty$, $u'(\bullet) > 0$, $u^{(2)}(\bullet) < 0$, $u^{(3)}(\bullet) \geq 0$, *and* $\lim_{c \to 0} u'(c) \to +\infty$.

Both the savings and the transfer are prevented from being negative in (1). For the former, this is the strictest borrowing constraint possible. For the latter, it implies that old individuals cannot extract wealth from the future generation. This is a direct implication of the non-unitary family model. If the parent were to receive a positive amount from its child, this would be a choice made by the latter, and not imposed by the former. But since the young is not altruistic towards its ancestor, this situation will not occur.

The star in the next period's state vector is here to recall that the old expects its child to best-respond to its decisions in the next stage of the game. Specifically, for all $u = t - T$, $t \in \{T+1, \ldots, 2T-1\}$, the child's optimal savings is the solution to[3]

$$\begin{aligned} V_u(\tilde{\boldsymbol{s}}) = \sup_{\tilde{a}'}\, & u(\tilde{c}) + \beta \mathbb{E}\left[V_{u+1}(\tilde{\boldsymbol{s}}'^\star) \mid \tilde{y},\, y\right], \\ \text{s.t.} \quad \tilde{\boldsymbol{s}} &= \left\{\tilde{a},\, a',\, b,\, \tilde{y},\, y\right\}, \\ \tilde{\boldsymbol{s}}'^\star &= \left\{\tilde{a}',\, a''^\star,\, b'^\star,\, \tilde{y}',\, y'\right\}, \\ \tilde{c} &= R\tilde{a} + b + \tilde{y} - \tilde{a}', \\ a''^\star &\in \mathcal{A}_{t+1}(a',\, \tilde{a}',\, y',\, \tilde{y}'), \\ b'^\star &\in \mathcal{B}_{t+1}(a',\, \tilde{a}',\, y',\, \tilde{y}'), \\ \tilde{a}' &\in [0,\, R\tilde{a} + b + \tilde{y}]. \end{aligned} \tag{2}$$

[3] The dimensionality of the Markov strategies can be reduced in the first period for both the parent and the child, as young people start off with no wealth.

A noticeable difference with the Markov perfect literature in macroeconomics is that both agents are strategic players in this game.[4] Specifically, the first player, here the old, knows that its child will best-respond in the second stage, but the latter also knows that its parent will play according to its Markov strategies in the first stage of the next period's game. There is thus intra and intertemporal strategic interactions.

The problems are slightly different for $t = 2T$, as both the old and the young are in the final period of their respective life stages. Specifically, in the next period, the old will die, leaving a bequest to the young, while the young will become old. Accordingly, for $t = 2T$, the problem of the old writes

$$\begin{aligned} V_{2T}(\boldsymbol{s}) = \sup_{a',\,b}\, & u\left(c\right) + \alpha u(\tilde{c}^{\star}) + \delta\beta\mathbb{E}\left[V_{T+1}\left(\boldsymbol{s}'^{\star}\right) \mid y,\, \tilde{y}\right], \\ \text{s.t.} \quad \boldsymbol{s} &= \left\{a,\, \tilde{a},\, y,\, \tilde{y}\right\}, \\ \boldsymbol{s}'^{\star} &= \left\{a' + \tilde{a}'^{\star},\, y',\, \tilde{y}'\right\}, \\ c &= Ra + y - b - a', \\ \tilde{c}^{\star} &= R\tilde{a} + \tilde{y} + b - \tilde{a}'^{\star}, \\ \tilde{a}' &\in \tilde{\mathcal{A}}_T\left(\tilde{a},\, a',\, b,\, \tilde{y},\, y\right), \\ a',\, b &\in [0,\, Ra + y], \end{aligned} \tag{3}$$

where δ represents the intensity of the bequest motive. As before, the young's savings is the solution to its own maximization program, given by

$$\begin{aligned} V_T(\tilde{\boldsymbol{s}}) = \sup_{\tilde{a}'}\, & u\left(\tilde{c}\right) + \beta\mathbb{E}\left[V_{T+1}(\boldsymbol{s}') \mid \tilde{y},\, y\right], \\ \text{s.t.} \quad \tilde{\boldsymbol{s}} &= \left\{\tilde{a},\, a',\, b,\, \tilde{y},\, y\right\}, \\ \boldsymbol{s}' &= \left\{\tilde{a}' + a',\, y',\, \tilde{y}'\right\}, \\ \tilde{c} &= R\tilde{a} + b + y - \tilde{a}', \\ \tilde{a}' &\in [0,\, R\tilde{a} + \tilde{y} + b]. \end{aligned} \tag{4}$$

Definition 3 (Markov strategies)**.** *The young's Markov strategies are functions, $\tilde{\mathcal{A}}_t : \tilde{\boldsymbol{s}}_t \mapsto \mathbb{R}_+$, that solve (2) or (4) given $\mathcal{A}_{t+T+1}$ and $\mathcal{B}_{t+T+1}$, for every $t \in \{1, \ldots T\}$. The old's Markov strategies are functions, $\mathcal{A}_t : \boldsymbol{s}_t \mapsto \mathbb{R}_+$ and $\mathcal{B}_t : \boldsymbol{s}_t \mapsto \mathbb{R}_+$, that solve (1) or (3) given $\tilde{\mathcal{A}}_{t-T}$, for every $t \in \{T+1, \ldots, 2T\}$.*

Finding the Markov strategies remains a fixed-point problem, with the small difference that the recursiveness is over $2T$ periods.

Equilibrium At any point in time, agents are different in their age, their income and their wealth, and the equilibrium is defined as the distribution of agents over their respective states. We restrict the set of equilibria to stationary

[4] For instance, in the time-consistent public policies literature (Klein and Ríos-Rull, 2003; Klein et al., 2008), only one player, the government, acts strategically. The other, the representative consumer, does not as it does not expect its behavior to have any effect on the government.

equilibrium, where the distribution of agents remains unchanged over time.

Let $\lambda_t(\boldsymbol{s}_t)$ denote the measure of households aged t with state $\boldsymbol{s}_t$, $t = 1, \ldots, 2T$, and $\Lambda_t(\boldsymbol{s}_t)$ the corresponding cumulative measure. Since this model does not feature population growth, and the life cycle is deterministic, the entire population is normalized to one and the mass of agents of a given age represents $1/2T$ of the overall population.

By construction, the stage game is deterministic once the income shocks are realized. Its Markov nature then implies that, along the equilibrium path, the state space of the young is a subset of the old's. Indeed, for each $t \in \{T+1, \ldots, 2T\}$, the latter is $\boldsymbol{s}_t = \{a,\, \tilde{a},\, y,\, \tilde{y}\}$, while the former writes $\tilde{\boldsymbol{s}}_{t-T} = \{\tilde{a},\, a',\, b,\, \tilde{y},\, y\} = \{\tilde{a},\, \mathcal{A}_t(\boldsymbol{s}_t),\, \mathcal{B}_t(\boldsymbol{s}_t),\, \tilde{y},\, y\} \subseteq \boldsymbol{s}_t$. Then, there exists a function that maps the measure of the old to the measure of the young, $g : \lambda_t \mapsto \lambda_{t-T}$, and similarly for the cumulative measure, $G : \Lambda_t \mapsto \Lambda_{t-T}$, for each $t \in \{T+1, \ldots, 2T\}$.

It follows that a stationary equilibrium of this dynamic game is solely defined by the measures of the old households. For each $t \in \{T+1, \ldots, 2T\}$, $u = t - T$, the law of motion of the measures writes

$$\lambda'_{t+1}(\boldsymbol{s}_{t+1}) = \int_{A^2 \times Y^2} \mathbf{1}_{a_{t+1} = \mathcal{A}_t(\boldsymbol{s}_t)} \mathbf{1}_{\tilde{a}_{u+1} = \tilde{\mathcal{A}}_u(\tilde{\boldsymbol{s}}_u)} \pi(y',\, \tilde{y}' \mid y,\, \tilde{y})\, \mathrm{d}\Lambda_t(\boldsymbol{s}_t),$$

while for $t = T$,

$$\lambda'_{T+1}(\boldsymbol{s}_{T+1}) = \int_{A^2 \times Y^2} \mathbf{1}_{a_{T+1} = \tilde{\mathcal{A}}_T(\tilde{\boldsymbol{s}}_T) + \mathcal{A}_{2T}(\boldsymbol{s}_{2T})} \pi(y',\, \tilde{y}' \mid y,\, \tilde{y})\, \mathrm{d}G(\Lambda_{2T}(\boldsymbol{s}_{2T})),$$

where $A = [0,\, \bar{a}]$, is the asset space, with $\bar{a} < \infty$, Y the support of the income distribution, and $\pi(\bullet \mid y,\, \tilde{y})$ the conditional income distribution.

Definition 4 (Steady-State Recursive Partial Equilibrium)**.** *Given a set of exogenously fixed prices, a steady-state recursive partial equilibrium is a set of Markov strategies, $\{\tilde{\mathcal{A}}_t\}_{t=1}^{T}$, $\{\mathcal{A}_t\}_{t=T+1}^{2T}$, $\{\mathcal{B}_t\}_{t=T+1}^{2T}$, and a set of measure $\{\lambda_t\}_{t=T+1}^{2T}$, such that*

- *given their state, players act optimally based on their Markov strategies,*
- *the measures are stationary.*

Note that a steady-state recursive partial equilibrium is also a Markov perfect equilibrium.

3. An illustration: two-period model

To illustrate the redistributive motive of intervivos transfer, and how it eventually affects the savings of the donor and the recipient, this section focuses on a simplified, two-period version of the model, thus abstracting from the life cycle.

Assumption 3.1.

- *Young and old live only for one period,*
- *There is no bequest motive, $\delta = 0$,*
- *The old does not receive any endowment.*

The model described in Section 2 then writes compactly as

$$V_o(a,\, y) = \sup_{b \,\in\, [0,\, Ra]} u(Ra - b) + \alpha u(\tilde{y} + b - \tilde{\mathcal{A}}\,[b,\, \tilde{y}]), \tag{5a}$$

$$\text{s.t.} \quad \tilde{\mathcal{A}}(b,\, \tilde{y}) \in \arg \sup_{\tilde{a}' \,\in\, [0, \tilde{y}+b]} u(\tilde{y} + b - \tilde{a}') + \beta \mathbb{E}_{\tilde{y}'}\left[V_o(\tilde{a}',\, \tilde{y}') \mid \tilde{y}\right]. \tag{5b}$$

3.1. Existence

Proposition 3.1. *This game admits a unique pair of Markov strategies, $\tilde{\mathcal{A}}(b,\, \tilde{y})$ and $\mathcal{B}(a,\, \tilde{y})$. These strategies are continuous and twice differentiable almost everywhere. $\tilde{\mathcal{A}}(b,\, \tilde{y})$ is nondecreasing and convex in b, and $\mathcal{B}(a,\, \tilde{y})$ is nondecreasing and convex in a. The first order conditions of the young and the old are respectively*

$$u'(\tilde{y} + b - \tilde{a}') = \beta R\, \mathbb{E}_{\tilde{y}'}\, u'\big(R\tilde{a}' - \mathcal{B}\,[\tilde{a}',\, \tilde{y}']\,\big), \tag{6}$$

$$u'(Ra - b) \geq \alpha u'\left(\tilde{y} + b - \tilde{\mathcal{A}}\,[b,\, \tilde{y}]\right)\left(1 - \frac{\partial \tilde{\mathcal{A}}}{\partial b}\right). \tag{7}$$

Proof. Lemma A.1 demonstrates the monotonicity of the Markov strategies in their state variable. Lemma A.2 shows that V_o is absolutely continuous, and Lemma A.3 that it is strictly increasing and concave. Based on this, Lemma A.4 derives the existence, uniqueness, differentiability and convexity of $\mathcal{A}$. This is then used in Lemma A.5 to prove the same results for $\mathcal{B}$. The first order condition follows from Lemma A.4 and Lemma A.5, and in particular from the differentiability of the Markov strategies.[5] □

Unique Markov strategies do not always imply a unique Markov perfect equilibrium. Finding the Markov perfect equilibria is yet another fixed point problem. Yet, knowing that the Markov strategies are unique will be helpful when computing the numerical solutions.

3.2. Homogeneous agent

In this simplified version of the model, the young will never be credit constrained due to the absence of endowment in their old age. Moreover, if income uncertainty is removed, the risk-sharing motive of intervivos transfer disappears, and only the redistributive motive remains. Then, heterogeneity among the family is essential for transfer to occur. Said differently, in an economy where generations are endowed similarly, there is no reason for intergenerational transfer.

[5] All these lemmas are exposed in Appendix A.1.

Assumption 3.2.

- *Let the income be deterministic and identical for all generations,*
- *Let the utility be logarithmic,* $u(\bullet) = \log(\bullet)$*,*
- *Let* $\beta R = 1$*.*

Proposition 3.2. *Under Assumption 3.2, if*

$$a_0 < \tilde{y}\left(\frac{1 + R(1-\alpha)}{\alpha}\right), \tag{8}$$

where a_0 *is the initial condition on wealth, the unique Markov perfect equilibrium is such that transfer will never occur.*

Proof. Guess that the transfer Markov strategy will be zero, for all level of assets. Then, solving the first order condition of the young, (6), taking into account our previous guess, we obtain

$$\tilde{\mathcal{A}}(b,\, \tilde{y}) = \frac{\tilde{y} + b}{1 + R}. \tag{9}$$

Going back to the old's program, the corner solution will indeed bind if

$$u'(Ra) > \alpha\left(1 - \left.\frac{\partial\tilde{\mathcal{A}}}{\partial b}\right|_{b=0}\right) u'\left(\tilde{y} - \tilde{\mathcal{A}}\left[0,\, \tilde{y}\right]\right).$$

Plugging the savings Markov strategy and solving the inequality, we obtain

$$\tilde{y} > \alpha R a.$$

Consider now the first agent of this dynasty, endowed with wealth a_0. Along the equilibrium path, this individual will save according to (9). If (8) is satisfied, then $\alpha R\tilde{\mathcal{A}}(a_0,\, \tilde{y}) < \tilde{y}$ holds, and the first agent decides not to transfer. Then, $b = 0$ and the second agent will save according to $\tilde{\mathcal{A}}(0,\, \tilde{y}) \leq \tilde{\mathcal{A}}(a_0,\, \tilde{y})$, therefore not transferring either. By recursivity, this constitutes a Markov perfect equilibrium. Finally, note that this equilibrium is unique given our guess, $\mathcal{B}(a,\, \tilde{y}) = 0$. Yet, Proposition 3.1 proved the uniqueness of the Markov strategies. Hence, $\mathcal{B}(a,\, \tilde{y}) = 0$ is the unique Markov strategy, and this Markov perfect equilibrium is unique. □

3.3. Heterogeneous agents

To understand the degree of heterogeneity needed for intergenerational transfer to occur, and the impact this will have on the savings of the donors and the recipients, we now let the endowment vary across the generations.

Assumption 3.3.

- *Let young agents born in even (odd) periods receive a high (low) endowment,* $\tilde{y}_R > \tilde{y}_P$*,*

- *Let the wage spread satisfy*

$$\frac{\tilde{y}_R}{\tilde{y}_P} > \frac{1+R}{\alpha R},$$

- *Let utility be logarithmic,* $u(\bullet) = \log(\bullet)$*,*
- *Let* $\beta R = 1$*.*

As in Section 3.2, there is thus no uncertainty in this model. In particular, a rich child perfectly knows that its future child will receive a low endowment. The assumed wage spread allows us to derive easily the Markov strategies and the Markov perfect equilibrium by preventing poor old from transferring to rich young people in equilibrium.

Proposition 3.3. *Under Assumption 3.3, in equilibrium, the heterogeneity reduces to four types: poor young, rich young, poor old and rich old. Their respective Markov strategies are*

$$\begin{aligned}
\tilde{\mathcal{A}}(\tilde{y}_P, b_R) &= \frac{\tilde{y}_P + b_R}{1+R}, \\
\tilde{\mathcal{A}}(\tilde{y}_R, b_P) &= \frac{(1+\alpha)(\tilde{y}_R + b_P) - \tilde{y}_P}{1+R+\alpha}, \\
\mathcal{B}(a_R, \tilde{y}_P) &= \begin{cases} \dfrac{\alpha R a_R - \tilde{y}_P}{1+\alpha} & \text{if } \alpha R a_R > y_P, \\ 0 & o.w. \end{cases} \\
\mathcal{B}(a_P, \tilde{y}_R) &= \quad 0 \qquad \text{if } \alpha a_P R^2 < \tilde{y}_P R + \tilde{y}_P.
\end{aligned}$$

At the unique Markov perfect equilibrium[6]*, savings and transfer are given by*

$$a_R = \frac{(1+\alpha)\tilde{y}_R - \tilde{y}_P}{1+\alpha+R}, \tag{10}$$

$$b_R = \frac{\alpha R \tilde{y}_R - (1+R)\tilde{y}_P}{1+\alpha+R}, \tag{11}$$

$$a_P = \frac{\alpha(\tilde{y}_P + R\tilde{y}_R)}{(1+R)(1+\alpha+R)}, \tag{12}$$

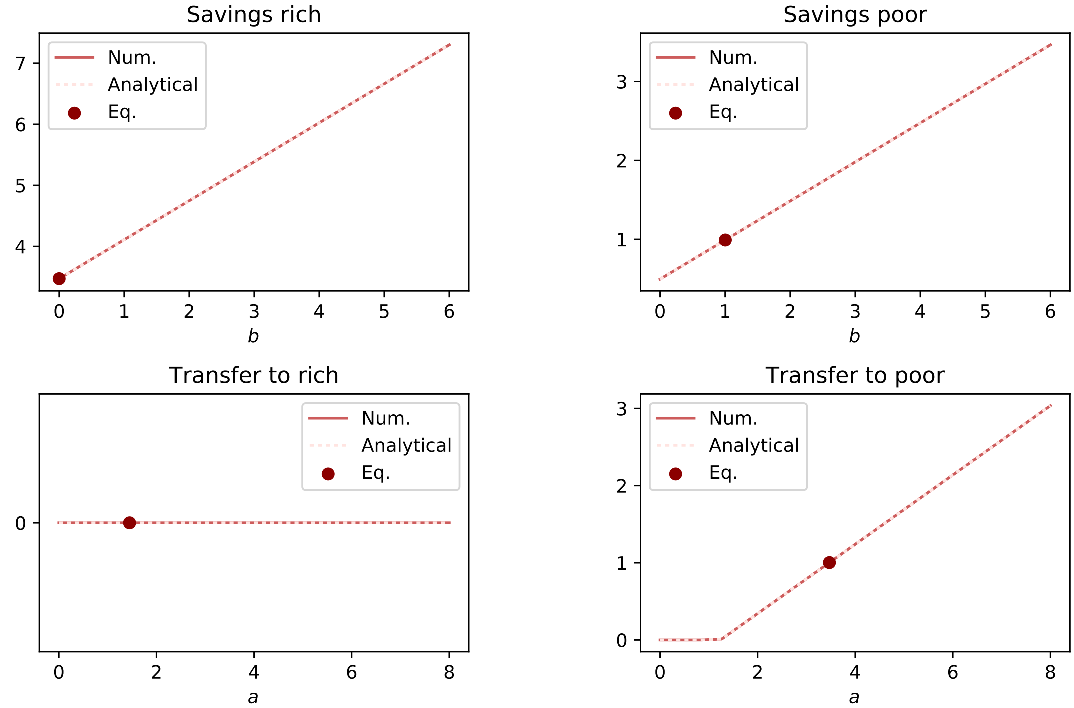
$$b_P = 0. \tag{13}$$

Proof. See Appendix A.2. □

In this setting, the wage spread is so large that the poor parent will never transfer to its rich child. As such, the savings of the poor are independent of the rich endowment. Yet, in equilibrium, the poor's savings will increase with the wage of the rich, as the transfer it received depends on the latter's endowment. On the contrary, the rich agent knows it will transfer a positive amount to its descendant, which then affects its savings decision. The poorer its child, the greater the transfer, and therefore the greater its savings. The rich child's savings are thus both affected by its own need and that of its future child. Uncertainty being absent, this increase in savings is solely due to redistributive concerns.

[6] This Markov perfect equilibrium is also a steady-state recursive partial equilibrium, as stated in Definition 4.

Figure 2: Policy function iteration solutions vs. analytical solutions



Calibration: $R = 1.01$, $\beta = 0.98$, $\alpha = 0.8$. The support of the income distribution is $\{1, 6\}$. The asset and transfer grids are discretized over 100 points.

This figure can be read as follows: when the young receive a low endowment, its parent transfers it around one (bottom right panel). With this transfer, and its own endowment, it saves around one (top right panel). With this one of savings, it has a wealth of R when old, based on which it transfers zero to its rich child (bottom left panel). Given that it received zero from its parent, the rich young saves around 3.5 (top left panel). It thus has $R \times 3.5$ when old, which pushes it to transfer one to their child (bottom right panel). And so on.

How effective are transfers as a redistributive tool? To see this, consider the same model without altruism,

$$\begin{aligned} &\max_{c_1,\, c_2,\, a'} \quad u(c_1) + \beta u(c_2) \\ &\text{s.t.} \quad c_1 = \tilde{y}_i - a', \quad \forall i \in \{P,\, R\}, \\ &\qquad\;\; c_2 = Ra', \end{aligned}$$

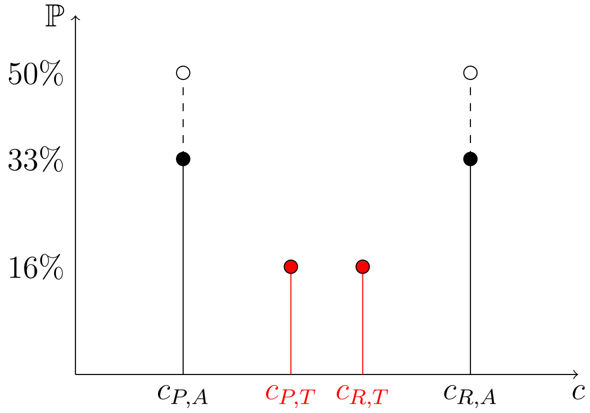
whose equilibrium behaviors are

$$c_{1,i} = c_{2,i} = \frac{R\tilde{y}_i}{1+R}, \quad a_i = \frac{\tilde{y}_i}{1+R}. \qquad \forall i \in \{P,\, R\}$$

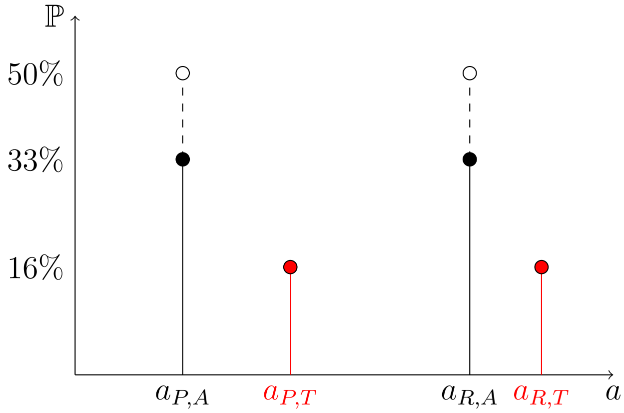
Let $w =: \tilde{y}_i + b_j$ be the young's wealth post-transfer, and ω_o the wedge of variable o between the rich and the poor. In the altruistic version of the model, we have

$$\omega_y = \frac{\tilde{y}_R}{\tilde{y}_P} \qquad \omega_w = \frac{w_R}{w_P} = \frac{\tilde{y}_R(2+\alpha)}{\alpha(\tilde{y}_P + \tilde{y}_R)}$$
$$\omega_c = \frac{c_R}{c_P} = \frac{2}{2+\alpha} \qquad \omega_a = \frac{a_R}{a_P} = \frac{2\,(\tilde{y}_R\,[1+\alpha] - \tilde{y}_P)}{\alpha\,(\tilde{y}_P + \tilde{y}_R)},$$

Figure 3: Aggregate consequences of intergenerational transfers



(a) Consumption



(b) Savings

Note: $\bullet_{P,\,A}$ denote the level of $\bullet$ for the poor agents in the dynasties with homogeneous members, as well as for the poor agents in the dynasties with heterogeneity in the endowment when transfer is not allowed. Similarly for $\bullet_{R,\,A}$ with rich individuals. $\bullet_{P,\,T}$ denote the level of $\bullet$ for the poor agents in the dynasties with alternating endowment, when transfer is allowed. Similarly for $\bullet_{R,\,T}$ for the rich agents.

while in the usual two-period OLG, $\omega_y = \omega_w = \omega_c = \omega_a$. For all variables, the spreads are smaller in the altruistic case. Moreover, the more altruistic the individuals, the lower the intergenerational inequality, as transfer, the redistributive tool here, increases.

Numerical solutions The existence of closed form solutions allows us to test the accuracy of the numerical algorithm used to solve the general model. We resort to policy function iteration, as described in Appendix B.1. Figure 2 plots the numerical solutions against the analytical ones. The linearity of the policy functions allows for a perfect fit.[7]

Additionally, we evaluate the accuracy of value function iteration (Figure 9). The latter is indeed the most used method in the macroeconomic literature, in part due to its simplicity.[8] Yet, its accuracy has rarely been tested in environments with strategic interactions. While it is relatively precise for the savings, the approximation errors tend to be significant for the transfer policy functions. This is primarily due to strategic interactions, mathematically translated by the partial derivative of the savings Markov strategy in (7). When iterating on the first order conditions, this information is added to the algorithm, yielding a greater accuracy. On the contrary, value function iteration struggles to understand the young's marginal reaction to variations in transfer, which eventually results in over-estimated Markov strategies.

[7] The code for the numerical solutions is available on Github.

[8] In particular, value function iteration does not require the derivation of the first order conditions.

3.4. Macroeconomic consequences

How would these intergenerational transfers shape the distribution of consumption and wealth in a world populated by different types of families? To answer this, consider an economy with a continuum of dynasties. Assume that the first third of this continuum is made of dynasties with homogeneous and poor individuals, the second is populated by dynasties with homogeneous and rich agents, while the remaining part is constituted of heterogeneous dynasties, as described in Section 3.3. The type of a given dynasty is determined *ex ante* and is fixed over time.

To start with, remove the possibility for parents to transfer wealth intergenerationally. In this setup, half of the population will be poor, and the other half rich ($c_{P,A}$, $c_{R,A}$, $a_{P,A}$ and $a_{R,T}$ in Figure 3). Then, reintroduce altruism in the old's utility. As shown in Section 3.2, this will not affect the decisions of the homogeneous dynasties. However, for the heterogeneous type, rich parents will start redistributing wealth towards their poor descendant. This will eventually raise the savings of all generations, while reducing consumption inequality, as described in Section 3.3 (the two red circles in Figure 3). Eventually, the wealth distribution becomes more skewed to the right, which was the primary motivation of this paper.

4. OLG, life cycle and strategic interactions: results

A trade-off appears when studying the general model of Section 2. On the one hand, multiple periods within each life stage allows for a more realistic life cycle. On the other hand, it multiplies the intertemporal strategic iterations, complicating the derivation of the first order conditions.[9] Yet, these first order conditions are necessary to compute the steady-state recursive partial equilibrium via policy function iteration. We cut this tradeoff short in favor of policy function iteration, and impose $T = 2$, such that the young and the old life stage lasts for two periods. For simplicity, we also align the bequest motive with the altruistic parameter, $\delta = \alpha$.

If the life cycle structure remains relatively simple, going from two to four periods already allows for more parent-child interactions. First, the young can receive a transfer in two periods, plus a bequest in the third. They thus need to take into account the effect of their present savings on future transfers. Second, the old generation cares about the future utility of their child. Finally, agents receive a positive endowment in every period, such that the borrowing constraint can occasionally bind.

Assumption 4.1. *There exists continuous and differentiable (a.e.) Markov strategies,* $\{\tilde{\mathcal{A}}_t\}_{t=1}^{2}$, $\{\mathcal{A}_t\}_{t=3}^{4}$ *and* $\{\mathcal{B}_t\}_{t=3}^{4}$.

[9] Specifically, the young's first order condition would include the parent's marginal reaction to a marginal change in the child's savings, from the current period to the latter's death.

Assumption 4.1 is in the spirit of Proposition 3.1, and allows us to derive the first oder conditions by using the envelope conditions of the problems.

Family (2,4) This game takes place during the final period of both agents' respective life stage. In particular, in the next period, the old will die, and it thus has to choose how much of its wealth to transfer in the current period, and how much to leave to its child as bequest.

For a child with a sufficiently large wealth, whether the intergenerational transmission of wealth occurs via an intervivos transfer or a bequest is however irrelevant. As long as the child is not credit constrained, it will indeed vary its savings to stay on its optimal consumption path. For instance, if its parent decides to transfer most of its wealth under the form of a bequest, the young will reduce its present savings, and eventually consume more today – thanks to lower savings – and tomorrow – thanks to the bequest. Dying in the next period, the parent's two choices, transfer and bequest, will only affect the consumption of the young. The latter being indifferent between the two, so is the parent.

Proposition 4.1. *For the (2,4) family, the first order condition of the young is*

$$u'(\tilde{c}_2) = \beta R \, \mathbb{E} \, u'(c_3^\star) + \lambda_{\tilde{a}_2'}, \tag{14}$$

where λ is the Kuhn-Tucker coefficient. For the old, the first order condition for savings and transfer are respectively

$$u'(c_4) = \beta R \, \mathbb{E} \, u'(c_3^\star) + \lambda_{a_4'}, \tag{15}$$

$$u'(c_4) = \alpha u'(\tilde{c}_2^\star) + \lambda_{b_4}. \tag{16}$$

Proof. See Appendix A.3 □

Proposition 4.2. *When the child is not credit constrained, both players are indifferent between wealth transmission under the form of intervivos transfer or bequest. On the contrary, when the young is credit constrained, the parent's preferred mean of wealth transmission is intervivos transfer.*

Proof. Combining (14), (15) and (16), one obtains

$$\frac{1}{\alpha}\left(\lambda_{a_4'} - \lambda_{b_4}\right) = \lambda_{\tilde{a}_2'}.$$

If the young saves a positive amount, such that $\lambda_{\tilde{a}_2'} = 0$, then $\lambda_{a_4'} = \lambda_{b_4}$ has to hold. Thus, either the parent's savings and transfer are nil, with $\lambda_{a_4'} = \lambda_{b_4} > 0$, or both are positive, in which case there is an infinity of possible allocation. If the young is credit constrained, i.e. $\lambda_{\tilde{a}_2'} > 0$, then $\lambda_{a_4'} > \lambda_{b_4}$ has to hold. Once more, two scenarios are possible. In one of them, both choices are at the constraint. In the other, the transfer is positive and the bequest is set to zero. □

Assumption 4.2. *Whenever multiple allocations are possible, assume that wealth transmission is entirely done via intervivos transfer. That is, let $a'^\star \to 0$ and $b^\star$ solves (16) given $a'^\star$.*

Assumption 4.2 could be justified in several ways. For instance, one could think of a tax scheme such that bequests are taxed but not intervivos transfers. To conclude, strategic interactions completely disappear in the game played by the (2,4) family. This is in part due to the old dying in the next period, so that the young cannot influence it after this point, and also because the parent's objective function includes the future value function of the young, such that their objectives are better aligned.

Family (1,3) In contrast to the (2,4) family, strategic interactions will affect the players' optimal behaviors in this game. When the young is considering how much to save, it indeed takes into account that a marginal change in its savings will result in a different transfer in the next period. Similarly, the old understands that an increase in its savings will result in an increase of the young's consumption, in anticipation of higher future transfer.

Proposition 4.3. *For the (1,3) family, the young's first order condition reads*

$$u'(\tilde{c}_1) - \beta R \mathbb{E} u'(\tilde{c}_2^\star) = \beta \mathbb{E}\left(u'(\tilde{c}_2^\star) \frac{\partial \mathcal{B}_4}{\partial \tilde{a}_1'} \right) + \lambda_{\tilde{a}_1'}. \tag{17}$$

The first order condition for savings and transfer of the age 3 old are respectively

$$u'(c_3) - \beta R \, \mathbb{E} \, u'(c_4^\star) = \alpha \frac{\partial \tilde{\mathcal{A}}_1}{\partial a_3'} \left(\beta R \mathbb{E} u'[\tilde{c}_2^\star] - u'[\tilde{c}_1^\star] \right) + \lambda_{a_3'}, \tag{18}$$

$$u'(c_3) - \alpha u'(\tilde{c}_1^\star) = \alpha \frac{\partial \tilde{\mathcal{A}}_1}{\partial b_3} \left(\beta R \mathbb{E} \, u'[\tilde{c}_2^\star] - u'[\tilde{c}_1^\star] \right) + \lambda_{b_3}, \tag{19}$$

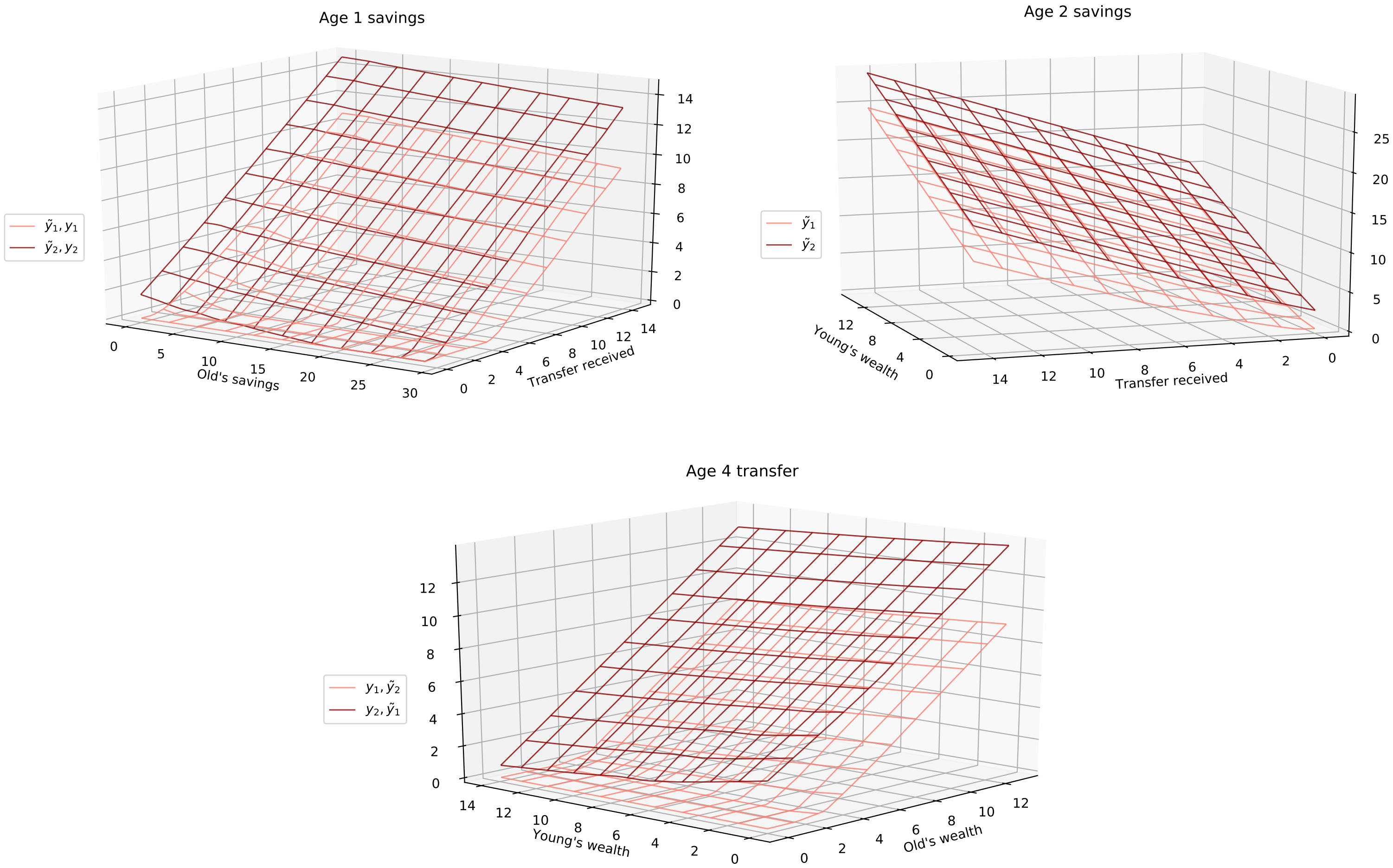
Proof. See Appendix A.3 □

If one guesses that transfers decrease with the young's wealth, (17) tells us that strategic interactions lead to under-saving for the child, in order to extract further wealth from the old in the future. Furthermore, when the young is credit constrained, the parent's first order conditions boil down to the no-interaction optimality conditions. In this case, a unique allocation of the parent's wealth exists (Figure 10).

However, when the constraint is not binding for the young, multiple allocations are possible (Figure 11), as in the (2,4) family.[10] The parents need to decide how much of their wealth to allocate for savings, and how much to transfer to their descendant. The transfer increases the young's current consumption, and helps it constitute its own buffer stock of wealth, whereas savings yield higher future consumption, and a better insurance against adverse shocks. The old will thus always want to save a positive amount for precautionary purposes. When not constrained, the young will however adjust its savings to the parent's allocation in order to remain on its optimal consumption path. Hence, once the parent constituted this buffer stock of wealth, it is indifferent between saving more, even if this requires higher transfer in the future if its descendant is hit by a negative shocks, or directly increasing the transfer. Both choices lead to the same consumption and the same level of insurance for the young and the old.

[10] We are not able to prove this multiplicity analytically, but the numerical solutions leave no doubt.

Figure 4: Markov strategies



Note: the calibration of the model is detailed in Appendix B.2. y denotes the parent's income, while $\tilde{y}$ stands for the young's endowment.

Assumption 4.3. *Whenever multiple allocations of the parent's wealth are achievable, pick the allocation in the middle of the solution set (see Figure 11).*

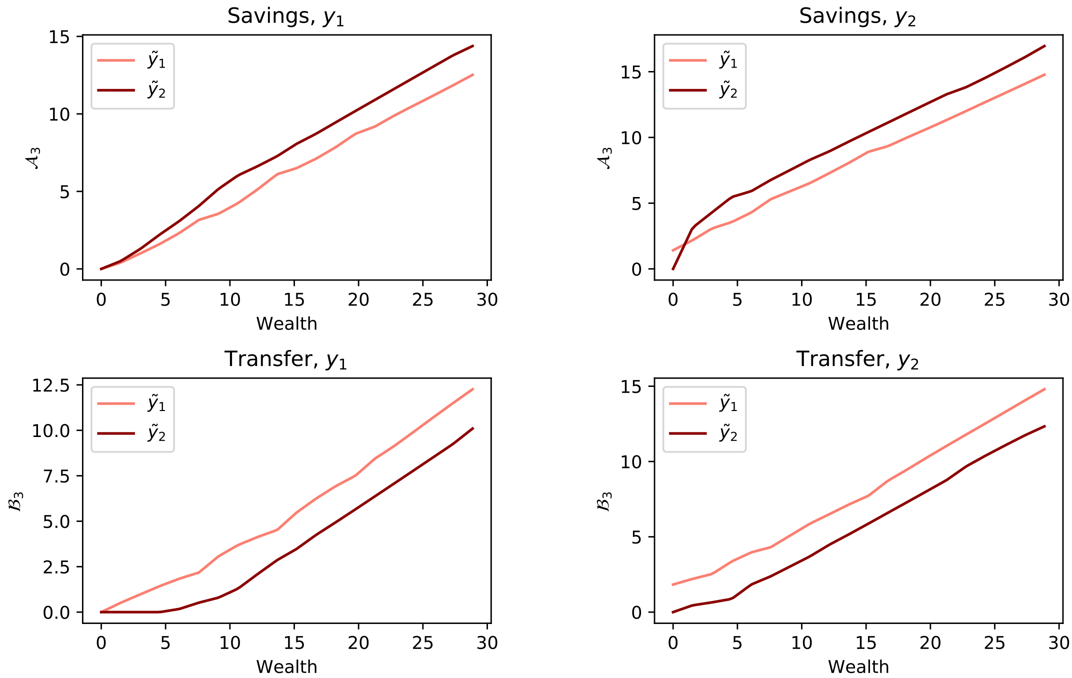
4.1. Markov strategies and strategic interactions

The Markov strategies are solved using policy function iteration as this algorithm proved its accuracy and efficiency.[11] The specific calibration used is detailed in Appendix B.2. Figures 4 and 5 plot the Markov strategies.

To understand the shape of the Markov strategies, consider the dynamic game played by a given family: the old 3 moves first, followed by the young 1, then

11 Alternatively, one could use an extended version of the endogenous grid method. In particular, because this method does not require to use root solver, it would circumvent some of the numerical problems mentioned in Appendix B.2. However, extending EGM to multidimensional problems, with occasionally binding constraints, is not an easy task – see for instance Hintermaier and Koeniger (2010), Ludwig and Schön (2013). In the long run, this model should be solved using an algorithm close to Druedahl and Jørgensen (2017).

Figure 5: Markov strategies, old 3



Note: the calibration of the model is detailed in Appendix B.2. y denotes the parent's income, while $\tilde{y}$ stands for the young's endowment. Approximation errors occur due to the multiple solutions to the old's problem, hence the bumps in the Markov strategies. These are discussed in Appendix B.2.

the old 4 plays and the game is concluded by the young 2. In the last stage of this game, the young 2 chooses its optimal savings. In the next period, its child will form a household of its own. If the latter is directly hit by a bad shock, and no transfer can be provided, it will have no means to smooth its consumption, and both itself and its parent will suffer a utility loss. Anticipating this, the young 2 saves a high fraction of its wealth to constitute a larger buffer stock.[12] Moreover, its consumption becomes relatively invariant in its own wealth.[13] In response, the transfer of the old 4 becomes relatively irresponsive to changes in the wealth of its descendant, unless the initial repartition of wealth in the family is already very skewed in favor of the young, in which case the transfer is nil.

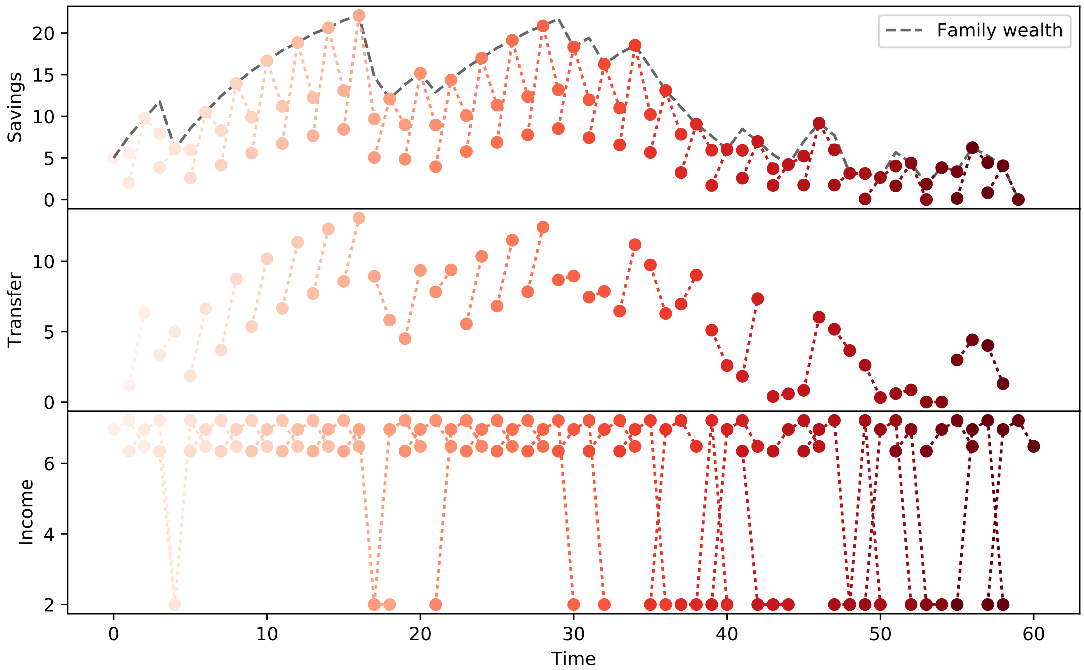
This insensitivity propagates backwards and eventually affects the savings of the youngest agents. In Section 4, we indeed argued that strategic interactions would incentivize them to overconsume in the first period. However, since the age 4 transfer is relatively independent from the young's wealth, these are minimized, and their savings are close to the first best. In short, risk aversion downplays strategic interactions.

Reaching the top of the tree, the old 3 decides not to transfer when its child is richer than itself, constituting instead a buffer stock. For richer parents,

[12] Since income uncertainty is resolving over time, most of these precautionary savings are indeed driven by the uncertainty on its future child's income.

[13] For instance, the young's propensity to save out of its wealth approximates 0.9.

Figure 6: Simulation, an example



Note: the calibration of the model is detailed in Appendix B.2. In the plots, one color represents one household. The savings of the age 4 player are not represented as there are always nil.

assembling this stock is however easier, and the player transfers a positive amount. When the young's endowment is lower than the parental one, a fraction of this transfer serves as a redistributive tool. Otherwise, transfers are used as a risk sharing tool to help the young constitute their own buffer stock.[14]

4.2. Steady-state equilibrium and empirical validation

To obtain the steady-state recursive partial equilibrium, we drew a large number of dynasties and let them play the stage game until the distributions converge – see Section B.2 for the detailed algorithm. As an example, Figure 6 displays the choices made by thirty successive generations of the same dynasty. In this simulation, phases of wealth accumulation alternate with periods of wealth decline when the members of the family are hit by successive bad shocks. Moreover, transfer tracks closely the evolution of the family wealth: expansion implies increasing transfer over the life cycle and over the generations, and the reverse holds as well.

[14] This is the sole consequence of Assumption 4.3. For the scenarios where the young's income is higher than the parental one, the set of solution contains the no-transfer case. As explained previously, the parent is indeed indifferent between transferring a positive amount in the present to help its child constitute its own buffer stock, or keeping the money and transferring it in the latter period.

Table 1: Stationary distribution, summary statistics

	All	Lifecycle 1	2	3	4
Consumption	5.309	5.053	5.145	5.900	5.140
	(1.018)	(0.886)	(0.670)	(1.175)	(1.031)
Beginning-of-period wealth	3.357	0.0	1.531	6.856	5.040
	(3.516)	(0.000)	(1.188)	(3.474)	(2.507)
Transfer received	3.375	2.073	4.697		
	(2.603)	(1.318)	(2.895)		
Transfer received (in perc.)	0.916	0.903	0.928		
Credit constrained (in perc.)	0.051	0.098	0.000	0.056	

Note: this table reports the mean, and the standard deviation in parenthesis, of the stationary distributions for several variables. The beginning-of-period wealth is defined net of transfer. The percentages are reported as decimals. The model is calibrated as explained in Appendix B.2.

Table 1 displays some statistics of the stationary distribution, for the calibration reported in Appendix B.2. As observed in the data, the mean and the standard deviation of consumption and wealth follow a hump-shaped profile over the life cycle (Browning and Lusardi, 1996; Gourinchas and Parker, 2002; Fernández-Villaverde and Krueger, 2007). Additionally, the standard deviation of consumption is smaller than the wealth standard deviation, suggesting a lower degree of inequality for the first variable (Heathcote et al., 2010; Krueger et al., 2010). Finally, while most of the young receive a transfer, its actual amount varies significantly across individuals.

Table 2 reports the mean and the standard deviation of the saving rates, for two calibrations of the model. In the altruistic case, α is set to 0.9, while it is equal to zero in the non-altruistic version. For all periods of the life cycle, the mean saving rates are higher when agents care about their descendants. These higher rates are due to the redistributive and the risk-sharing motives. Transferring a positive amount to achieve a more equal allocation of resources across the household is indeed costly and requires to save more in the first place – the redistributive

Table 2: Saving rates

	All	Lifecycle 1	2	3	4
Altruism	0.271	0.197	0.530	0.357	0.000
	(0.225)	(0.133)	(0.130)	(0.124)	(0.000)
No altruism	0.114	0.143	0.185	0.128	0.000
	(0.102)	(0.123)	(0.081)	(0.032)	(0.000)

Note: this table reports the mean, and the standard deviation in parenthesis, of the savings rate at the steady-state recursive partial equilibrium, as defined by $a'/(p.R \times a + y)$ for the old, and $a'/(p.R \times a + y + b)$ for the young. The percentages are reported as decimals. The model is calibrated as explained in Appendix B.2.

Table 3: The timing of transfers

	Dependent: b		Dependent: Prob($b > 0$)	
	OLS (1)	OLS (2)	Logit (3)	Logit (4)
Parent's asset	0.299 (0.001)	0.659 (0.002)	0.524 (0.005)	1.270 (0.018)
Parent's income	0.465 (0.001)	0.584 (0.002)	1.207 (0.014)	0.162 (0.006)
Young's income	-0.564 (0.001)		-0.605 (0.018)	
Young's income (FD)		-0.274 (0.001)		-0.254 (0.007)
Young's wealth		-0.509 (0.004)		
Age	1.912 (0.012)			
Age $\times$ Young's income	0.403 (0.002)			
R^2	0.915	0.968	0.571	0.622
N	100 000	50 000	100 000	50 000

Note: the standard errors are reported in parentheses. None of the regressions include an intercept. The pseudo R^2 used for the logistic regression is the McFadden's adjusted R-squared. Information is at the family level: there are N families and $2N$ individuals in the sample. For regressions (2) and (4), the sample size is reduced as it focuses on those individuals that had age 1 in the first period of the panel to be able to compute their first difference in income.

motive. Additionally, one's child may encounter a negative shock in the future and be incapable of smoothing its consumption due to the borrowing constraint, forcing the parent to transfer a higher amount – the risk-sharing motive.

To measure the extent to which intergenerational transfers improve risk-sharing, we simulate a two-period panel of $100,000$ families along the Markov perfect equilibrium of the model, with and without altruism. Based on these simulated data, we test whether the timing of the transfers is in line with the one empirically observed. McGarry (2016) finds that transfers are often made in conjunction with specific events in the child's life, and in particular, that parents frequently respond to negative shocks to the child's income. She additionally reports that the effect of a child's current income on transfers is large and significantly different from zero.

Two logistic regressions are run, whose dependent variable is the indicator function equal to one whenever a transfer occurred. The first regression includes the entire sample and has for explanatory variable the young's income, whereas the second studies the impact of a change in the young's income. Both regressions control for the parents' wealth. Table 3 reports the estimated coefficients and their standard errors. For both regressions, the estimates related to the young's income are statistically significant, with the signs expected from the empirical evidence.

Table 4: Intergenerational transfers and risk sharing

	Dependent: Δc	
	OLS Altruism	OLS No altruism
Young's income variation	-0.185	-0.692
	(0.001)	(0.003)
Young's wealth	-0.030	-2.456
	(0.002)	(0.012)
Parent's income	-0.104	-0.001
	(0.002)	(0.003)
Parent's wealth	-0.496	0.019
	(0.004)	(0.018)
R^2	0.430	0.496
N	50 000	50 000

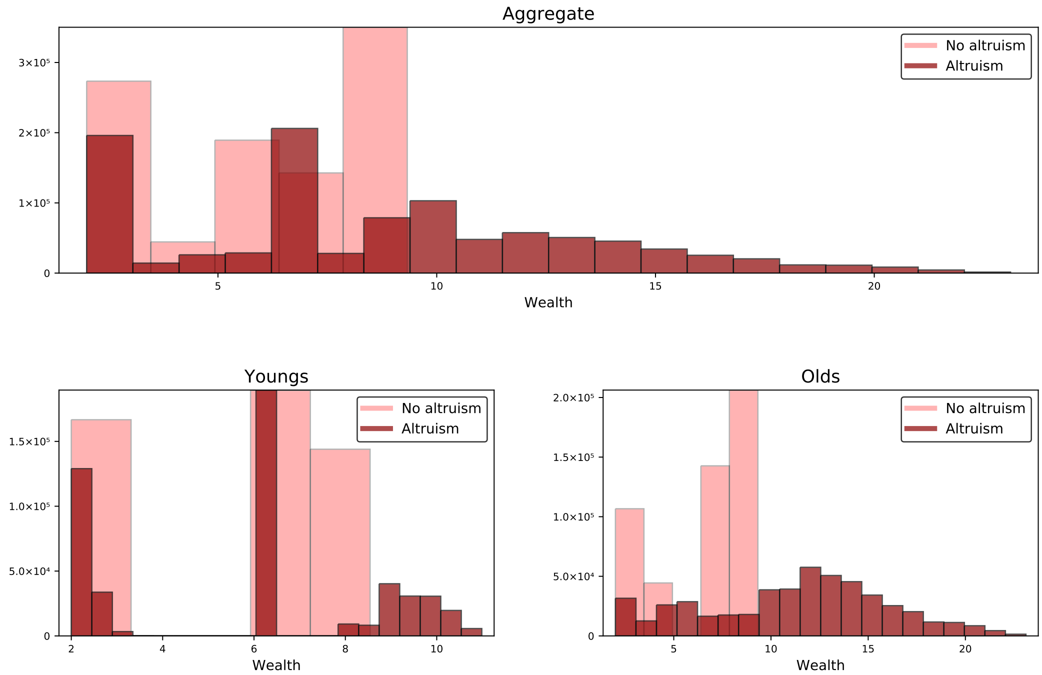
Note: the standard errors are reported in parentheses. Both regressions include an intercept whose coefficients is not reported in the table. Information is at the family level: there are N families and $2N$ individuals in the sample. The sample size is reduced to the individuals that had age 1 in the first period of the panel to be able to compute their first difference in income.

Additionally, two ordinary least square regressions are run to analyze the effect of the parent's and child's wealth on the actual size of the transfer, adding as controls a dummy variable for the age and an interaction term between the young's income and the old's age. Here, as in the empirical data, the estimated coefficients indicate that transfers flow from wealth- and income-rich agents to poor recipients (McGarry and Schoeni, 1995, 1997; Berry, 2008). Besides, the sign of the interaction indicates that this is particularly true in the middle of the life cycle, while transfers are less responsive to the young's income at the end of the parents' life, in agreement with the findings of Dunn and Phillips (1997).

Finally, we regress the change in consumption on the change in the young's income, controlling for the young's and the parent's wealth, for the models with and without altruism. In an economy with perfect risk sharing, the estimated coefficient would be zero, while in an autarkic world, it would be one. Table 4 reports the results of these regressions. While both estimates are negative and statistically significantly different from zero, the coefficient when parents are altruistic is significantly lower than the one in the other model, and is closer to the empirical estimates (Hall and Mishkin, 1980; Zeldes, 1989). This, combined with our result on the timing of the transfer, suggests that the latter is indeed used as an effective risk sharing devise.

The initial motivation of this paper was to understand whether one-sided altruism could generate a wealth distribution with a higher skewness. To see this, Figure 7 plots the histogram of this distribution in the stationary steady state, both for the model with and without altruism. When α is set to zero, the

Figure 7: Stationarity distributions of wealth



Note: the calibration of the model is detailed in Appendix B.2. Wealth is defined as the sum of beginning-of-period assets and income. For both models, the sample size is $500,000$ families, that is $1,000,000$ individuals.

dynamic game boils down to a standard four-period life cycle model. Agents start off with no wealth and can only accumulate wealth during their lifetime, hence the very concentrated distribution.[15] When older generations care about their descendants however, the distribution is more dispersed, and a relatively thick right tail appears in the aggregate distribution. This greater skewness emerges from two forces. First, both the saving rates and the actual amount saved are higher throughout the life cycle. Second, parents can transfer a fraction of their wealth to the future generations, allowing for intergenerational accumulation of wealth.[16] Although the skewness of the wealth distribution remains far from the empirical one, we believe that a more accurate calibration of the model, and in particular a more realistic income profile, would allow to get closer to the data.

5. Conclusion

This paper developed a heterogeneous agent model with overlapping-generations, a deterministic life cycle, and strategic interactions between the different generations. While its qualitative predictions are in line with the empirical evidence, some of its quantitative predictions remain far from the data. In particular, the

[15] A better comparison would be established by comparing our results to an actual infinite-horizon model, for instance Aiyagari (1994).

[16] As an example, with the current calibration of the model, old people transfer on average 41.2% of their wealth before dying.

fraction of individuals at the credit constraint is too low, the percentage of young people receiving a transfer is too high, and the right tail of the wealth distribution is not long enough to match the empirical one. However, the qualitative results presented in Section 4 seem to indicate that there is room for improvement. In particular, the current calibration of the model is far from perfect, and enhancing it would yield more accurate predictions.

On the theoretical side, the model is currently suffering from the multiple solutions to the parents' problems. Refinements should be integrated to the model in order to obtain clear predictions on the allocation of the old's wealth.

Finally, the current model is missing one aspect of family risk sharing, that would eventually help to obtain a more skewed distribution. By definition, young people have access to different degrees of family insurance. When facing an adverse shock, a young with wealthy parents can stay on its optimal consumption path thanks to its parents' transfer, while a young with a poor family is going to be liquidity constrained. In a model in which risky decisions are to be made, these heterogeneous insurances would result in different risk-taking behaviors. As an example, the exogenous income process could be endogenized by a job search in the first stage of the game. Young people with wealthier families would have a higher reservation wage, thus waiting to obtain better jobs. This would *in fine* bolster the disparity between rich and poor families, resulting in a wealth distribution with a thicker right tail.

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Appendices

A. Proofs

A.1. Proof of Proposition 3.1

To prove the existence of unique Markov strategies, we first show that the value function of the old is absolutely continuous, increasing and concave in a. We then use this result to establish that the Markov strategy of the young is unique, continuous, twice differentiable almost everywhere, increasing and convex in b. We then go back to the old problem to conclude by demonstrating that the old Markov strategy is also unique, continuous, differentiable twice almost everywhere, increasing and convex in a.

Assumption A.1. *Restrict the space of a and b to $[\underline{a},\, \bar{a}]$ and $[0,\, \bar{b}]$ respectively, with $\underline{a} > 0$, and $\bar{a},\, \bar{b} < \infty$.*

Lemma A.1. *The Markov strategies of the young and the old are nondecreasing in b and a respectively.*

Proof. From the problem of the young and the old, we have

$$\frac{\partial U_y}{\partial \tilde{a}' \partial b} = -u''(\tilde{y} + b - \tilde{a}') > 0 \qquad \frac{\partial U_o}{\partial b \partial a} = -Ru''(Ra - b) > 0,$$

where U_y and U_o are their respective objective function. Thus, U_y has increasing differences in $(\tilde{a}',\, b)$ while U_o has increasing difference in $(b,\, a)$. By Topki's theorem, we conclude that $\tilde{\mathcal{A}}$ and $\mathcal{B}$ are respectively nondecreasing in b and a. $\square$

Lemma A.2. *There exists a V_o absolutely continuous, differentiable everywhere in a, for all $a \in (\underline{a},\, \bar{a})$.*

Proof. Recall the problem of the old,

$$\begin{aligned} V_o(a,\, y) &= \sup_{b \in \mathbb{B}(a)} u(Ra - b) + \alpha u(\tilde{y} + b - \tilde{a}^\star), \\ \text{s.t.} \quad & \tilde{a}^\star \in \arg \sup_{\tilde{a}' \in \mathbb{A}(b,\, \tilde{y})} u(\tilde{y} + b - \tilde{a}') + \beta \mathbb{E}\left[V_o(\tilde{a}',\, \tilde{y}') \mid \tilde{y}\right], \end{aligned}$$

where $\mathbb{A}(b,\, \tilde{y}) = [0,\, b + \tilde{y}]$ and $\mathbb{B}(a) = [0,\, Ra]$, for all $a \in [\underline{a},\, \bar{a}]$. The strict positivity of the endowment, $\min Y > 0$, implies that $\mathbb{A}$ is non empty, for all $b \in [0,\, \bar{b}]$. Therefore, there exists a set of maximizers, $\tilde{\mathcal{A}}^\star$, solving the constraint. Then, $\forall \tilde{\mathcal{A}} \in \tilde{\mathcal{A}}^\star$, the problem rewrites

$$V_o(a,\, y) = \sup_{b \in \mathbb{B}(a)} u(Ra - b) + \alpha u\big(\tilde{y} + b - \tilde{\mathcal{A}}\,[b,\, y]\,\big).$$

We cannot say anything about the continuity of the objective function, even less about its differentiability. Nevertheless, showing that $V_o(a,\, y)$ is continuous and differentiable in a boils down to the usual Envelope theorem. The only difference with the Envelope theorem of Milgrom and Segal (2002) is the presence of the parameter, a, in the choice set.[17] For simplicity, let $f(a,\, b) =: u(Ra-b)+\alpha u\big(\tilde{y}+b-\tilde{\mathcal{A}}\,[b,\, y]\,\big)$. Note that $f(\bullet,\, b)$ is continuously differentiable in a, and $\mathcal{B}^* \neq \emptyset$ for all $a \in [\underline{a},\, \bar{a}]$. Moreover, for $\varepsilon > 0$ small, there exists an integrable function, $b : [\underline{a},\, \bar{a}] \mapsto \mathbb{R}_+$ s.t. $|\, f_a(a,\, b)\,| \leq b(a)$ for all $b \in [0,\, Ra-\varepsilon]$ and all $a \in [\underline{a},\, \bar{a}]$.[18]

Because the supremum over a set is no smaller than the supremum over a subset, we can write

$$\sup_{b\,\in\mathbb{B}(a'')} \big(f(a',\, b)+f(a'',\, b)\big) \leq \sup_{b\,\in\mathbb{B}(a'')} f(a',\, b) + \sup_{b\,\in\mathbb{B}(a'')} f(a'',\, b) \tag{20}$$

Here, the choice set is maintained fixed. In our case, however, $\mathbb{B}(a)$ expands in a, i.e. $\mathbb{B}(a') \subset \mathbb{B}(a'')$, for $a' < a''$. In general, this implies

$$\sup_{b\,\in\,\mathbb{B}(a')} f(a',\, b) \leq \sup_{b\,\in\,\mathbb{B}(a'')} f(a',\, b). \tag{21}$$

If, however,

$$\sup_{b\,\in\,\mathbb{B}(a')} f(a',\, b) \geq \sup_{b\,\in\,\mathbb{B}(a'')\setminus\mathbb{B}(a')} f(a',\, b),$$

then (21) will hold as an equality. Now, recall that $f(a,\, b) = u(Ra-b)+\alpha u(\tilde{y}+b-\tilde{\mathcal{A}})$. Hence, $\forall b \in \mathbb{B}(a'') \setminus \mathbb{B}(a')$, the old agent would consume a negative amount, which is impossible by definition.[19] (21) thus holds as an equality. Using this, (20) implies

$$\sup_{b\,\in\mathbb{B}(a'')} \big(f(a',\, b)+f(a'',\, b)\big) \leq \sup_{b\,\in\mathbb{B}(a')} f(a',\, b) + \sup_{b\,\in\mathbb{B}(a'')} f(a'',\, b).$$

Finally, this inequality can in turn be used to obtain

$$\begin{aligned}
\sup_{b\,\in\mathbb{B}(a'')} f(a'',\, b) - \sup_{b\,\in\mathbb{B}(a')} f(a',\, b) &\leq \sup_{b\,\in\mathbb{B}(a'')} \left(f(a'',\, b)-f(a',\, b)\right) \\
\Leftrightarrow \qquad |\, V_o(a'')-V_o(a')\,| &\leq \sup_{b\,\in\mathbb{B}(a'')} |\, f(a'',\, b)-f(a',\, b)\,|.
\end{aligned}$$

The remaining of the proof is identical to Milgrom and Segal (2002). For all $a' < a''$ in $[\underline{a},\, \bar{a}]$, we have

$$\begin{aligned}
|\, V_o(a'')-V_o(a')\,| &\leq \sup_{b\,\in\mathbb{B}(a'')} |\, f(a'',\, b)-f(a',\, b)\,| \\
&= \sup_{b\,\in\mathbb{B}(a'')} \left| \int_{a'}^{a''} f_a(t,\, b)\,\mathrm{d}t \right| \\
&\leq \int_{a'}^{a''} \sup_{b\,\in\mathbb{B}(a'')} |\, f_a(t,\, b)\,|\,\mathrm{d}t \\
&\leq \int_{a'}^{a''} b(t)\,\mathrm{d}t.
\end{aligned}$$

[17] We cannot simply do a change of variable, because the state, a, would then appear in the Markov strategy of the young, complicating further the problem

[18] We indeed need to restrict the set of b as $\lim_{b\to Ra} u'(Ra-b) \to \infty$ from Assumption 2. This is not a problem as the Inada condition ensures that the old will consume a positive amount, $b < Ra$.

[19] The formal rational differs depending on the type of utility function used. Focusing on CRRA, we have $u(\bullet) = (\bullet^{1-\sigma}-1)/(1-\sigma)$, $\sigma > 0$. Then, for all $\sigma \neq \mathbb{N}_+$, $\forall\, b \in \mathbb{B}(a'') \setminus \mathbb{B}(a')$, $u(Ra-b)$ is not defined, and we could set it arbitrarily to $-\infty$, in line with $\lim_{b\to Ra} u(Ra-b) = -\infty$.

This proves that V_o is absolutely continuous. This implies in particular that, for any $\mathcal{B}(a, y) \in \mathcal{B}^*$, we have

$$\begin{aligned} V_o(a, \tilde{y}) &= V(\underline{a}, \tilde{y}) + \int_{\underline{a}}^{a} f_a(t, \mathcal{B}[t, \tilde{y}])\,\mathrm{d}t \\ &= V(\underline{a}, \tilde{y}) + R\int_{\underline{a}}^{a} u'(Rt - \mathcal{B}[t, \tilde{y}])\,\mathrm{d}t. \end{aligned} \tag{22}$$

Moreover, since $f_a(\bullet, b)$ is continuously differentiable for all $a_0 \in [\underline{a}, \bar{a}]$ from Assumption 2, and the Inada condition ensures that $\mathcal{B}(a) < Ra$, for all $a \in [\underline{a}, \bar{a}]$, V_o is differentiable for all $a_0 \in (\underline{a}, \bar{a})$ – Theorem 3 of Milgrom and Segal (2002). □

Lemma A.3. *V_o is strictly increasing and concave in a.*

Proof. The differentiability of V_o at $a_0 \in (\underline{a}, \bar{a})$ implies $V'(a_0) = f_a(a_0, \mathcal{B}[a_0]) = Ru'(Ra_0 - \mathcal{B}[a_0])$, for all $\mathcal{B}(a) \in \mathcal{B}^*$ – Theorem 1 of Milgrom and Segal (2002). Assumption 2 then implies $V'(a_0) > 0$, for all $a_0 \in (\underline{a}, \bar{a})$.

Regarding the concavity of V_o, for $a_1, a_2 \in (\underline{a}, \bar{a})$, $a_1 < a_2$, concavity requires

$$V'(a_1) \geq V'(a_2) \quad \Rightarrow \quad u'(Ra_1 - \mathcal{B}[a_1]) \geq u'(Ra_2 - \mathcal{B}[a_2])$$

where the second line follows from the strict concavity of $u(\bullet)$. Hence, if the consumption of the old is non-decreasing in a, V_o is concave.

First, note that at the corner solution, consumption is strictly increasing in a. Second, at the interior solution, the old agent needs to be indifferent between transferring an additional ε or consuming it. Since $\mathcal{B}$ is non-decreasing in a, this indifference criterion has to hold when a increases. If, however, the old agent decreases his consumption and increases his transfer, his level of marginal utility will go down while the marginal utility of his kid will increase. But then the old agent would no longer be indifferent. Hence consumption cannot be strictly decreasing in consumption. □

Lemma A.4. *There exists a unique Markov strategy for the young agent, $\tilde{\mathcal{A}}(b, \tilde{y})$. $\tilde{\mathcal{A}}$ is continuous, twice differentiable (a.e.) and convex in b.*

Proof. Recall that the young is looking to maximize

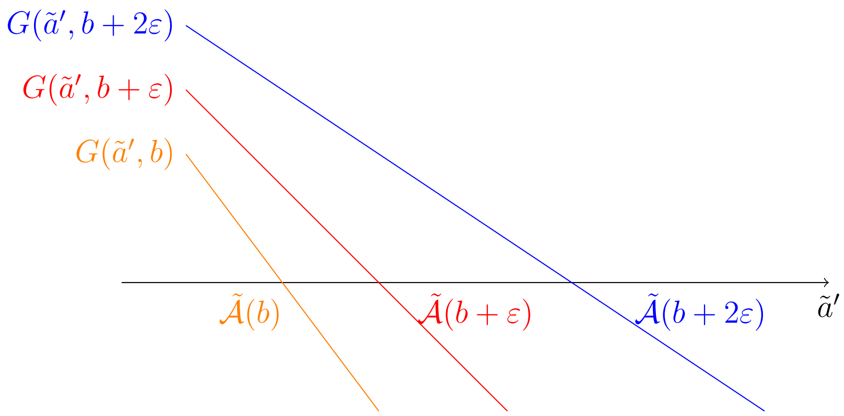
$$\sup_{\tilde{a}' \in [0, \tilde{y}+b]} \quad u(\tilde{y} + b - \tilde{a}') + \beta \int_{\tilde{y}' \in Y} V_o(\tilde{a}', \tilde{y}') f(\tilde{y}')\,\mathrm{d}\tilde{y}', \tag{23}$$

for all $b \in [0, \bar{b}]$. Let $E(\tilde{a}') =: \mathbb{E}_{\tilde{y}'} V_o(\tilde{a}', \tilde{y}')$, and $U_y(b, \tilde{y}, \tilde{a}') =: u(\tilde{y}+b-\tilde{a}') + E(\tilde{a}')$. Note that $\tilde{y}' \perp \tilde{a}'$. Hence $E(\tilde{a}')$ preserves the continuity of V_o in $\tilde{a}'$. Moreover, by Leibniz rule

$$\frac{\partial E(\tilde{a}')}{\partial \tilde{a}'} = \int_{\tilde{y}' \in Y} \frac{\partial V_o(\tilde{a}', \tilde{y}')}{\partial \tilde{a}'} f(\tilde{y}')\,\mathrm{d}\tilde{y}'.$$

Since $f(\tilde{y}') \geq 0$, $E(\tilde{a}')$ remains increasing in $\tilde{a}'$.The same argument holds for the concavity of $E(\tilde{a}')$, even though V_o is not twice differentiable.

Figure 8: Graphical representation of the young's FOC



Note: G are represented as linear functions to ease the interpretation. Our results do not change if we make these functions concave or convex.

Moreover, a direct implication of Assumption 2 is that $u(\tilde{y}+b-\tilde{a}')$ is continuous and concave for $\tilde{a}' \in [0, \tilde{y}+b)$, while the Inada condition ensures that $\tilde{a}' < \tilde{y}+b$ in equilibrium. Therefore, $U_y(b,\, \tilde{y},\, \tilde{a}')$ is continuous and concave in $\tilde{a}'$. This implies in particular that the first order condition is a sufficient condition to (23), and that the set of maximizer is single-valued. Then, by Berge's theorem, $\tilde{\mathcal{A}}(b,\, \tilde{y})$ is a continuous functions of b, where

$$\tilde{\mathcal{A}}(b,\, \tilde{y}) = \arg \sup_{\tilde{a}' \in [0,\, \tilde{y}+b]} \quad u(\tilde{y}+b-\tilde{a}') + \beta E(\tilde{a}'),$$

for all $b \in [0,\, \bar{b}]$, for all $\tilde{y} \in Y$.

From the Inada condition, $\lim_{c\to 0} u'(c) = +\infty$. Moreover, Lemma A.3 implies $\lim_{\tilde{a}'\to 0} E'(\tilde{a}') = +\infty$. Thus, $\mathcal{A}$ must lie in the interior of the choice set, ruling out any corner solution. The first order condition to (5b) then writes,

$$G(b,\, \tilde{y},\, \tilde{a}') =: \frac{\partial U_y}{\partial \tilde{a}'} = -u'(\tilde{c}) + \beta E'(\tilde{a}') = 0.$$

Combined with the concavity of U_y, this implies the downward shape of the G functions in Figure 8. Additionally, Lemma A.1 implies $G(\tilde{a}',\, b) < G(\tilde{a}',\, b+\varepsilon) < G(\tilde{a}',\, b+2\varepsilon)$, for all $\tilde{a}',\ \varepsilon > 0$. Note nevertheless that consumption is also nondecreasing in b. To see this, assume the opposite, and consider the first order condition

$$u'(y+b-\tilde{\mathcal{A}}[b]) = \beta E'\big(\tilde{\mathcal{A}}[b]\big).$$

Let b increase by ε. We showed that $\tilde{\mathcal{A}}(b)$ is nondecreasing in b, such that the first order condition has to hold at $b+\varepsilon$. Yet, under the assumption that $\tilde{c}$ is decreasing in b, its left-hand side goes up, while its right-hand side goes down by the concavity of E. This is a contradiction, and $\tilde{c}$ is nondecreasing in b as well.

To conclude the comparative statics, by Assumption 2, note that, for all $b,\, b'$, such that $b' > b$,

$$\frac{\partial^2 G}{\partial \tilde{a}' \partial b} = u^{(3)}(\tilde{c}) > 0 \quad \Leftrightarrow \quad \frac{\partial G(b',\, \tilde{a}')}{\partial \tilde{a}'} > \frac{\partial G(b,\, \tilde{a}')}{\partial \tilde{a}'}$$

which holds for all $\tilde{a}' \in [0,\, \tilde{y}+b]$. This implies

$$\begin{aligned} & \tilde{\mathcal{A}}(b+2\varepsilon) - \tilde{\mathcal{A}}(b+\varepsilon) > \tilde{\mathcal{A}}(b+\varepsilon) - \tilde{\mathcal{A}}(\varepsilon) \\ \Leftrightarrow \qquad & \frac{1}{2}\big(\tilde{\mathcal{A}}[b+2\varepsilon] + \tilde{\mathcal{A}}[\varepsilon]\big) > \tilde{\mathcal{A}}(b+\varepsilon), \end{aligned}$$

for $\varepsilon \neq 0$, such that $\tilde{\mathcal{A}}$ is midpoint-convex in b. But from Berge's theorem, we also know that $\tilde{\mathcal{A}}$ is continuous in b. Thus $\tilde{\mathcal{A}}$ is convex in b. Finally, by Alexandrov theorem, the twice differentiability (a.e.) of $\tilde{\mathcal{A}}$ in b is obtained from its convexity. □

Lemma A.5. *There exists a unique Markov strategy for the old agent, $\mathcal{B}(a,\, \tilde{y})$. $\mathcal{B}$ is continuous, twice differentiable (a.e.) and convex in a.*

Proof. The old household solves their problem, taking into account that the young agent will best respond to their transfer. Let $U_p(a,\, \tilde{y},\, b) =: u(Ra - b) + \alpha u(\tilde{y} + b - \tilde{\mathcal{A}}[\tilde{y},\, b])$. $U_p(a,\, \tilde{y},\, b)$ is continuous in b. First because the Inada condition on u ensures that $b < Ra$. Second because $\tilde{\mathcal{A}}$ is continuous, and $\tilde{\mathcal{A}} \in (0,\, \tilde{y}+b)$ from Lemma A.4. Since the sum of continuous functions is continuous, $U_p(b)$ is continuous in b.

Regarding its concavity, it is clear that its first element is concave in b. For the second, we have

$$\frac{\partial^2 u(\tilde{y} + b - \tilde{\mathcal{A}}(b))}{\partial b^2} = u^{(2)}(\tilde{c})\left(1 - \frac{\partial \tilde{\mathcal{A}}(b)}{\partial b}\right)^2 - u'(\tilde{c})\frac{\partial^2 \tilde{\mathcal{A}}(b)}{\partial b^2} < 0,$$

from Assumption 2 and the convexity of $\tilde{\mathcal{A}}$.

Therefore, here as well, we are maximizing a continuous and concave function over a compact set, such that the first order condition is sufficient, and the solution to the old program is unique. By Berge's theorem, $V_o(a,\, \tilde{y})$ and $\mathcal{B}(a,\, \tilde{y})$ are continuous function of b, where

$$\begin{aligned} V_o(a,\, \tilde{y}) &= \sup_{b\,\in[0,\, Ra]} \quad U_p(a,\, \tilde{y},\, b), \\ \mathcal{B}(b,\, \tilde{y}) &= \arg \sup_{b\,\in[0,\, Ra]} \quad U_p(a,\, \tilde{y},\, b). \end{aligned}$$

Along the same line as Lemma A.4, let

$$H(a,\, \tilde{y},\, b) =: \frac{\partial U_p}{\partial b} = -u'(c) + \alpha u'(\tilde{c})\left(1 - \frac{\partial \tilde{\mathcal{A}}}{\partial b}\right).$$

Note that $\lim_{b\to Ra} H(a,\, \tilde{y},\, b) = -\infty$, such that $\mathcal{B}$ lies in $[0,\, Ra)$. However, $|H(a,\, \tilde{y},\, 0)| < \infty$, such that a corner solution is possible whenever $H(a,\, \tilde{y},\, 0) < 0$, $\mathcal{B} = 0$.

Finally,

$$\frac{\partial^2 H}{\partial a \partial b} = Ru^{(3)}(c) > 0.$$

By the same argument as in Lemma A.4, $\mathcal{B}$ is thus convex in a. Finally, the convexity of $\mathcal{B}$ also implies its twice differentiable almost everywhere on any open set included within its domain, by Alexandrov theorem.[20] □

Corollary A.6. *The second derivative of V_o exists almost everywhere, with*

$$\frac{\partial^2 V_o}{\partial a^2} = Ru^{(2)}(c)\left(R - \frac{\partial \mathcal{B}}{\partial a}\right). \tag{24}$$

Proof. The Envelope theorem of Lemma A.3 and the differentiability of $\mathcal{B}$. □

A.2. Proof of Proposition 3.3

The first order condition of the parent suggests that there should exist a wage spread such that the old that received a low endowment when young will not want to transfer to their (rich) child in equilibrium. This implies that this economy will feature only four kinds of agents in equilibrium: the poor young, the rich young, the poor old and the rich old. To see this, consider first the case in which the first agent to ever exist in this economy receives a high endowment. In $t = 1$, it will save a given amount, and transfer a fraction of its wealth in $t = 2$. The child's wealth in $t = 2$ will then be equal to its (low) endowment plus the transfer. It will save a given amount, and not transfer anything to its kid in $t = 3$, given that the wage spread is sufficiently large. But then its child will have the same wealth as agent 1, thus save the same amount. If instead the first agent has a low endowment, it will then have a different wealth than any of the other "poor" individuals since no agent rich will be there before them. Providing that its initial endowment is lower than the transfer a poor young would received otherwise, this agent will not transfer intergenerationally. The second agent is then rich with no transfer, such that we are back to the previous case.

When the equilibrium features a countable degree of heterogeneity, a Markov perfect equilibrium is solved in the same manner as a Nash equilibrium. First, solve for the Markov strategies. Then, find the MPE by forcing all agents to play their best response. To get started with the first step, we need to assume that indeed the poor old will not want to transfer, $\mathcal{B}(a_P,\, \tilde{y}_R) = 0$.[21] Starting with the first order condition of the poor young,

$$u'(\tilde{y}_P + b_R - \tilde{a}'_P) = u'\left(R\tilde{a}'_P\right) \quad \Rightarrow \quad \tilde{\mathcal{A}}(\tilde{y}_p,\, b_R) = \frac{\tilde{y}_P + b_R}{1+R}, \tag{25}$$

[20] Our results on the differentiability of our policy functions is less nice than the one we could obtain by using the implicit function theorem. In particular, the differentiability is only almost everywhere, and the policy functions are not continuously differentiable. However, it is impossible to rely on the implicit function theorem here, as it would result in a non-ending cycle. Specifically, showing $\mathcal{B} \in \mathcal{C}^k$ requires $\tilde{\mathcal{A}} \in \mathcal{C}^{k+1}$. Yet, $\tilde{\mathcal{A}} \in \mathcal{C}^{k+1}$ requires $V_o \in \mathcal{C}^{k+2}$. Unfortunately, to prove that $V_o \in \mathcal{C}^{k+2}$ holds, we need $\mathcal{B} \in \mathcal{C}^{k+1}$.

[21] Without using this hint, we are back to a fixed point problem, which requires the guess-and-verify method.

where the solution uses $u(\bullet) = \log(\bullet)$ and $\mathcal{B}(a_P,\, \tilde{y}_R) = 0$. Turning to the first order condition of the rich parent, plugging (25) for $\tilde{\mathcal{A}}(\tilde{y}_P,\, b_R)$ and its partial derivative, we have

$$u'\left(Ra_R - b_R\right) \geq \frac{\alpha}{1+\beta} u'\left(\frac{\tilde{y}_P + b_R}{1+\beta}\right).$$

Taking into account the non-negativity constraint on b, the Markov strategy is then

$$\mathcal{B}(a_R,\, \tilde{y}_P) = \begin{cases} \dfrac{\alpha R a_R - \tilde{y}_P}{1+\alpha} & \text{if } \alpha R a_R > \tilde{y}_P \\ 0 & \text{o.w.} \end{cases} \tag{26}$$

Moving to the rich young's condition, and substituting (26) for $\mathcal{B}(a_R,\, \tilde{y}_P)$,

$$\begin{aligned} u'(\tilde{y}_R + b_P - \tilde{a}'_R) &= u'\left(R\tilde{a}'_R - \mathcal{B}\left[\tilde{a}'_R,\, \tilde{y}_P\right]\right) \\ \Leftrightarrow \qquad \tilde{y}_R + b_P - \tilde{a}'_R &= \tilde{a}'_R - \frac{\alpha R \tilde{a}'_R - \tilde{y}_P}{1+\alpha} \mathbf{1}\left\{\alpha R \tilde{a}'_R > \tilde{y}_P\right\}. \end{aligned}$$

Several remarks are of importance here. First, because we are solving for the Markov strategy, we let b_P be different from 0. Second, note that the indicator variable on the right hand side include the choice variable, a'_R. This forces us to consider the two cases separately.

Case 1, $\mathcal{B}(a_R,\, \tilde{y}_P) = 0$ Then,

$$\tilde{a}'_R = \frac{\tilde{y}_R + b_P}{1+R}.$$

For $\mathcal{B}(a_R,\, \tilde{y}_P) = 0$ to hold, this requires

$$\alpha R \tilde{a}'_R \leq \tilde{y}_P \qquad \Rightarrow \qquad \tilde{y}_R + b \leq \frac{\tilde{y}_P(1+R)}{\alpha R}.$$

Case 2, $\mathcal{B}(a_R,\, \tilde{y}_P) > 0$ Here,

$$\tilde{a}'_R = \frac{(1+\alpha)(\tilde{y}_R + b_P) - \tilde{y}_P}{1+\alpha+R}.$$

Once more, for $\mathcal{B}(a_R,\, \tilde{y}_P) > 0$ to hold,

$$\alpha R \tilde{a}'_R > \tilde{y}_P \qquad \Rightarrow \qquad \tilde{y}_R + b_P > \frac{(1+R)\tilde{y}_P}{\alpha R}.$$

However, recall Assumption 3.3 and the non-negativity constraint on b. Therefore, case 1 will never occur, and the Markov strategy is

$$\tilde{\mathcal{A}}(b_P,\, \tilde{y}_R,\, \tilde{y}_P) = \frac{(1+\alpha)(\tilde{y}_R + b_P) - \tilde{y}_P}{1+\alpha+R}. \tag{27}$$

It finally remains to check whether the Markov strategy of the poor parent is indeed $\mathcal{B}(a_P,\, \tilde{y}_R) = 0$, as initially assumed. For the non-negativity to bind, it has to be that the first order condition of the poor old holds as a strict inequality when $b_P = 0$, i.e.

$$u'\left(Ra_P\right) > \alpha u'\left(\tilde{y}_R - \tilde{\mathcal{A}}(0,\, \tilde{y}_R,\, \tilde{y}_P)\right)\left(1 - \left.\frac{\partial \tilde{\mathcal{A}}}{\partial b_P}\right|_{b_P=0}\right), \qquad \forall a_P.$$

Using the Markov strategy (27) for $\tilde{\mathcal{A}}$, this inequality rewrites

$$a_P < \frac{\tilde{y}_R R + \tilde{y}_P}{\alpha R^2}. \tag{C1}$$

This inequality will not hold for any arbitrary level of wealth. However, it will hold in equilibrium. To see this, we compute the Markov perfect equilibrium, in which all the players are "best responding" to each other. Hence the savings of the young rich is given by (27), but substituting for $\mathcal{B}(a_P,\, \tilde{y}_R) = 0$,

$$\tilde{\mathcal{A}}_R^\star = \frac{(1+\alpha)\tilde{y}_R - \tilde{y}_P}{1+\alpha+R}, \tag{28}$$

with $\tilde{\mathcal{A}}^\star > 0$ from A.3.3. Plugging the optimal rich savings into (26) yields

$$\mathcal{B}_R^\star = \frac{\alpha R\tilde{y}_R - (1+R)\tilde{y}_P}{1+\alpha+R}, \tag{29}$$

since $\alpha R\tilde{\mathcal{A}}_R^\star > \tilde{y}_P$ holds by A.3.3. The savings of the poor young is finally given by (25), but substituting (29) for b_R,

$$\tilde{\mathcal{A}}_P^\star = \frac{\alpha(\tilde{y}_P + R\tilde{y}_R)}{(1+R)(1+\alpha+R)}. \tag{30}$$

Coming back to (C1), two cases are possible. Either the first agent ever born has a low endowment. Then $a_P = \tilde{y}_P/(1+R)$, from (25) and $b = 0$, which satisfies (C1). Or the first newborn of this economy has a high endowment, in which case (30) describes the savings of the second (poor) newborn. But then the inequality becomes

$$\alpha R < \sqrt{(1+R)(1+\alpha+R)},$$

which hold as well. Hence, in equilibrium, we indeed have $b_P^\star = 0$.

As in Section 3.2, this is the unique Markov perfect equilibrium as it is the unique equilibrium given our guess, $\mathcal{B}(a_P,\, \tilde{y}_R) = 0$, and Proposition 3.1 showed that this game admitted a unique Markov strategy.

A.3. Proof of Propositions 4.1 and 4.3

To start with, consider the problem of the age 2 young,

$$V_2(\tilde{a},\, a',\, b) = \sup_{\tilde{a}'\geq 0} u(R\tilde{a} + \tilde{y} - b - \tilde{a}') + \beta\mathbb{E}V_3\left(\tilde{a}' + a'\right),$$

where we removed the income from the state space to ease the notation. Assumption 4.1 implies the twice differentiability (a.e.) of the value functions, such that the first order condition to this problem writes

$$-u'(\tilde{c}_2) + \beta \mathbb{E} \frac{\partial V_3}{\partial \hat{a}} + \lambda_{\tilde{a}'_2} = 0, \tag{31}$$

where $\hat{a} =: \tilde{a}' + a'$. Additionally, the envelope conditions are

$$\frac{\partial V_2}{\partial \tilde{a}'} = Ru'(\tilde{c}_2^\star), \quad \frac{\partial V_2}{\partial b} = u'(\tilde{c}_2^\star), \quad \text{and} \quad \frac{\partial V_2}{\partial a'} = \beta \mathbb{E} \frac{\partial V_3}{\partial \hat{a}}. \tag{32}$$

Moving to age 4 old, whose problem is

$$V_4(a,\, \tilde{a}) = \sup_{a',\, b \geq 0} u(Ra + y - b - a') + \alpha u(R\tilde{a} + \tilde{y} + b - \tilde{a}'^\star) + \alpha\beta \mathbb{E}\, V_3(\hat{a}^\star),$$

with $\tilde{a}'^\star = \tilde{\mathcal{A}}_2\,(\tilde{a},\, a',\, b)$, and $\hat{a}^\star = a' + \tilde{a}'^\star$, the savings and transfer first order conditions are respectively

$$-u'(c_4) + \alpha\beta \mathbb{E} \frac{\partial V_3}{\partial \hat{a}} + \alpha \frac{\partial \tilde{\mathcal{A}}_2}{\partial a'} \left(\beta \mathbb{E} \frac{\partial V_3}{\partial \hat{a}} - u'(\tilde{c}_2^\star) \right) + \lambda_{a'_4} = 0,$$

$$-u'(c_4) + \alpha u'(\tilde{c}_2^\star) + \alpha \frac{\partial \tilde{\mathcal{A}}_2}{\partial b} \left(\beta \mathbb{E} \frac{\partial V_3}{\partial \hat{a}} - u'(\tilde{c}_2^\star) \right) + \lambda_{b_4} = 0.$$

Regarding the terms inside the parentheses, two scenarios are possible. Either the child's savings are positive. Then $\lambda_{\tilde{a}'_2} = 0$, and the two parentheses evaluate at 0 from (31). Or the young is constrained on the credit market. But then $\partial \tilde{\mathcal{A}}_2 / \partial a' = \partial \tilde{\mathcal{A}}_2 / \partial b = 0$. Therefore, in both cases, the two first order conditions simplify to

$$u'(c_4) = \alpha\beta \mathbb{E} \frac{\partial V_3}{\partial \hat{a}} + \lambda_{a'_4} \qquad \text{and} \qquad u'(c_4) = \alpha u'(\tilde{c}_2^\star) + \lambda_{b_4}. \tag{33}$$

A similar reasoning yields the envelope conditions of the problem,

$$\frac{\partial V_4}{\partial a} = Ru'(c_4^\star) \qquad \text{and} \qquad \frac{\partial V_4}{\partial \tilde{a}} = \alpha R u'(\tilde{c}_2^\star). \tag{34}$$

Turning to the problem of youngest old, age 3,

$$V_3(a) = \sup_{a',\, b \geq 0} u(Ra + y - b - a') + \alpha u(\tilde{y} + b - \tilde{a}'^\star) + \beta \mathbb{E} V_4(a',\, \tilde{a}'^\star),$$

with $\tilde{a}'^\star = \tilde{\mathcal{A}}_1(a',\, b)$, the savings and transfer first order conditions are

$$-u'(c_3) - \alpha u'(\tilde{c}_1^\star) \frac{\partial \tilde{\mathcal{A}}_1}{\partial a'} + \beta \mathbb{E} \left(\frac{\partial V_4}{\partial a'} + \frac{\partial V_4}{\partial \tilde{a}'} \frac{\partial \tilde{\mathcal{A}}_1}{\partial a'} \right) + \lambda_{a'_3} = 0,$$

$$-u'(c_3) + \alpha u'(\tilde{c}_1^\star) \left(1 - \frac{\partial \tilde{\mathcal{A}}_1}{\partial b} \right) + \beta \frac{\partial \tilde{\mathcal{A}}_1}{\partial b} \mathbb{E} \frac{\partial V_4}{\partial \tilde{a}'} + \lambda_{b_3} = 0.$$

Plugging (34) in the relevant places yields the age 3 first order conditions. Additionally, the envelope condition gives

$$\frac{\partial V_3}{\partial a} = Ru'(c_3^\star). \tag{35}$$

Substituting (35) for the relevant partial derivatives in (31) and (33) generates the first order conditions of the age 2 and age 4 individuals.

It remains only to compute the first order condition of the youngest agent, whose problem is

$$V_1(a',\, b) = \sup_{a' \geq 0} u(\tilde{y} + b - \tilde{a}') + \beta \mathbb{E} V_2(\tilde{a}',\, \mathcal{A}_4\left[a',\, \tilde{a}'\right],\, \mathcal{B}_4\left[a',\, \tilde{a}'\right]).$$

The first order conditions writes

$$-u'(\tilde{c}_1) + \beta \mathbb{E}\left(\frac{\partial V_2}{\partial \tilde{a}'} + \frac{\partial V_2}{\partial a''}\frac{\partial \mathcal{A}_4}{\partial \tilde{a}'} + \frac{\partial V_2}{\partial b'}\frac{\partial \mathcal{B}_4}{\partial \tilde{a}'}\right) + \lambda_{\tilde{a}'_1} = 0.$$

Using (32), (35) and Assumption 4.3, such that $\mathcal{A}_4(a,\, \tilde{a}) = 0$, one obtains (17).

B. Numerical methods

B.1. *Two-period model*

Policy function iteration To implement policy function iteration, we iterate on the first order condition of the young and the old given in (6) and (7) respectively. We use a non-linear solver to find their respective root. Specifically,

1. Build two grids, $\boldsymbol{A} =: \{0, \ldots, a_n\}$ and $\boldsymbol{B} =: \{0, \ldots, a_m\}$. Guess $\mathcal{B}_0(a,\, y)$.
2. Given $\mathcal{B}_0(a,\, y)$, solve for $\tilde{\mathcal{A}}(b_k,\, y_j)$ according to (6), for each $b_k \in \boldsymbol{B}$, each $y_j \in Y$.
3. Given $\tilde{\mathcal{A}}(b,\, y)$, solve for $\mathcal{B}_1(a_i,\, y_j)$ in (7), for each $a_i \in \boldsymbol{A}$, each $y_j \in Y$.
4. Given $\tilde{\mathcal{A}}(b,\, y)$ and $\mathcal{B}_1(a,\, y)$, check for convergence. If $\max|\, \mathcal{B}_1 - \mathcal{B}_0\,| < \varepsilon$, for ε small, then the iteration has converged. Otherwise, set $\mathcal{B}_0 = \mathcal{B}_1$, and go back to 2.

Value function iteration Value function iteration is based on the Bellman equation (5). To find the numerical solutions, follow

1. Build two grids, $\boldsymbol{A} =: \{0, \ldots, a_n\}$ and $\boldsymbol{B} =: \{0, \ldots, a_m\}$. Guess $V_1^0(a,\, y)$.
2. Given $V_1^0(a,\, y)$, solve for $\tilde{\mathcal{A}}(b_k,\, y_j)$ according to (5b), for each $b_k \in \boldsymbol{B}$, each $y_j \in Y$.
3. Given $\tilde{\mathcal{A}}(b,\, y)$, solve for $\mathcal{B}(a_i,\, y_j)$ based on (5a), for each $a_i \in \boldsymbol{A}$, each $y_j \in Y$. Once $\mathcal{B}(a_i,\, y_j)$ is found, compute the updated value function by plugging $\mathcal{B}(a_i,\, y_j)$ into (5a). This yields $V_1^1(a,\, y)$.
4. Check for convergence. If $\max|\, V_1^1(a,\, y) - V_1^0(a,\, y)\,| < \varepsilon$, for ε small, then the iteration has converged. Otherwise, set $V_1^0 = V_1^1$, and go back to step 2.

B.2. Four-period model

Policy function iteration To solve for the Markov strategies in the general model, we used policy function iteration on (14:19). Thanks to Assumption 4.2, we do not have to solve for the bequest function, $\mathcal{A}_4$, and remove it from the state space of the young 2. Then, we follow

1. Build two grids, $\boldsymbol{A} =: \{0, \ldots, a_n\}$ and $\boldsymbol{B} =: \{0, \ldots, b_m\}$. Guess the savings function of the age 2 agent, $\tilde{\mathcal{A}}_2^0(\tilde{a},\, b,\, \tilde{y},\, y)$, as well as the savings and transfer function of the old 3, $\mathcal{A}_3(a,\, y,\, \tilde{y})$ and $\mathcal{B}_3(a,\, y,\, \tilde{y})$.

2. Given $\tilde{\mathcal{A}}_2^0$, $\mathcal{A}_3$ and $\mathcal{B}_3$, for each $a_i \in \boldsymbol{A}$, each $\tilde{a}_i \in \boldsymbol{A}$, each $y_j \in Y$, each $\tilde{y}_j \in Y$, solves for $\mathcal{A}_4(a_i,\, \tilde{a}_i,\, y_j,\, \tilde{y}_j)$ and $\mathcal{B}_4(a_i,\, \tilde{a}_i,\, y_j,\, \tilde{y}_j)$, solving (15) and (16).

3. Given $\tilde{\mathcal{A}}_2^0$, $\mathcal{A}_3$, $\mathcal{B}_3$ and $\mathcal{B}_4$, solves for $\tilde{\mathcal{A}}_1(a_i',\, b_k,\, \tilde{y}_j,\, y_j)$ using (17), for each $a_i' \in \boldsymbol{A}$, each $b_k \in \boldsymbol{B}$, each $y_j \in Y$, each $\tilde{y}_j \in Y$.

4. Given $\mathcal{A}_2^0$, $\tilde{\mathcal{A}}_1$ and $\mathcal{B}_4$, for each $a_i \in \boldsymbol{A}$, each $y_j \in Y$, each $\tilde{y}_j \in Y$, solves for $\mathcal{A}_3(a_i,\, y_j,\, \tilde{y}_j)$ and $\mathcal{B}_3(a_i,\, y_j,\, \tilde{y}_j)$ in (18) and (19).

5. Given $\mathcal{A}_3$ and $\mathcal{B}_3$, solves for $\tilde{\mathcal{A}}_2^1(\tilde{a}_i,\, b_k, \tilde{y}_j,\, y_j)$ in (14), for each $\tilde{a}_i, \in \boldsymbol{A}$, each $b_k \in \boldsymbol{B}$, each $y_j \in Y$, each $\tilde{y}_j \in Y$.

6. If $\max|\tilde{\mathcal{A}}_2^1 - \tilde{\mathcal{A}}_2^0| < \varepsilon$, for ε small, stops. Otherwise, update $\mathcal{A}_2^0 = \mathcal{A}_2^1$, and go back to step 2.

Numerical challenges Whenever required, the policy functions are interpolated along their endogenous states. To solve for the root of the first order conditions, the trust region method is used. The Jacobians of the functions are computed via automatic differentiation. The shape of the first order conditions, and in particular the presence of the policy functions derivatives combined with occasionally binding constraints, complicates the root finding task. Even if the Markov strategies are continuous, their gradient will indeed not be if the constraints ever bind. At this point, the partial derivatives will feature a downward or upward jump, propagating the discontinuity to the first order conditions. This is a problem, as finding numerically the roots of discontinuous functions is computationally costly.

Numerically, the Markov strategies are computed on a discrete grid, and then interpolated using splines. The choice of the splines is eventually what determines the smoothness of the first order conditions. As an example, Figure 12 plots the first order condition of the young 1 for a given state. In the left panel, linear splines are used to interpolate the Markov strategies. By definition, a linear interpolant has a piecewise constant gradient. This magnifies the discontinuities. In this case, a fast algorithm, e.g. Newton or trust-region, does not find the solution. Instead, in the right panel, the same first order condition is plotted, but using cubic splines. Here, the partial derivatives are oversmoothed. If cubic splines allow to solve this particular problem, they also bring greater approximation, which then complicates subsequent computations. As a result, quadratic splines are used. If extrapolation is needed, we assume a linear behavior of the Markov strategy outside the grid to prevent explosive behaviors of the interpolants.

Another numerical challenges emerge with the presence of multiple solutions to the old's problem, as explained in Section 4. As an example, Figure 10 and 11 plot the contour lines of the two first order conditions, (18) and (19), of the old 3, for two different states. In the first figure, a unique solution exists, while in the second one, a infinity of solutions satisfies the optimality conditions. To bypass this issue, Assumption 4.3 selects arbitrarily as solution to this problem the allocation of the parent's wealth that lies in the middle of the solution set – green dot in Figure 11. To find it numerically, we find the extreme solutions to (18) and (19) – black dots in Figure 11. These form the upper and lower bound of the solution set. We then pick the pair in the middle of set, and verify that the two first order conditions evaluate to zero.

If this assumption allows to circumvent the multiplicity issue, it however increases the approximation errors, as the root-finding algorithm sometimes miscompute the two extreme roots. This forces us to restrict the tolerance level to $1e^{-3}$ to obtain a convergence of the policy function iteration.

Stationary distributions To find the stationary distribution, we iterate on the Markov strategies until the distributions of agents remain unchanged over time. Specifically,

1. Draw N households, with N large, such that $N/2$ are (1,3) families and the other half is made of (2,4) families. Compute a first set of moments for the two distributions, $\boldsymbol{M}_0$.

2. Draw the old's and the young's incomes, and let both agents play according to the Markov strategies.

3. Compute the moments of the new distributions, $\boldsymbol{M}_1$.

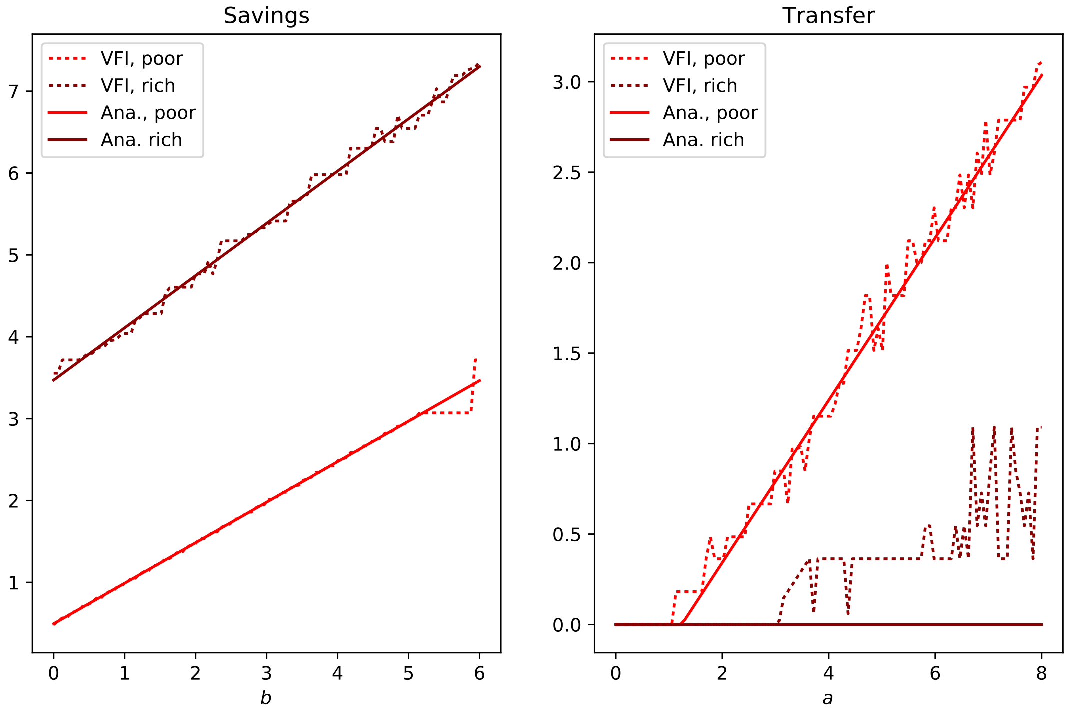
4. Check for convergence. If $\max|\boldsymbol{M}_0 - \boldsymbol{M}_1| < \varepsilon$, for ε small, stop. Otherwise, go back to step 2.

We use $N = 100\,000$, $\varepsilon = 2^{-3}$, and the mean, the standard deviation, the skewness, the kurtosis, and four percentiles (20%, 40%, 60%, 80%) as moments for the two distributions.

Calibration For the Markov strategies and the stationary distribution studied in Section 4.1 and 4.2, we set $R = 1.01$, $\beta = 0.98$, $\alpha = 0.9$. The utility is a CRRA utility function, with a degree of relative risk aversion set to three. To speed the computation, we assume that income follows a two-state Markov chain. To mimic the life cycle profile of income, we let the mean and the standard deviation of the Markov chain evolves. Specifically, the mean is hump-shaped and the standard deviation is U-shaped over the life cycle (Gourinchas and Parker, 2002; Feigenbaum and Li, 2012). We assume that next period's income is only dependent on the current income. That is, the income of the family members are independent, except for the income of the youngest individual, age 1, for whom its endowment depend on the income of its parent, age 3. Finally, the asset and transfer grids are respectively defined from zero to four times and twice the highest income, with a grid size of twenty.

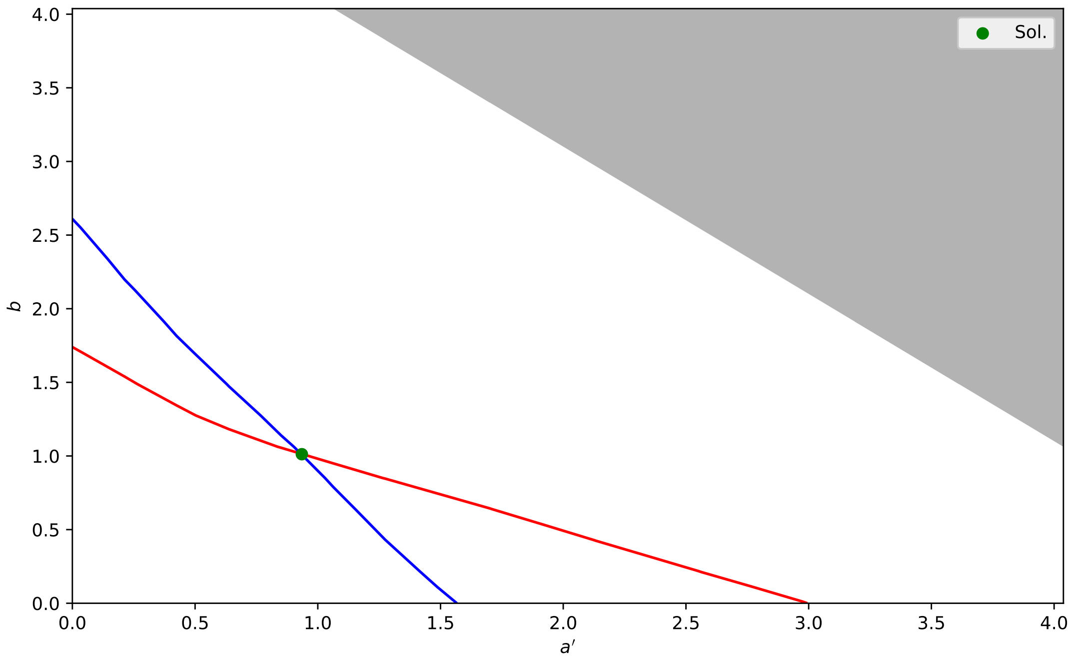
C. Figures

Figure 9: Value function iteration vs. analytical solutions



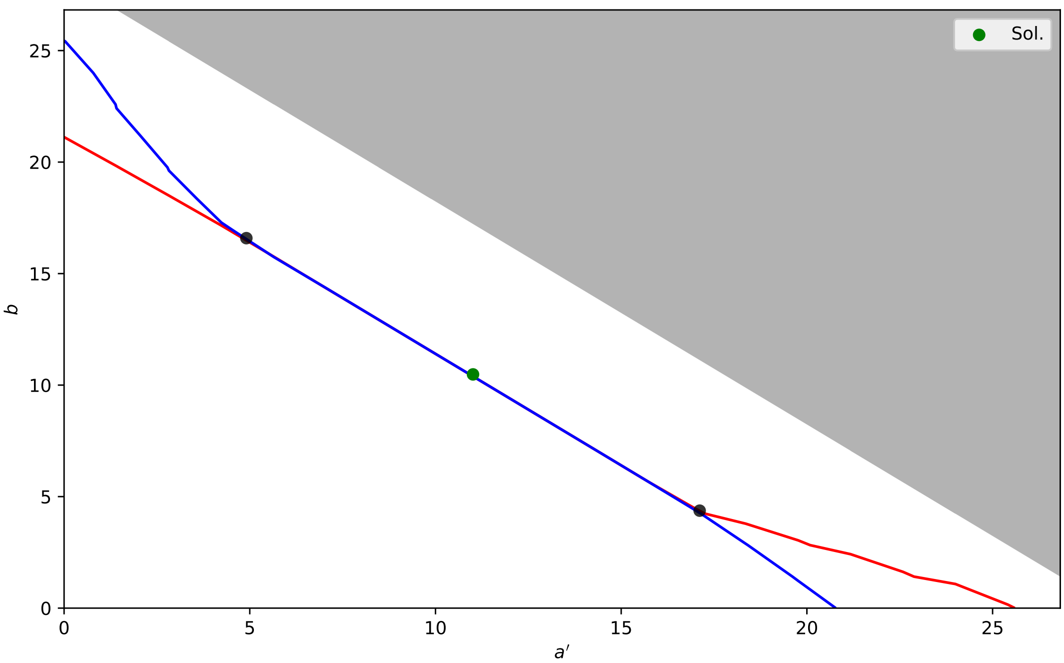
Calibration: $R = 1.01$, $\beta = 0.98$, $\alpha = 0.8$. The support of the income distribution is $\{1, 6\}$. The asset and transfer grids are discretized over 100 points. Value function iteration has converged with a tolerance level set to 10^{-8}.

Figure 10: Age 3 first order condition, unique solution



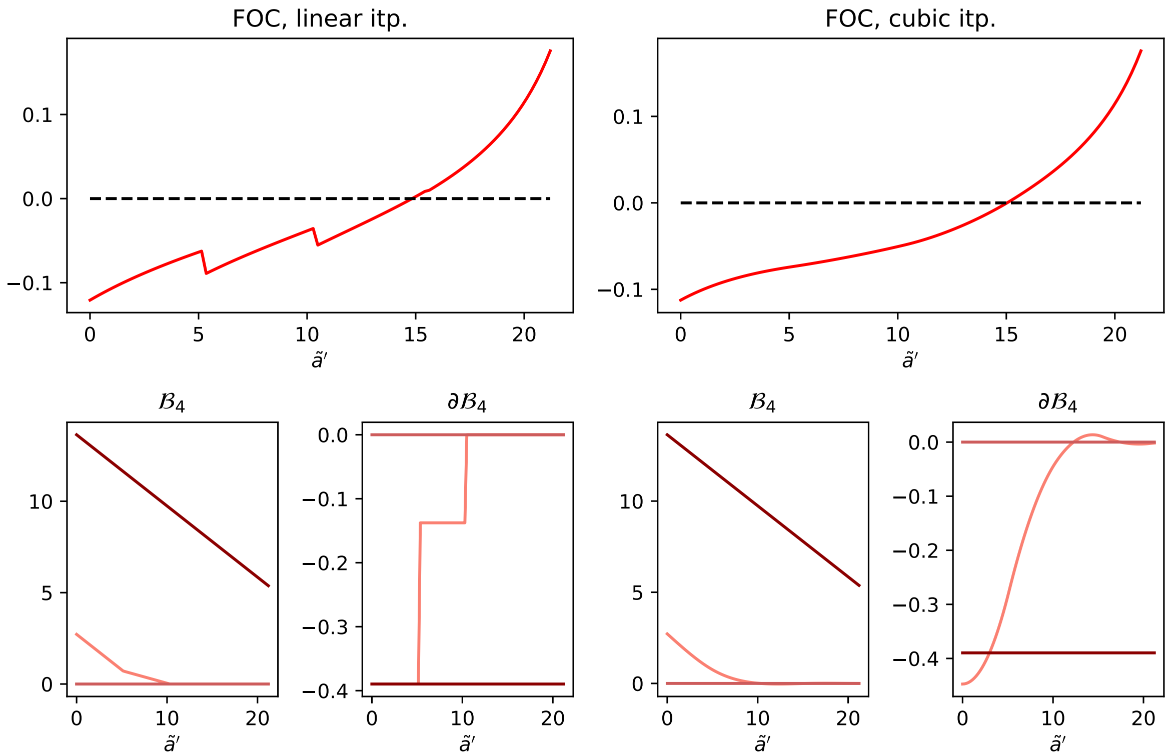
Note: the blue line is the zero-contour line of the savings first order condition, (18), while the red line is with respect to the transfer, (19). The white area represents the feasibility set as defined by the budget and the non-negativity constraints.

Figure 11: Age 3 first order condition, multiple solutions



Note: the blue line is the zero-contour line of the savings first order condition, (18), while the red line is with respect to the transfer, (19). The white area represents the feasibility set as defined by the budget and the non-negativity constraints.

Figure 12: Interpolation and first order condition



Note: the top panels represent the first order condition of the young 1, when its state is $\boldsymbol{s}_1 = \{46.4, 18, 2, 9.9\}$. On the left, linear splines are used to interpolate the parent's Markov strategy. On the right, cubic splines. In period 1, only the current wage of the young and the old are known. Yet, the partial derivatives, $\partial\mathcal{B}_4/\partial a'$, is defined in terms of future wages. Hence the three curves in the bottom panels. Each of them represents a potential future wage combination, $(\tilde{y}', y')$.